Nonlinear Perturbation Theory for the Large Scale Structure

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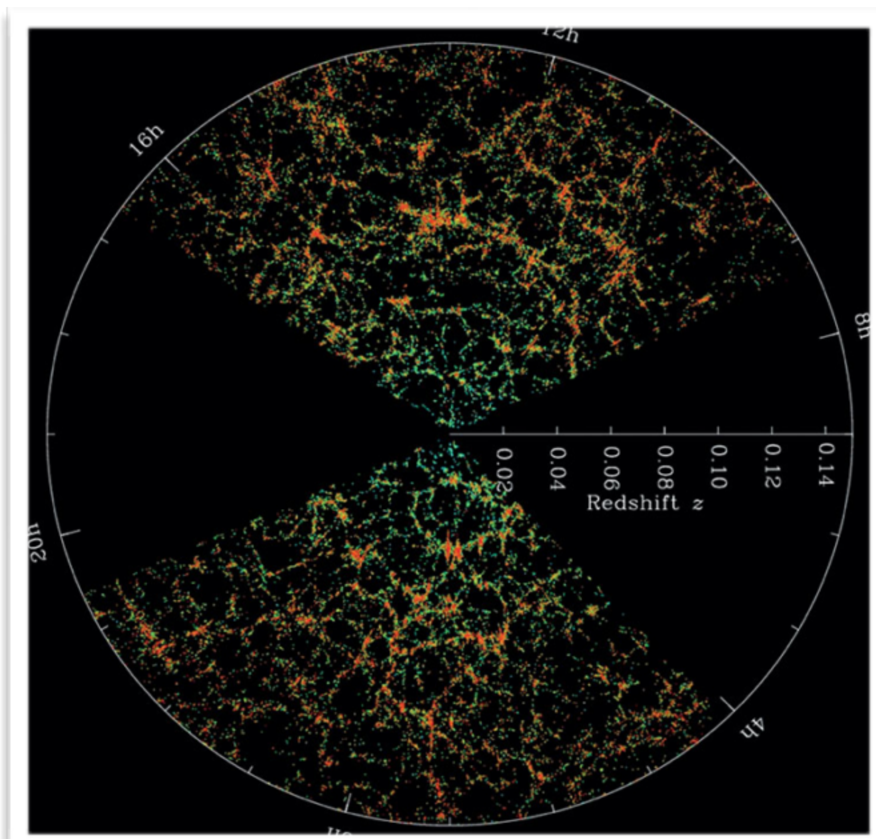
Lecture 1

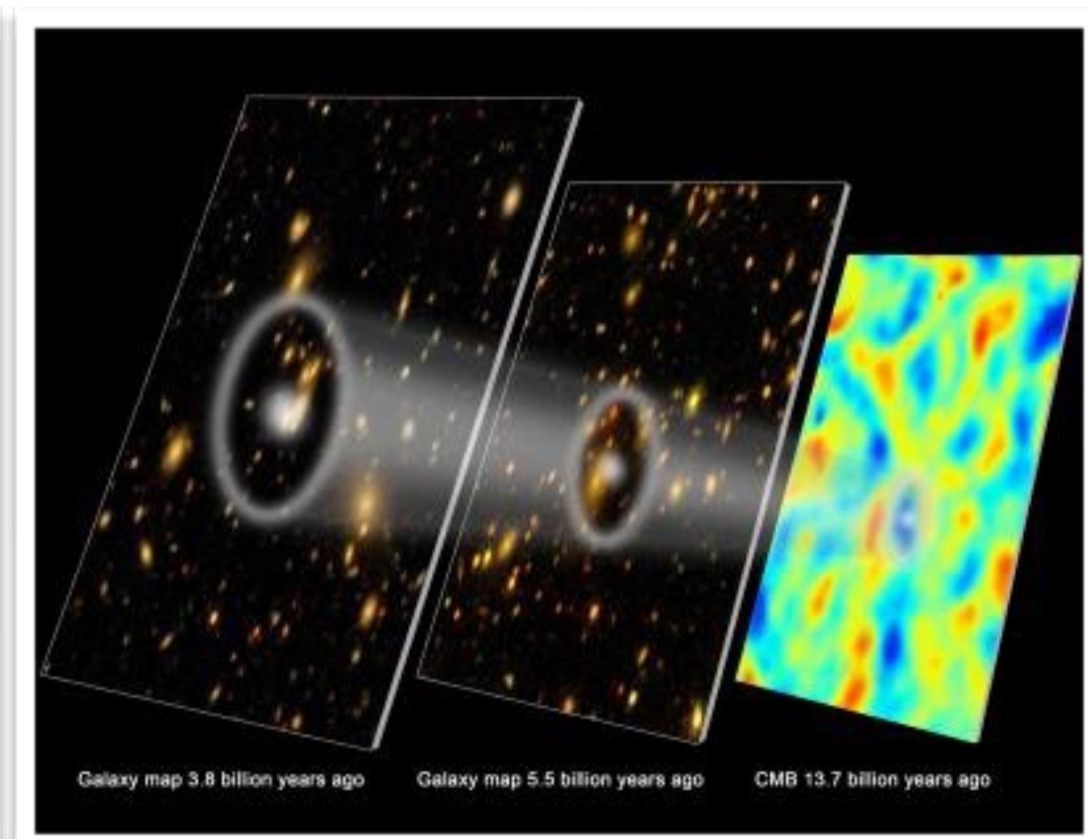
Motivation

Extract cosmological information from measurements of the Large Scale Structure (LSS)

- **Galaxy Clustering**
- **Weak Lensing**
- **21 cm**
- **Ly alpha**
- **...**

BOSS, DES, DESI, EUCLID, LSST, ...





PHYSICS GOAL

Probe density/velocity perturbations in matter, galaxy, … any other tracer at scales ~ CMB, but at late timesn in a matter/DE dominated universe

GOALS:

- **Constrain ΛCDM parameters;**
- **Test ΛCDM assumptions;**
- **Measure neutrino masses, primordial non gaussianity,…;**
- **Test gravity!**
- **Discover new physics beyond ΛCDM!!**

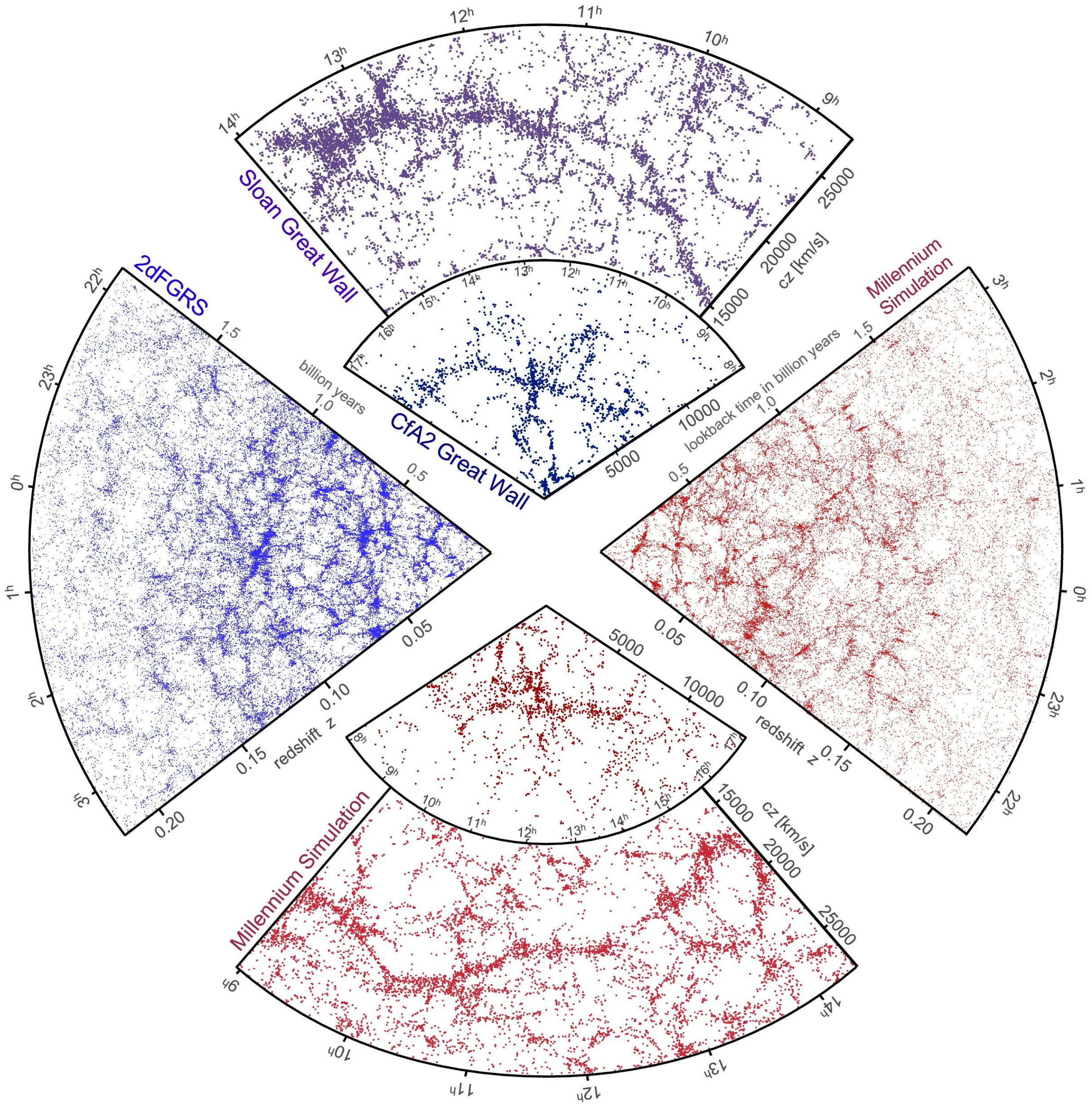
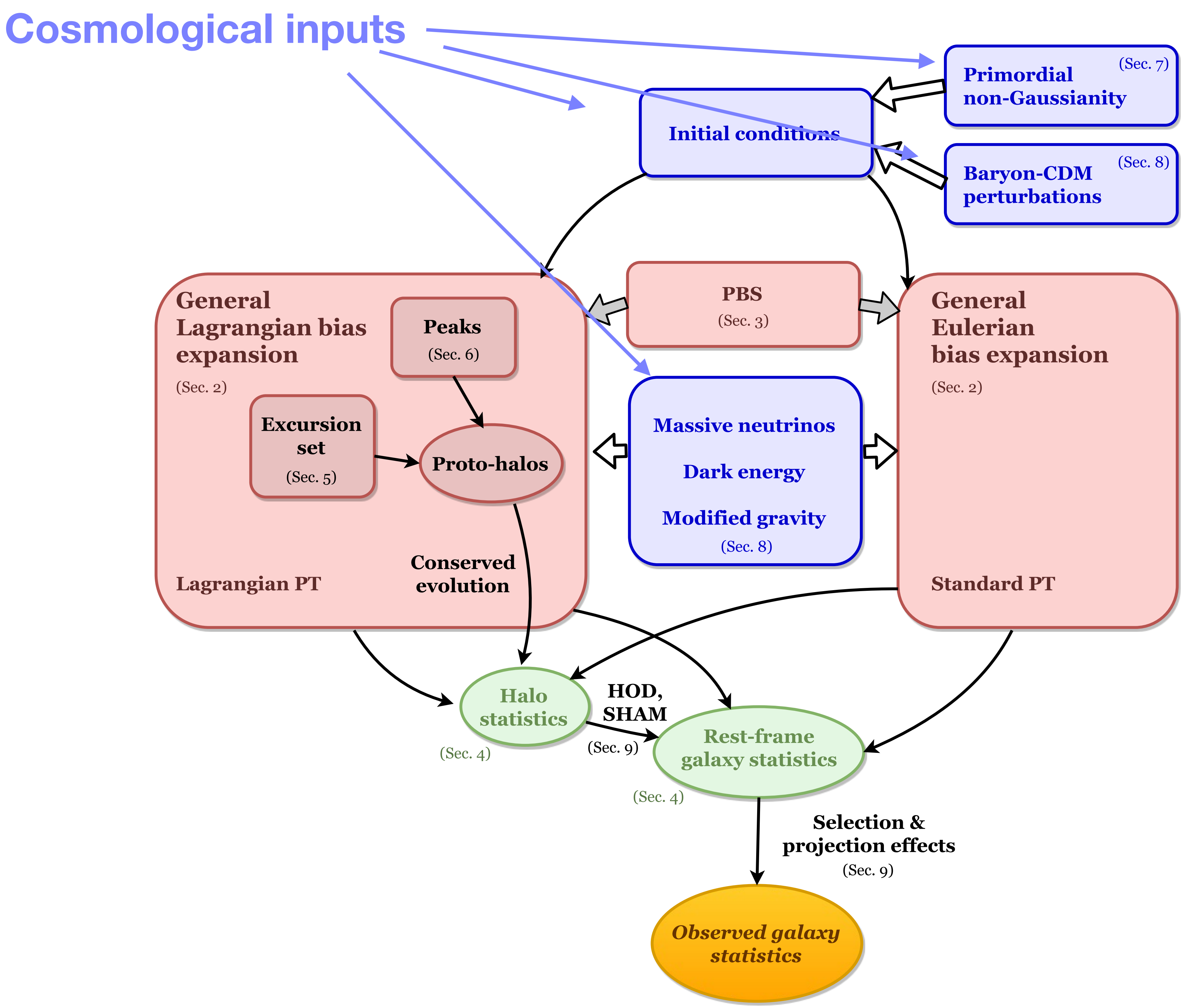


Figure 1: Two-dimensional slice projections (pie diagram) of the measured locations of galaxies in the CfA2, 2dF, and SDSS galaxy redshift surveys (top half). The bottom half shows the location of galaxies which were assigned to dark matter halos in the *Millennium* gravity-only N-body simulation using a semi-analytical prescription. It is apparent that the simulation, which assumes a flat ΛCDM cosmology, qualitatively reproduces the observed large-scale structure of the Universe very well. From [31].



Desjacques et al 1611.09787

Why going (semi)analytical?

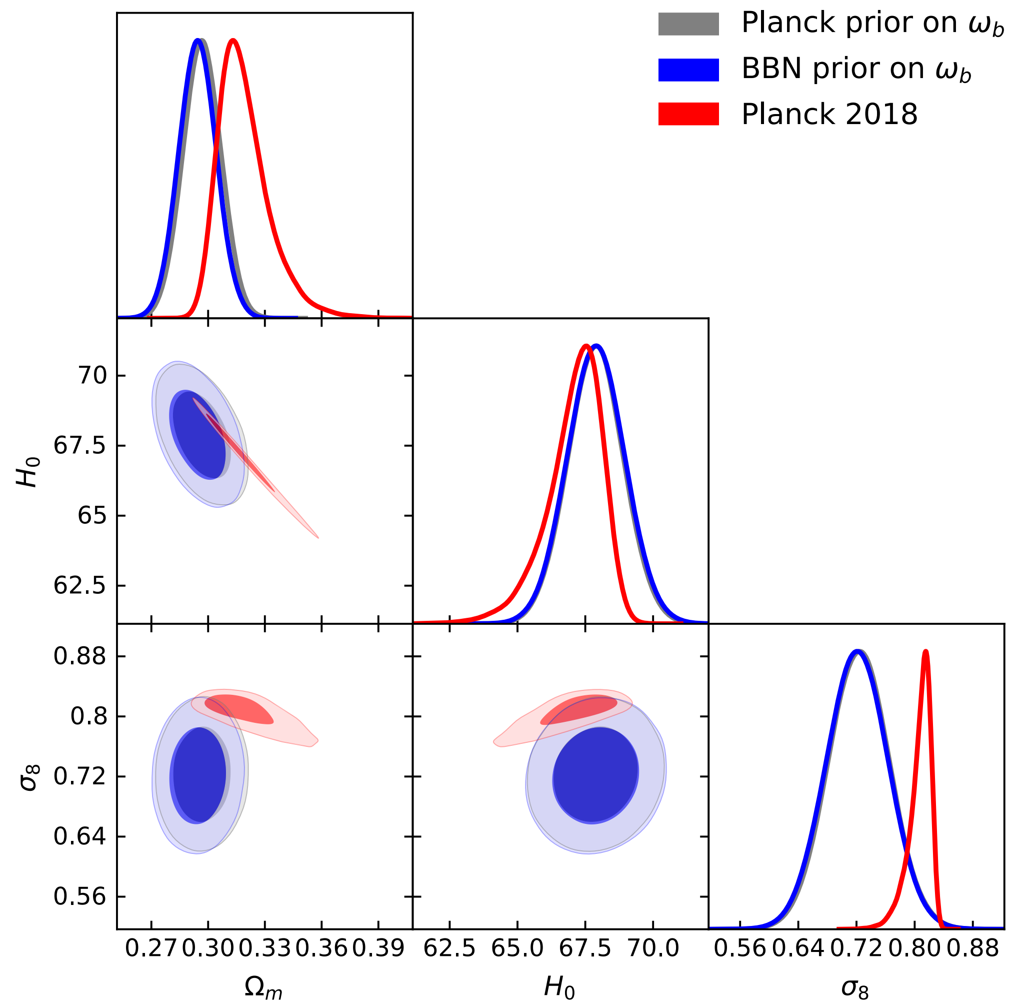
What we need:

a theoretical modelling for the data analysis pipeline which is:

- **accurate;**
- **fast;**
- **flexible**

Models scan:

- **forget about N-body alone;**
- **emulators;**
- **semi-analytical.**



Simonovic et al. 1909.05277

Why going (semi)analytical?

<u>Physical insight on the evolution of the LSS:</u>

- **bias;**
- **redshift space distortions;**
- **reconstruction;**
- **consistency relations;**
- **shell-crossing, virialization …**

Outline

- brief review of statistical field theory
- the setup: Eulerian vs Lagrangian, equations of motion
- structure formation in the LineLand (1+1 dimensions)
- Standard Perturbation Theory
- performance and problems of SPT
- IR effects: resummations and BAO’s
- UV behavior: Effective approaches
- From matter to biased tracers
- Redshift space distortions
- Putting all together (state of the art)
- Beyond PT: consistency relations
- Beyond PT: shell-crossing
- [Beyond CDM: Axions and ALP’s]
- [Beyond LCDM: neutrinos, PNG, non-standard growth and mode-coupling]

Statistical field theory for the LSS

Initial conditions on perturbations provided by
quantum fluctuations during inflation: **statistical description of *random fields***

Cosmological Principle: ***Statistical homogeneity and isotropy*** of random fields on spatial slices at fixed proper time

Fair Sample Hypothesis: "distant" patches of the Universe ~ independent realizations of the same statistical process. ***Ergodic Hypothesis.***
In ΛCDM this is valid above ~ O(100) Mpc/h [—> trade ensemble averages for spatial averages above these scales]

Random scalar field

(density, velocity divergence, gravitational potential,...)

scale factor	conformal time	comoving coordinates

$$ds^2 = a^2(\tau)\left[-(1+2\Phi)d\tau^2 + (1-2\Psi)\delta_{ij}dx^i dx^j\right]$$

$d\tau = 0$ 3D spatial slices

random variables: $\varphi(\mathbf{x})$

+ n-point probability distribution functions: $\mathcal{P}_n\left(\varphi(\mathbf{x}_1),\ \varphi(\mathbf{x}_2),\cdots,\varphi(\mathbf{x}_n)\right)$

Ensemble averages

n-point correlation functions:

$$\langle\varphi(\mathbf{x}_1)\cdots\varphi(\mathbf{x}_n)\rangle = \int d\varphi(\mathbf{x}_1)\cdots d\varphi(\mathbf{x}_n)\,\mathcal{P}_n\left(\varphi(\mathbf{x}_1),\cdots,\varphi(\mathbf{x}_n)\right)\ \varphi(\mathbf{x}_1)\cdots\varphi(\mathbf{x}_n)$$

2-point function

mean: $\langle \varphi(\mathbf{x}) \rangle = \bar{\varphi}$ (statistical homogeneity)

two point function: $\langle \varphi(\mathbf{x}_1)\varphi(\mathbf{x}_2) \rangle = \bar{\varphi}^2 \left[1 + \xi^{(2)}(\mathbf{x}_1, \mathbf{x}_2) \right]$

2-point correlation function: deviation from a uncorrelated random distribution

homogeneity isotropy

$$\xi^{(2)}(\mathbf{x}_1, \mathbf{x}_2) = \xi^{(2)}(\mathbf{x}_1 - \mathbf{x}_2) = \xi^{(2)}(|\mathbf{x}_1 - \mathbf{x}_2|) = \langle \delta(\mathbf{x}_1)\delta(\mathbf{x}_2) \rangle$$

$$[\varphi(\mathbf{x}) \equiv \bar{\varphi}\,(1 + \delta(\mathbf{x}))]$$

n-point functions

$$\langle\delta(\mathbf{x}_1)\rangle = 0$$

$$\langle\delta(\boldsymbol{x}_1)\delta(\boldsymbol{x}_2)\rangle = \xi^{(2)}(\boldsymbol{x}_1, \boldsymbol{x}_2)$$

$$\langle\delta(\boldsymbol{x}_1)\delta(\boldsymbol{x}_2)\delta(\boldsymbol{x}_3)\rangle = \xi^{(3)}(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3)$$

$$\langle\delta(\boldsymbol{x}_1)\delta(\boldsymbol{x}_2)\delta(\boldsymbol{x}_3)\delta(\boldsymbol{x}_4)\rangle = \xi^{(2)}(\boldsymbol{x}_1, \boldsymbol{x}_2)\xi^{(2)}(\boldsymbol{x}_3, \boldsymbol{x}_4) + \text{perm.} + \xi^{(4)}(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3, \boldsymbol{x}_4)$$

connected correlators: $\langle\delta(\boldsymbol{x}_1)\delta(\boldsymbol{x}_2)\cdots\delta(\boldsymbol{x}_n)\rangle_c \equiv \xi^{(n)}(\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_n)$

Power Spectrum

$$\delta(\mathbf{k}) = \int d^3x \, e^{i\mathbf{k}\cdot\mathbf{x}} \, \delta(\mathbf{x})$$

$$\delta(\mathbf{x}) = \int \frac{d^3k}{(2\pi)^3} \, e^{-i\mathbf{k}\cdot\mathbf{x}} \, \delta(\mathbf{k}) \equiv \int_{\mathbf{k}} e^{-i\mathbf{k}\cdot\mathbf{x}} \, \delta(\mathbf{k})$$

$$\langle \delta(\mathbf{k})\delta(\mathbf{k}') \rangle = (2\pi)^3 \delta_D(\mathbf{k} + \mathbf{k}') P(k)$$

homogeneity → $\delta_D(\mathbf{k} + \mathbf{k}')$; isotropy → $P(k)$

$$\xi^{(2)}(r) = \int_{\mathbf{k}} e^{-i\mathbf{k}\cdot\mathbf{r}} \, P(k) = \frac{1}{2\pi^2} \int_0^\infty dk \, k^2 P(k) j_0(kr)$$

$$\left(j_0(x) = \frac{\sin(x)}{x} \right)$$

$$\sigma_\delta^2 = \langle \delta(\mathbf{x})^2 \rangle = \xi^{(2)}(0) = \int_0^\infty \frac{dk}{k} \Delta^2(k)$$

variance

$$\Delta^2(k) \equiv \frac{k^3 P(k)}{2\pi^2}$$

dimensionless Power Spectrum

Higher order correlators

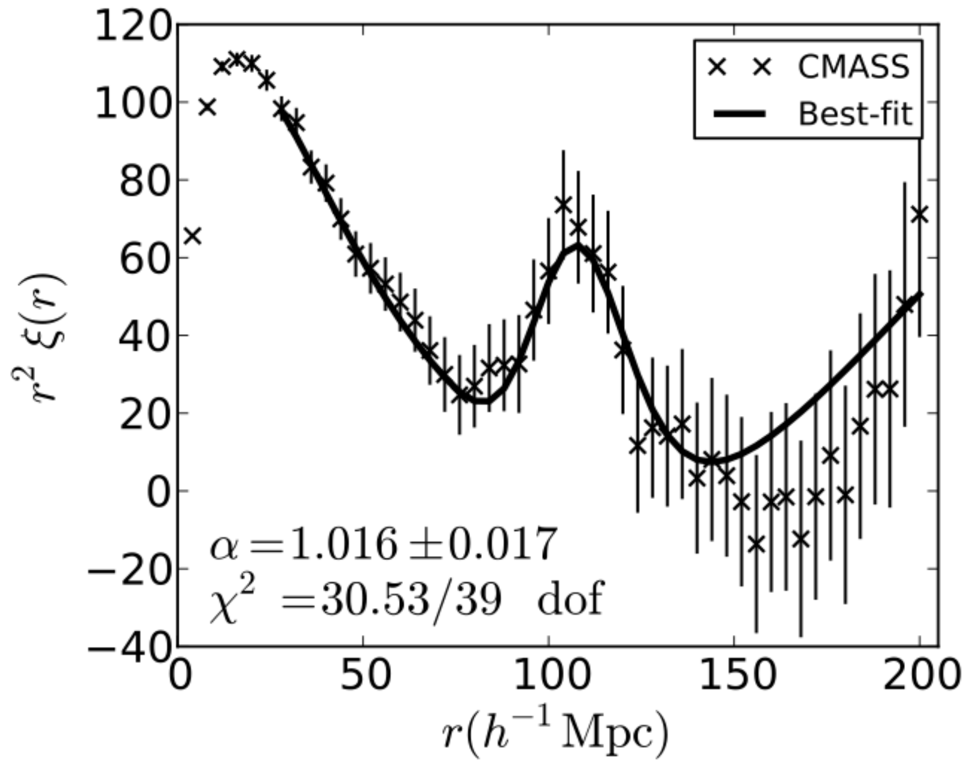
$$\langle \delta(\mathbf{k}_1) \cdots \delta(\mathbf{k}_n) \rangle' \equiv \frac{\langle \delta(\mathbf{k}_1) \cdots \delta(\mathbf{k}_n) \rangle}{(2\pi)^3 \delta_D(\mathbf{k}_1 + \cdots + \mathbf{k}_n)}$$

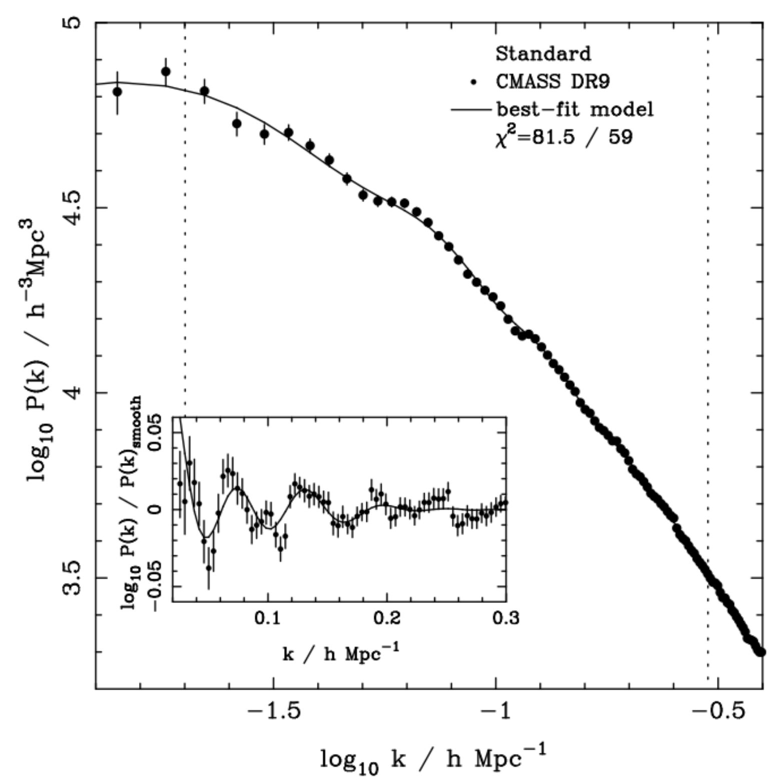
$$\langle \delta(\mathbf{k}_1) \delta(\mathbf{k}_2) \delta(\mathbf{k}_3) \rangle' \equiv B(k_1, k_2, k_3)$$ bispectrum

$$\langle \delta(\mathbf{k}_1) \delta(\mathbf{k}_2) \delta(\mathbf{k}_3) \delta(\mathbf{k}_4) \rangle'_c \equiv T(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4)$$ trispectrum

...

Configuration or Fourier space?





more intuitive (local in real space)

~insensitive to sky cuts

mix of linear and non linear scales

correlated errors

each (linear) scale evolves independently

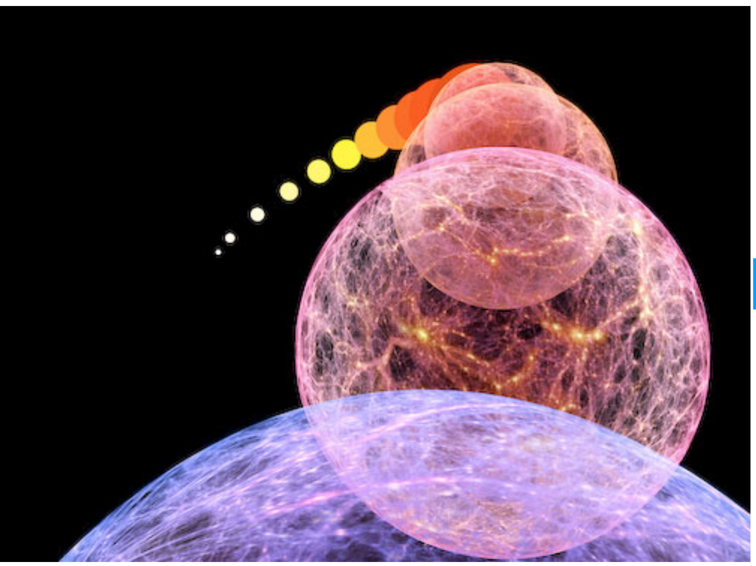
uncorrelated errors (for linear scales!)

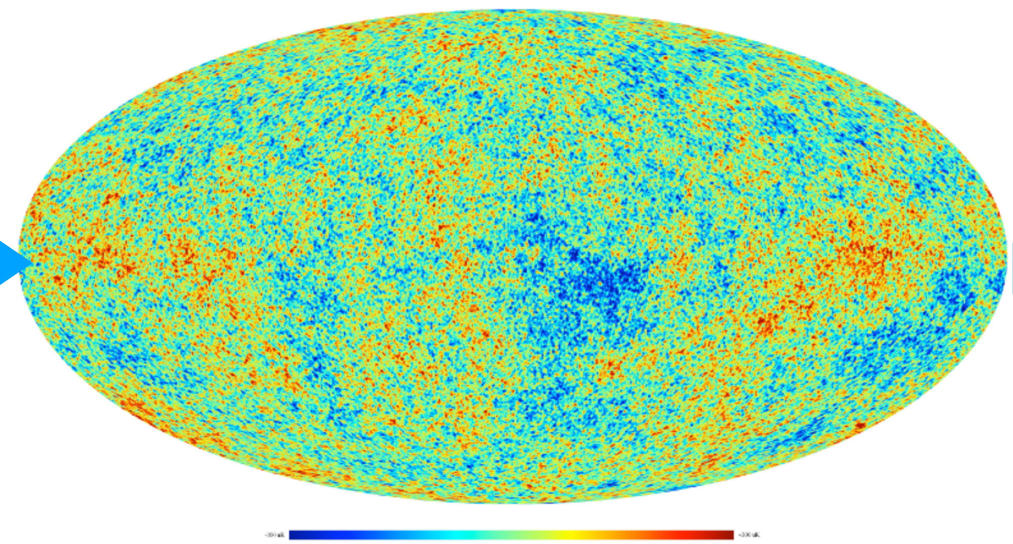
~very sensitive to sky cuts

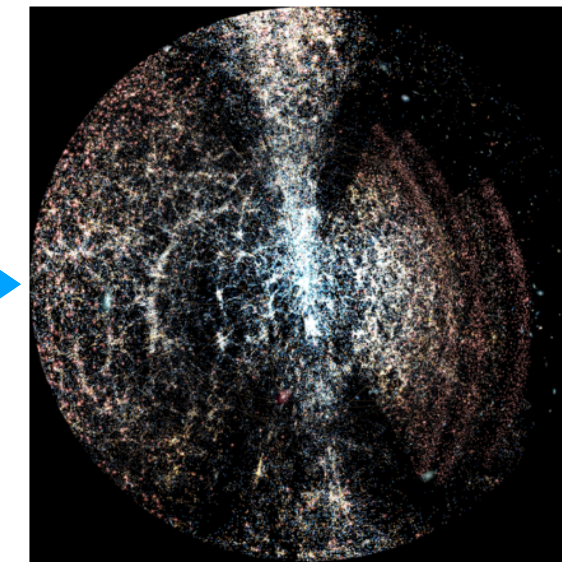
non-local in real space

Evolution of the Power Spectrum

Inflation | **Decoupling** | **Today**



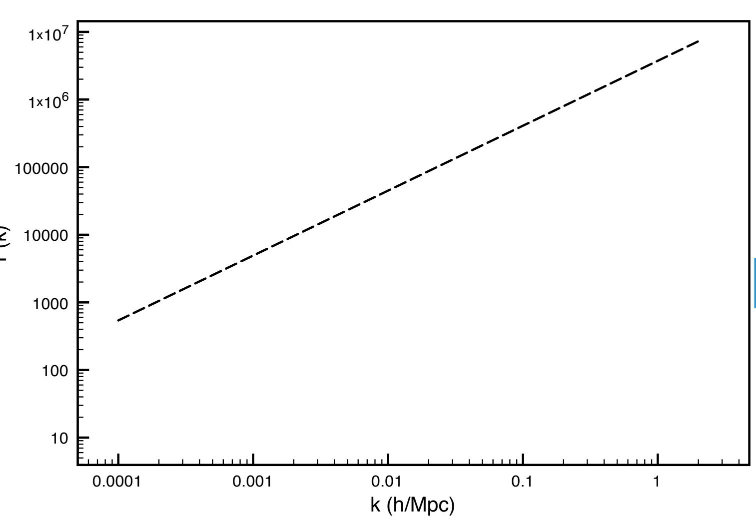




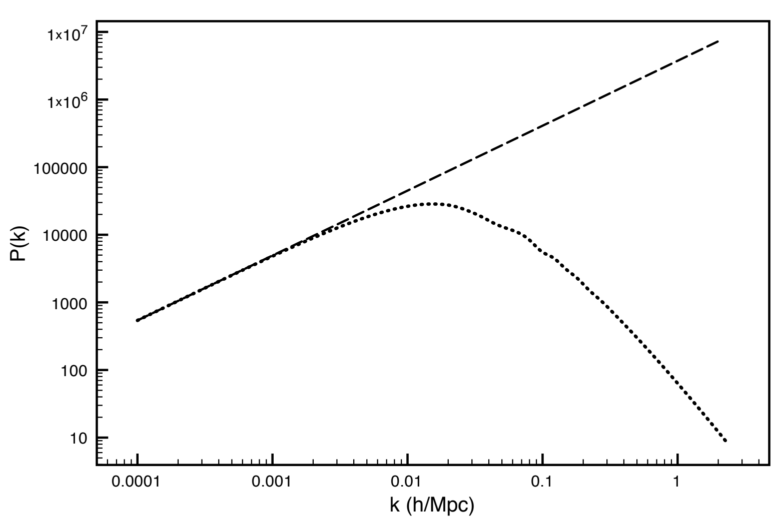
Linear, Gaussian | **Linear, Gaussian** | **non-Linear, non-Gaussian**

z >> 10^{10} | **z=O(1000)** | **z=O(1)**

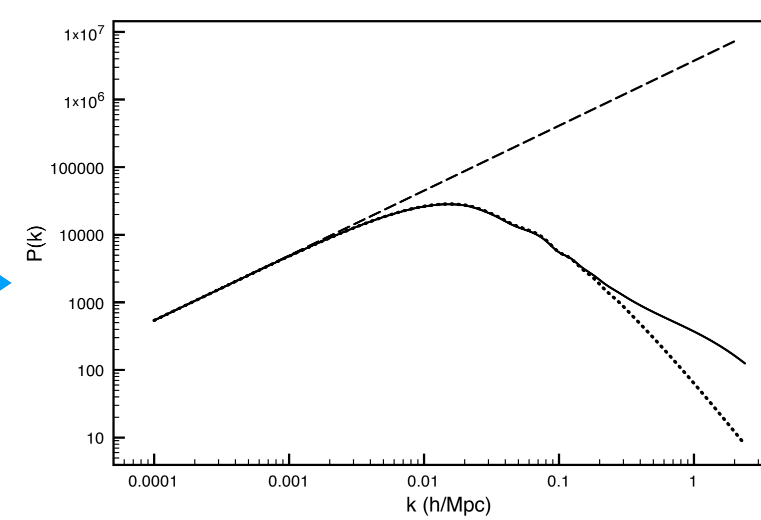
primordial density perturbations



fluid of photons-baryons-DM + …

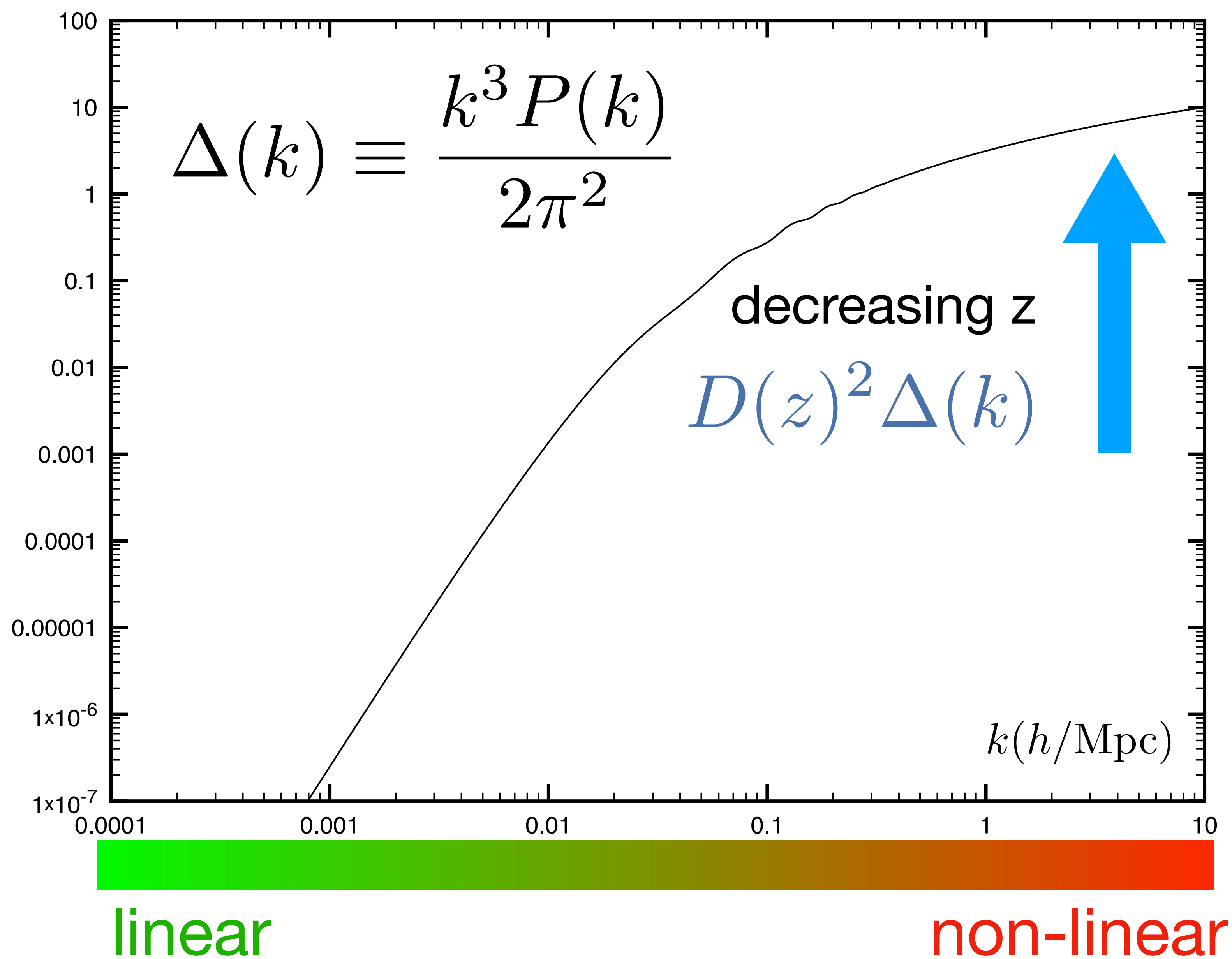


non-relativistic matter



Linear and non-linear scales

linear Power Spectrum in ΛCDM



Steps

- **Non-linear evolution of Cold Dark Matter (non-relativistic, pressureless);**
- **IR corrections (resummation);**
- **UV corrections (CGPT, EFToLSS);**
- **From CDM to *tracers* (halos, galaxies, …);**
- **From real to redshift space;**

Equations of motion for Cold Dark Matter

$$\mathcal{H} \equiv \frac{\dot{a}}{a} = \frac{8\pi}{3} G \rho_{tot}$$

$$\nabla_x^2 \phi(\mathbf{x}, \tau) = \frac{3}{2} \mathcal{H}^2 \Omega_m \, \delta(\mathbf{x}, \tau)$$

comoving density

$$\rho_m(\mathbf{x}, \tau) = \bar{\rho} \, (1 + \delta(\mathbf{x}, \tau))$$

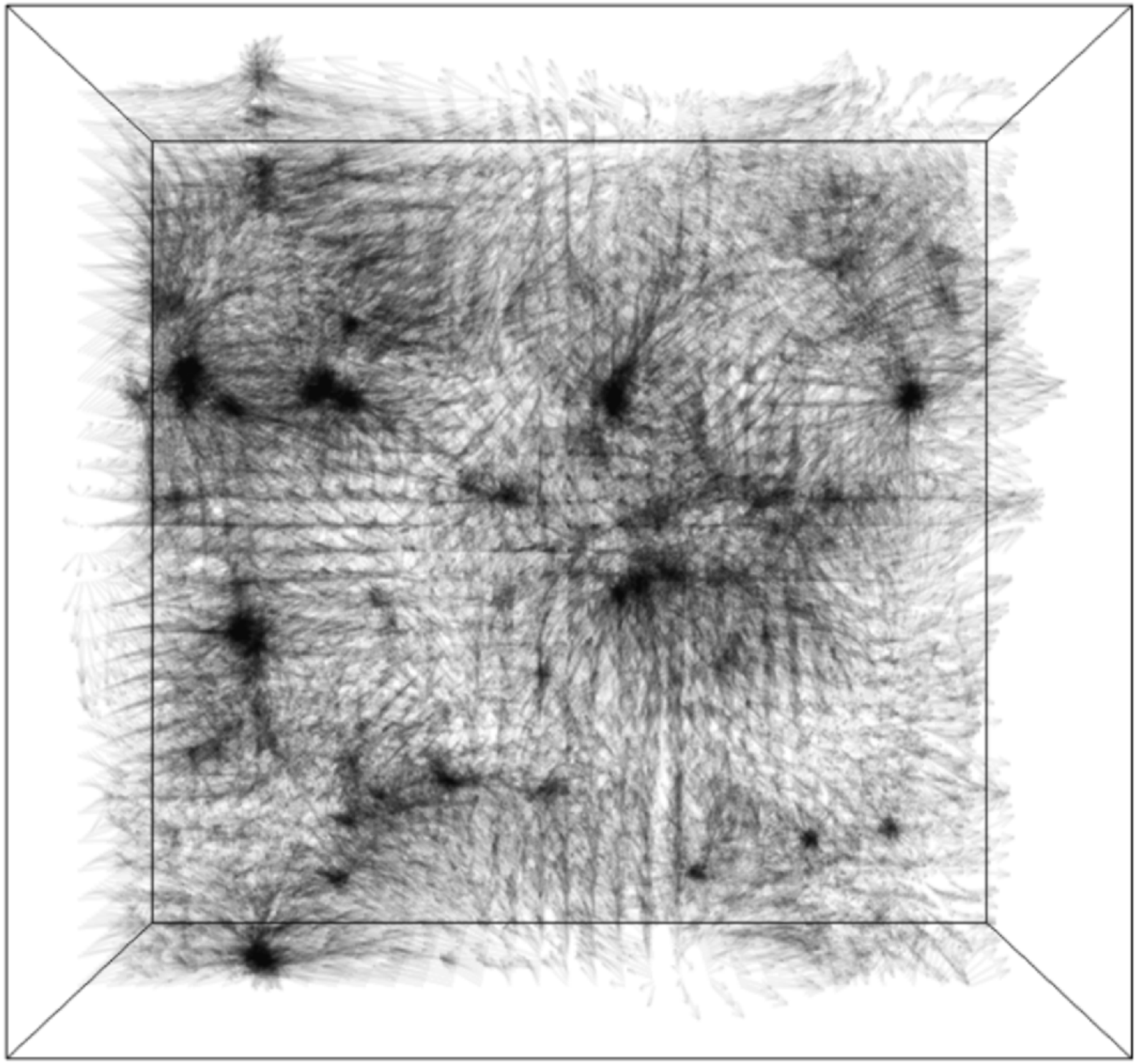
non-relativistic particles interacting only through gravity

$$\mathbf{p} = am \, \dot{\mathbf{x}}$$

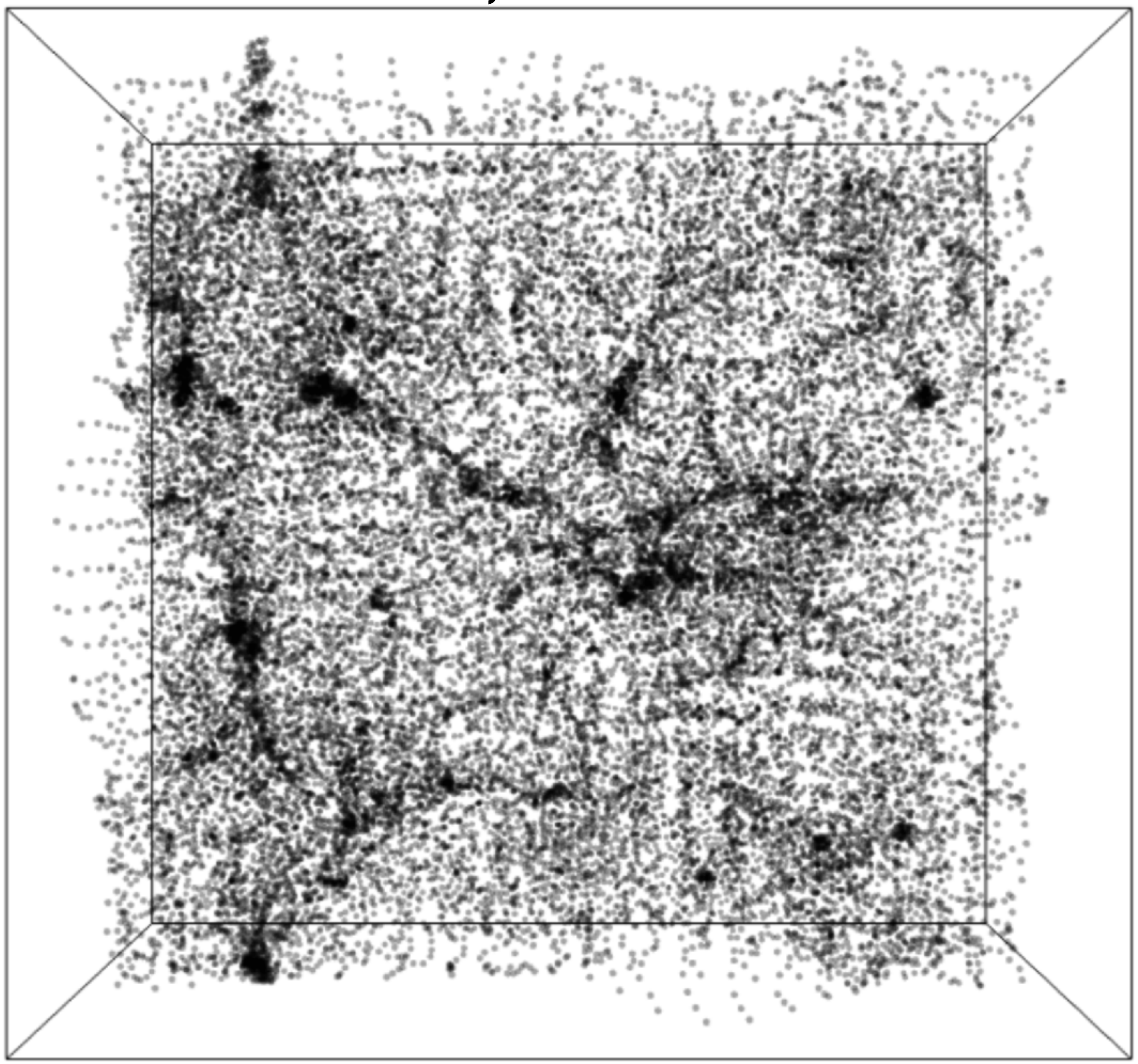
$$\dot{\mathbf{p}} = -am \, \nabla_x \phi$$

Lagrangian vs Eulerian

He et al, 1811.06533



displacement field



density field

Lagrangian viewpoint

lagrangian position $\mathbf{q} = \mathbf{x}(\tau_{in})$ initial position of a given particle (or fluid element)

eulerian position $\mathbf{x} = \mathbf{x}(\tau)$ final position

$$\mathbf{x} = \mathbf{q} + \psi(\mathbf{q}, \tau) \longleftarrow$$ displacement field

$$\mathbf{p} = am\,\dot{\mathbf{x}}$$
$$\dot{\mathbf{p}} = -am\,\nabla_x \phi$$

equation for the displacement + Poisson

$$\ddot{\psi}(\mathbf{q}, \tau) + \mathcal{H}\dot{\psi}(\mathbf{q}, \tau) = -\,\nabla_x \phi(\mathbf{x}, \tau)\big|_{\mathbf{x}=\mathbf{q}+\psi(\mathbf{q},\tau)}$$

displacement is "lagrangian" force is "eulerian"

$$1 + \delta(\mathbf{x}, \tau) = \int d^3q\, \delta_D(q + \psi(\mathbf{q}, \tau) - \mathbf{x}) \qquad (\delta(\mathbf{q}, \tau_{in}) \to 0)$$

in general, *nonlinear* (in ψ) and *nonlocal* (in lagrangian space)

Eulerian viewpoint

$f(\mathbf{x}, \mathbf{p}, \tau)$ distribution function in phase space

Vlasov equation + Poisson

$$\frac{d}{d\tau} f(\mathbf{x}, \mathbf{p}, \tau) = \left(\frac{\partial}{\partial \tau} + \frac{p^i}{am} \frac{\partial}{\partial x^i} - am \frac{\partial \phi(\mathbf{x}, \tau)}{\partial x^i} \frac{\partial}{\partial p^i} \right) f(\mathbf{x}, \mathbf{p}, \tau) = 0$$

$$1 + \delta(\mathbf{x}, \tau) = \frac{1}{\bar{\rho}} \int d^3p \, f(\mathbf{x}, \mathbf{p}, \tau)$$

nonlinear, local, but in phase space (6 dim): take moments (see later)

Lagrange-Euler equivalence

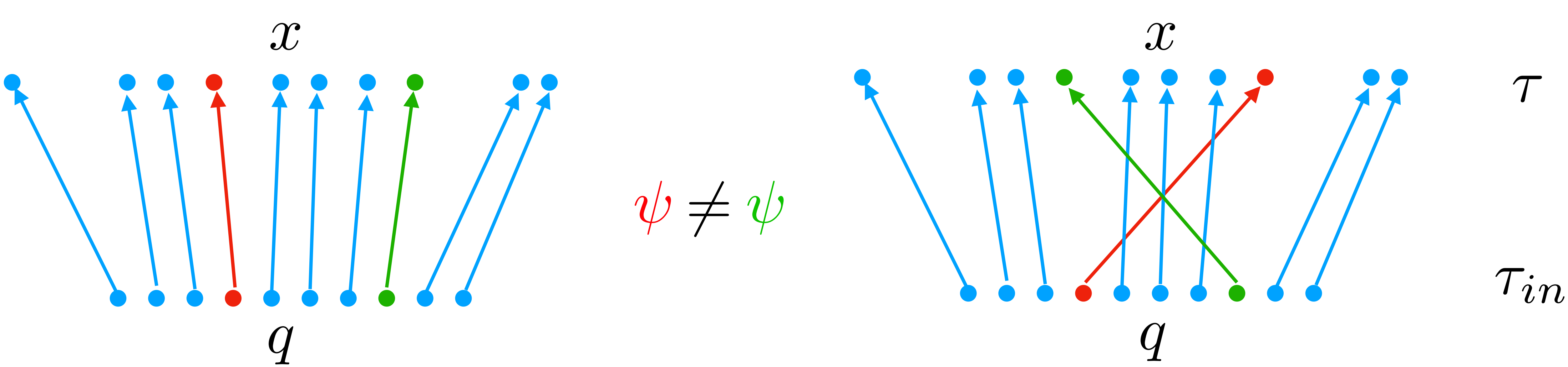
The distribution function

$$f(\mathbf{x},\mathbf{p},\tau) = \bar{\rho} \int d^3q\, \delta_D\left(\mathbf{q} + \psi(\mathbf{q},\tau) - \mathbf{x}\right)\, \delta_D\left(\mathbf{p} - am\dot{\psi}(\mathbf{q},\tau)\right)$$

solves the Vlasov equation if ψ solves the Lagrangian equation

notice: $\psi(\mathbf{q},\tau), \quad \dot{\psi}(\mathbf{q},\tau) \quad \Longrightarrow \quad f(\mathbf{x},\mathbf{p},\tau)$

but the opposite is in general, not possible



Same final distribution obtained from different displacements

Symmetry: Extended galilean invariance (a.k.a. the equivalence principle)

$$\ddot{\psi}(\mathbf{q},\tau) + \mathcal{H}\dot{\psi}(\mathbf{q},\tau) = -\left.\nabla_x \phi(\mathbf{x},\tau)\right|_{\mathbf{x}=\mathbf{q}+\psi(\mathbf{q},\tau)}$$

is invariant under the combined transformation

$$\psi(\mathbf{q},\tau) \to \psi(\mathbf{q},\tau) + \mathbf{d}(\tau) \qquad \nabla_x\phi(\mathbf{x},\tau) \to \nabla_x\phi(\mathbf{x},\tau) - \ddot{\mathbf{d}}(\tau) - \mathcal{H}\,\dot{\mathbf{d}}(\tau)$$

with $\mathbf{d}(\tau)$ uniform, but time dependent translation

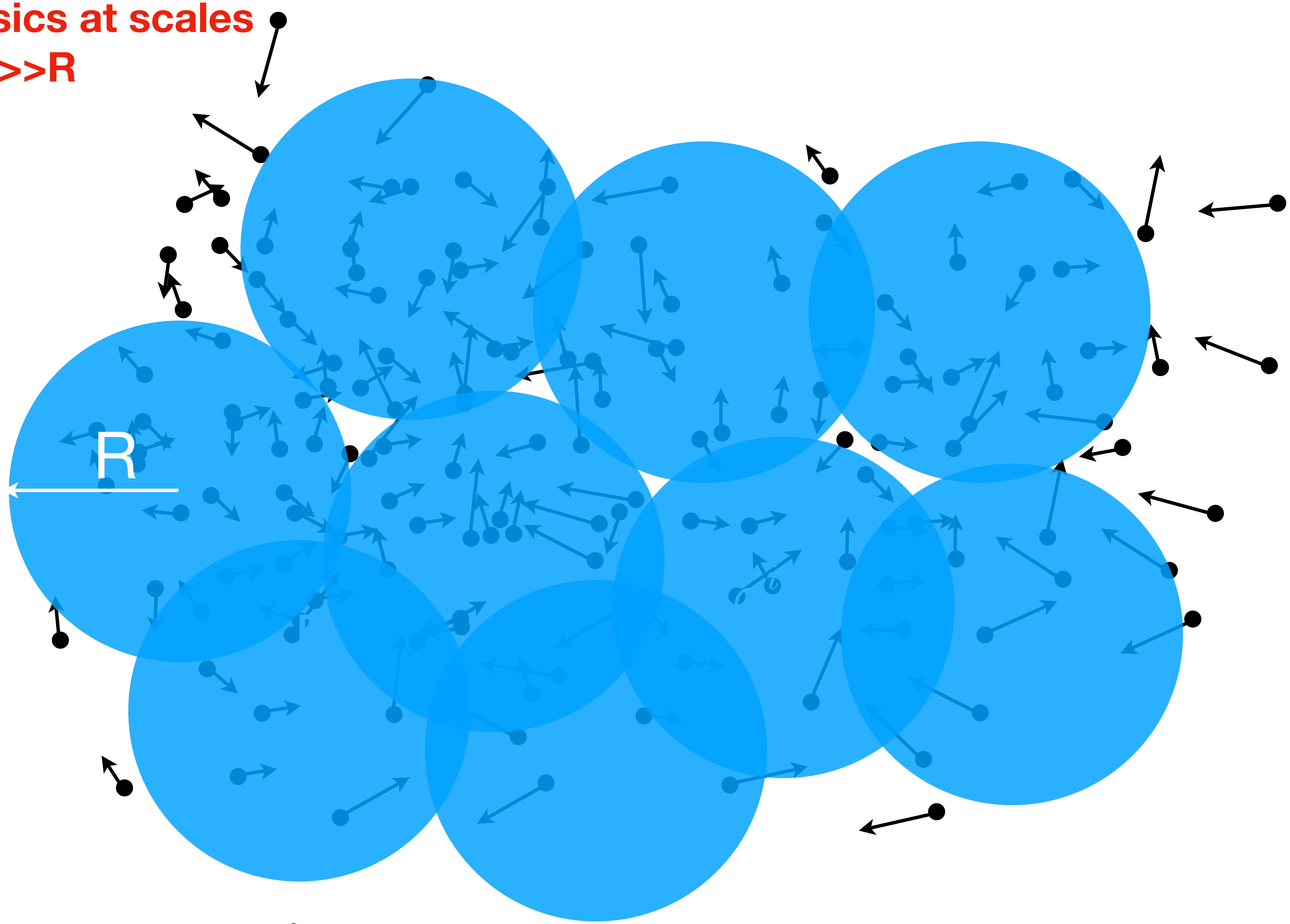
correspondingly, the distribution function is transformed as

$$f(\mathbf{x},\mathbf{p},\tau) \to \tilde{f}(\mathbf{x},\mathbf{p},\tau) = f(\mathbf{x}-\mathbf{d}(\tau), \mathbf{p} - am\dot{\mathbf{d}}(\tau), \tau)$$

NOTE: <u>THIS SYMMETRY IS EXACT</u> (no approximation yet):
consequences on the structure of exact correlators [consistency relations]

$$f(\mathbf{x},\mathbf{p},\tau) = \bar{\rho}\int d^3q\,\delta_D\left(\mathbf{q}+\psi(\mathbf{q},\tau)-\mathbf{x}\right)\,\delta_D\left(\mathbf{p}-am\dot{\psi}(\mathbf{q},\tau)\right)$$

Study physics at scales L>>R



$$f_R(\mathbf{x},\mathbf{p},\tau) = \int d^3y\,\mathcal{W}\left[\frac{y}{R}\right]\,f_R(\mathbf{x}-\mathbf{y},\mathbf{p},\tau) \qquad \left(\mathcal{W}\left[\frac{y}{R}\right] = \frac{1}{(2\pi)^{3/2}R^3}e^{-\frac{y^2}{2R^2}}\right)$$

Coarse-grained Vlasov equation

$$\left(\frac{\partial}{\partial \tau} + \frac{p^i}{am} \frac{\partial}{\partial x^i} - am \frac{\partial \phi_R(\mathbf{x}, \tau)}{\partial x^i} \frac{\partial}{\partial p^i} \right) f_R(\mathbf{x}, \mathbf{p}, \tau) =$$

$$am \left[\langle \frac{\partial \phi}{\partial x^i} \frac{\partial}{\partial p^i} f \rangle_R(\mathbf{x}, \mathbf{p}, \tau) - \frac{\partial \phi_R(\mathbf{x}, \tau)}{\partial x^i} \frac{\partial}{\partial p^i} f_R(\mathbf{x}, \mathbf{p}, \tau) \right]$$

effect of the short scales

$$\left(g_R(\mathbf{x}) \equiv \langle g \rangle_R(\mathbf{x}) \equiv \int d^3y \, \mathcal{W} \left[\frac{y}{R} \right] \, g(\mathbf{x} + \mathbf{y}) \right)$$

Moments

density fluctuation

$$1 + \delta_R(\mathbf{x}, \tau) = \frac{1}{\bar{\rho}} \int d^3p \, f_R(\mathbf{x}, \mathbf{p}, \tau)$$

velocity

$$(1 + \delta_R(\mathbf{x}, \tau)) \, v_R^i(\mathbf{x}, \tau) = \frac{1}{\bar{\rho}} \int d^3p \, \frac{p^i}{am} f_R(\mathbf{x}, \mathbf{p}, \tau)$$

velocity dispersion

$$(1 + \delta(\mathbf{x}, \tau)) \left(v_R^i(\mathbf{x}, \tau) v_R^j(\mathbf{x}, \tau) + \sigma_R^{ij}(\mathbf{x}, \tau) \right) = \frac{1}{\bar{\rho}} \int d^3p \, \frac{p^i}{am} \frac{p^j}{am} f_R(\mathbf{x}, \mathbf{p}, \tau)$$

...

take time-derivatives and use the coarse-grained Vlasov equation …

(drop the time dependence)

$$\frac{\partial}{\partial \tau}\delta_R(\mathbf{x}) + \frac{\partial}{\partial x^i}\left[(1+\delta_R(\mathbf{x}))v_R^i(\mathbf{x})\right] = 0$$

continuity eq.

$$\frac{\partial}{\partial \tau}v_R^i(\mathbf{x}) + \mathcal{H}v_R^i(\mathbf{x}) + v_R^k(\mathbf{x})\frac{\partial}{\partial x^k}v_R^i(\mathbf{x}) = -\nabla_x^i\phi_R(\mathbf{x}) - J_\sigma^i(\mathbf{x}) - J_1^i(\mathbf{x})$$

Euler eq.

short-distance effects

$$J_\sigma^i(\mathbf{x}) \equiv \frac{1}{1+\delta_R(\mathbf{x})}\frac{\partial}{\partial x^k}\left((1+\delta_R(\mathbf{x}))\sigma_R^{ki}(\mathbf{x})\right)$$

$$J_1^i(\mathbf{x}) \equiv \frac{1}{1+\delta(\mathbf{x})}\left(\langle(1+\delta)\nabla^i\phi\rangle_R(\mathbf{x}) - (1+\delta_R)(\mathbf{x})\nabla^i\phi_R(\mathbf{x})\right)$$

To close the system, we must provide information on the short-distance effects

Buchert, Dominguez, '05, Pueblas Scoccimarro, '09, Baumann et al. '10

M.P., G. Mangano, N. Saviano, M. Viel, 1108.5203, Carrasco, Hertzberg, Senatore,1206.2976 ....