Nonlinear Perturbation Theory for the Large Scale Structure

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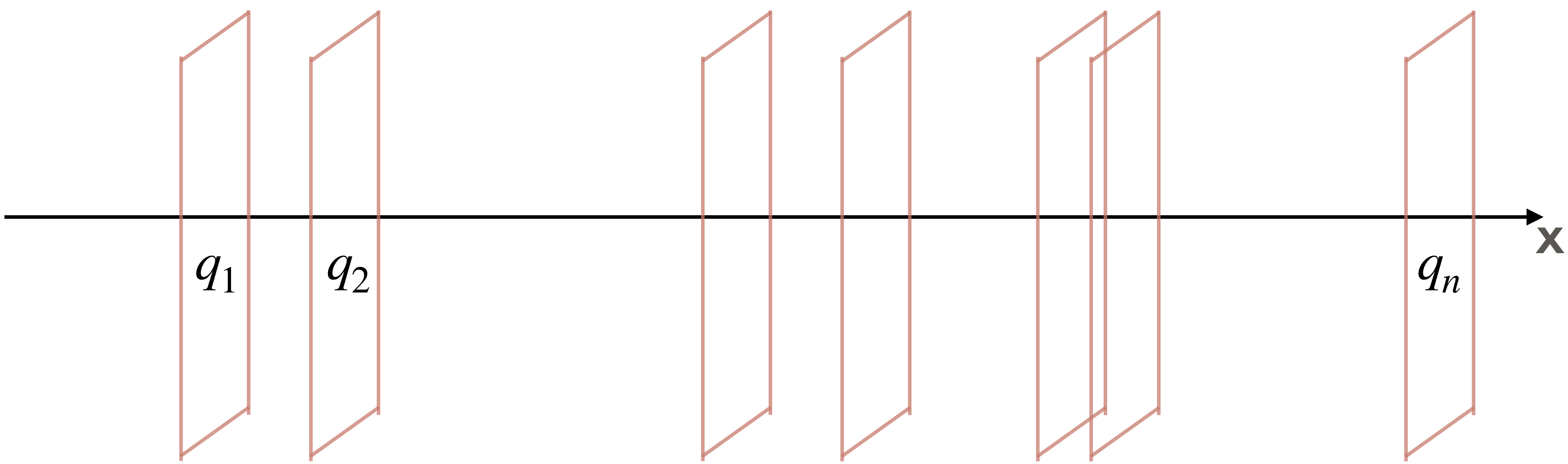
Lecture 2

Outline

- brief review of statistical field theory
- the setup: Eulerian vs Lagrangian, equations of motion
- structure formation in the LineLand (1+1 dimensions)
- Standard Perturbation Theory
- performance and problems of SPT (response functions)
- IR effects: resummations and BAO's
- UV behavior: Effective approaches
- From matter to biased tracers
- Redshift space distortions
- Putting all together (state of the art)
- Beyond PT: consistency relations
- Beyond PT: shell-crossing
- [Beyond CDM: Axions and ALP's]
- [Beyond LCDM: neutrinos, PNG, non-standard growth and mode-coupling]

References:

Structure formation in the LineLand (1+1 dim)



- Force is independent on distance
- Linear (Zel'dovich) approximation and PT valid up to shell-crossing
- Clear isolation of shell-crossing effects

(Mc Quinn, White, 1502.07389; Taruya, Colombi, 1701.09088; Rampf, Frisch, 1705.08456; McDonald, Vlah, 1709.02834, Pajer, van der Woude, 1710.01736, MP 1804.09140, Rampf et al 1912.00868…)

LAGRANGIAN TO EULERIAN MAPPING

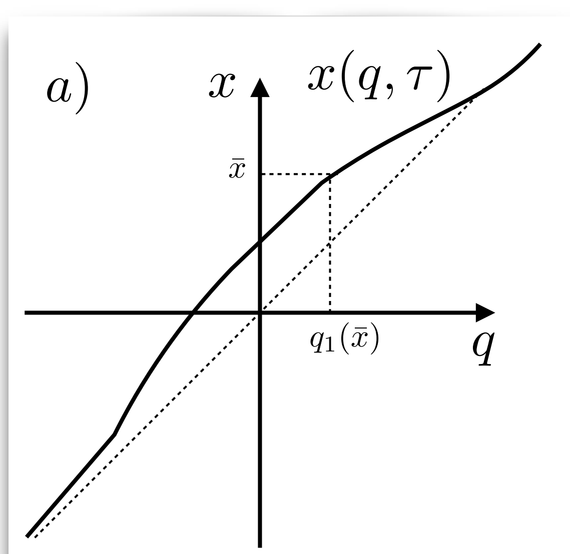
$$f(x,p,\tau) = \bar{\rho} \int dq\, \delta_D\left(q+\psi(q,\tau)-x\right)\, \delta_D\left(p - am\dot{\psi}(q,\tau)\right)$$

$$\frac{1}{\bar{\rho}} \int dp f(x,p,\tau) = 1+\delta(x,\tau) = \int dq\, \delta_D(q+\psi(q,\tau)-x) = \sum_{\text{roots}} \frac{1}{|1+\psi'(q_i,\tau)|}$$

$$(\psi'(q) \equiv \partial_q \psi(q))$$

roots: $q_i(x,\tau)$ such that $q_i + \psi(q_i,\tau) = x$

Case a): only one root in x (one stream)



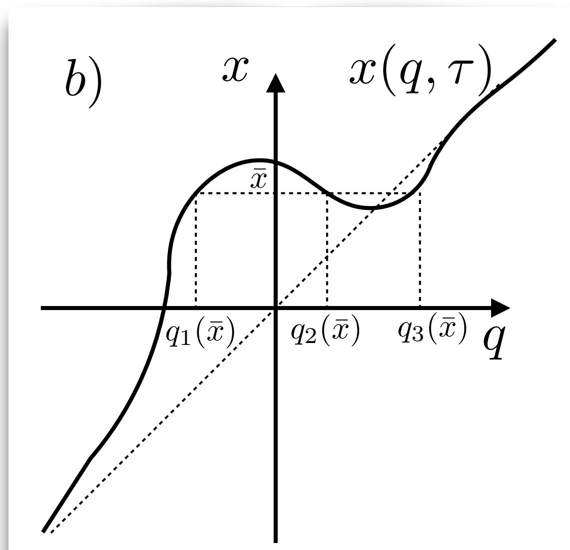
$$1+\delta(x,\tau) = \frac{1}{1+\psi'(q_1(x),\tau)}$$

nonlinear relation between δ and ψ

$$v(x,\tau) = \dot{\psi}(q_1(x),\tau)$$

no velocity dispersion: *single stream regime*

Case b): three roots in x (three streams)



$$1+\delta(x,\tau) = \frac{1}{1+\psi'(q_1(x),\tau)} - \frac{1}{1+\psi'(q_2(x),\tau)} + \frac{1}{1+\psi'(q_3(x),\tau)}$$

$$(1+\delta(x))\, v(x) = \frac{\dot\psi(q_1)}{1+\psi'(q_1(x))} - \frac{\dot\psi(q_2)}{1+\psi'(q_2(x))} + \frac{\dot\psi(q_3)}{1+\psi'(q_3(x))}$$

$$(1+\delta(x))\left(v^2(x)+\sigma(x)\right) = \sum_{i=1}^{3} \frac{\dot\psi(q_i)^2}{|1+\psi'(q_i(x))|}$$

velocity dispersion

- non-locality in lagrangian space
- non-zero velocity dispersion: three *streams*

Equation of motion

$$\ddot{\psi}(q) + \mathcal{H}\dot{\psi}(q) = -\partial_x \phi(x(q)) \qquad (x(q) = q + \psi(q))$$

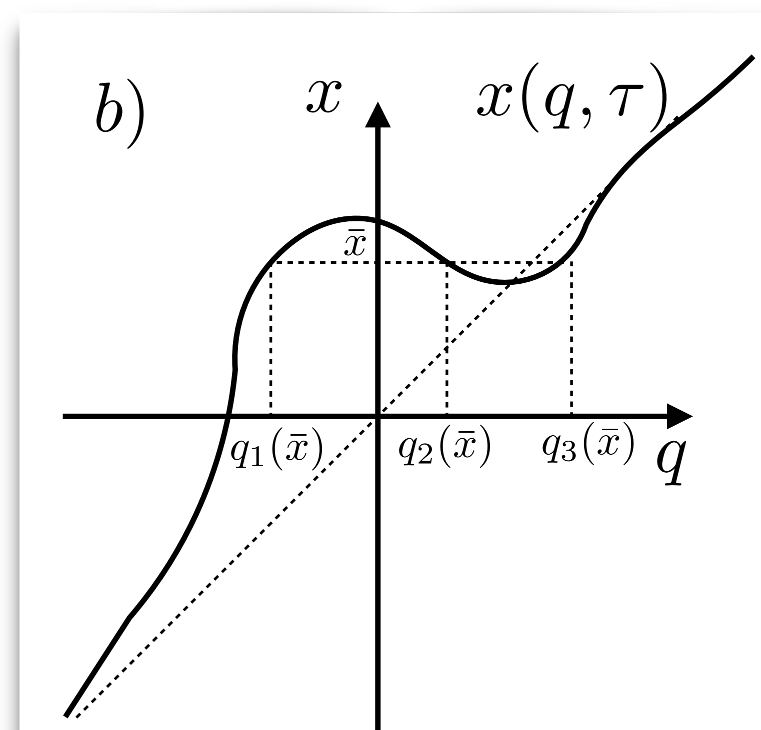
EdS: $\Omega_m = 1$

$$\partial_x^2 \phi(x) = \frac{3}{2}\mathcal{H}^2 \delta(x) = \frac{3}{2}\mathcal{H}^2 \int dq' \; (\delta_D(q' + \psi(q') - x) - \delta_D(q' - x))$$

Exact force:

$$-\partial_x \phi(x) = \frac{3}{2}\mathcal{H}^2 \int dq' \; (\Theta(q' + \psi(q') - x) - \Theta(q' - x)) = \frac{3}{2}\mathcal{H}^2 \sum_i (-1)^{i+1} \psi(q_i(x))$$

$$\ddot{\psi}(q) + \mathcal{H}\dot{\psi}(q) = \frac{3}{2}\mathcal{H}^2 \sum_i (-1)^{i+1} \psi(q_i(x(q)))$$



Single stream regime

only one root: $q_1(x(q)) = q$ → $\ddot{\psi}(q) + \mathcal{H}\dot{\psi}(q) = \frac{3}{2}\mathcal{H}^2\psi(q)$

Zel'dovich dynamics is exact in single stream regime (only in 1+1)

solution (growing mode): $\psi_Z(q,\tau) = a(\tau)\psi_Z(q,\tau_0)$ $\qquad (a(\tau_0) = 1)$

q=0 fixed (always possible by a time-dependent boost): $\delta_Z(0,\tau) = -\frac{\psi'_Z(0,\tau)}{1+\psi'_Z(0,\tau)}$

Perturbation Theory expansion: $\psi'_Z(0,\tau) \equiv -\delta_{\rm lin}(0,\tau) = -a(\tau)\delta_{\rm lin,0}$

$$\delta_{\rm SPT}(0,\tau) = \sum_{n=1}^{\infty} a(\tau)^n \delta^n_{\rm lin,0}$$

Convergence of SPT

$$\delta_{\text{SPT}}(0,\tau) = \sum_{n=1}^{\infty} a(\tau)^n \delta_{\text{lin},0}^n \overset{?}{\to} \frac{a(\tau)\delta_{\text{lin},0}}{1 - a(\tau)\delta_{\text{lin},0}}$$

mathematically, it converges if $|a(\tau)\delta_{\text{lin},0}| = |\delta_{\text{lin}}(0,\tau)| < 1$

the true answer is $\delta_{\text{true}}(0,\tau) = -1 + \sum_{i=1}^{N_{\text{streams}}} \frac{1}{|1 + \psi'(q_i(0),\tau)|}$

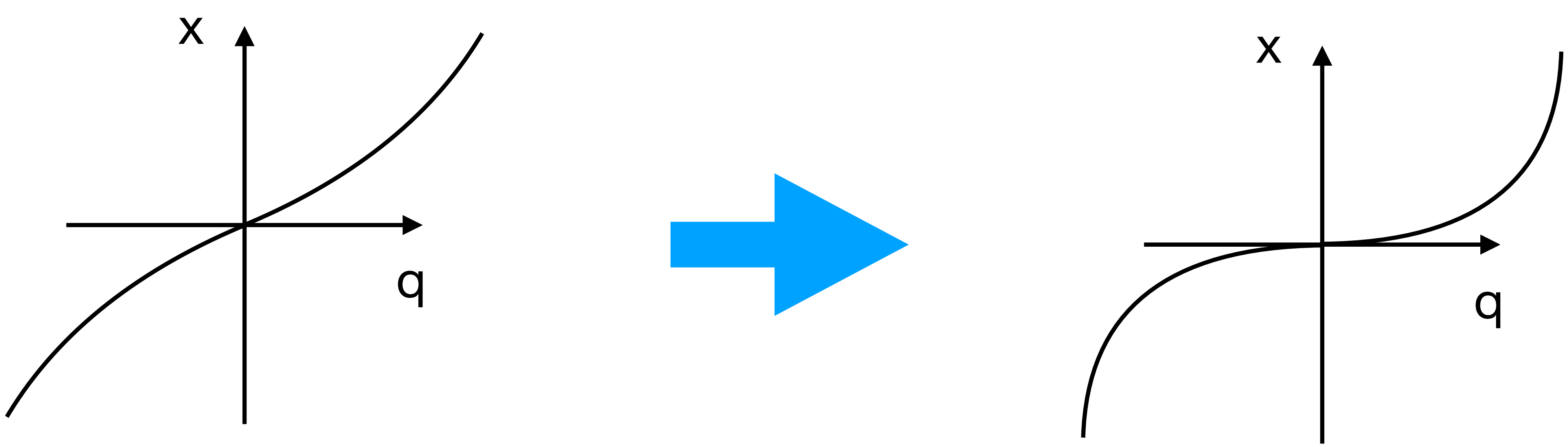
if also $N_{\text{streams}} = 1$, $\delta_{\text{SPT}}(0,\tau) \to \delta_{\text{true}}(0,\tau) = \delta_Z(0,\tau) = -\frac{\psi'_Z(0,\tau)}{1 + \psi'_Z(0,\tau)}$

SPT does not converge to the true answer if there is multi-streaming and for $|a(\tau)\delta_{\text{lin},0}| \geq 1$

What happens when $|\psi'_Z(0,\tau)| = |a(\tau)\delta_{\text{lin},0}| = 1$?

Shell-crossing

$$-\psi'_Z(0,\tau) = a(\tau)\delta_{\mathrm{lin},0} = 1$$

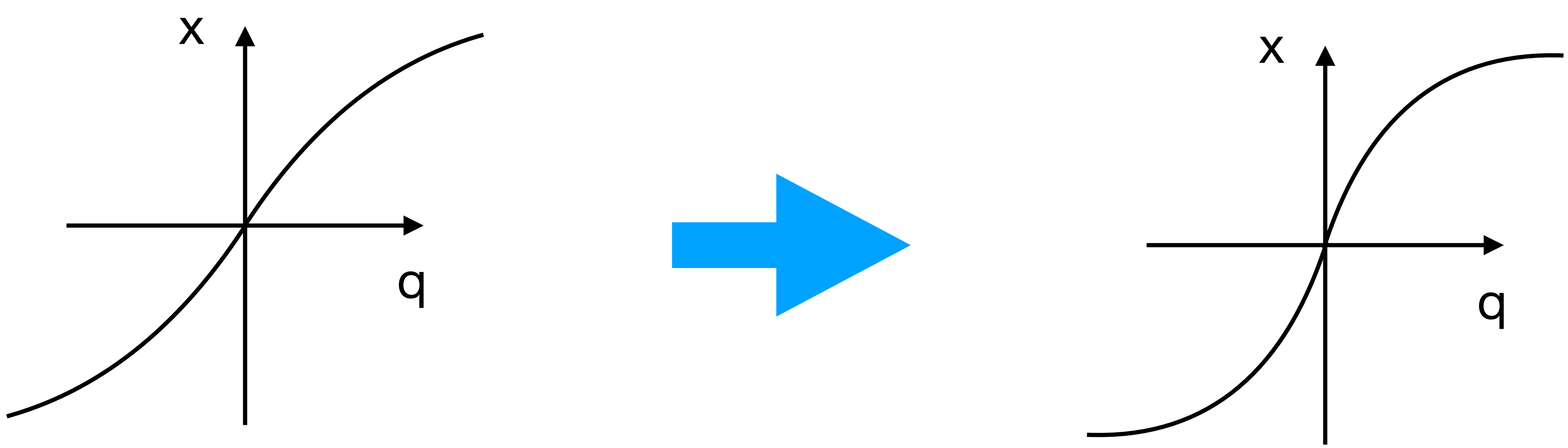


$$\frac{dx}{dq} = 1 + \psi'(q(x)) > 0 \qquad\qquad \frac{dx}{dq} = 1 + \psi'(q(x)) = 0$$

- the mapping becomes singular and the distribution function diverges
- the perturbative expansion does not converge any more
- after shell-crossing, *non-locality* in lagrangian space

Voids

$$-\psi'_Z(0,\tau) = a(\tau)\delta_{\mathrm{lin},0} = -1$$



$$0 < \frac{dx}{dq} = 1 + \psi'(q(x)) < 2$$

$$\frac{dx}{dq} = 1 + \psi'(q(x)) = 2$$

- the linear density contrast becomes unphisical $(\delta \geq -1)$
- but the true density contrast is still meaningful $(\delta_{\mathrm{true}} = -\frac{1}{2})$
- the PT expansion can be analytically continued as

$$\delta_{\mathrm{SPT}}(0,\tau) = \sum_{n=0}^{\infty} a(\tau)^n \delta_{\mathrm{lin},0}^n \rightarrow \frac{a(\tau)\delta_{\mathrm{lin},0}}{1 - a(\tau)\delta_{\mathrm{lin},0}}$$

Exact dynamics

$$\partial_\eta \Psi(q,\eta) = \chi(q,\eta)\,,$$

$$\partial_\eta \chi(q,\eta) = -\frac{1}{2}\chi(q,\eta) + \frac{3}{2}\sum_{i=1}^{N_s(x,\eta)} (-1)^{i+1}\Psi(q_i(x,\eta),\eta)$$

$$\eta = \log\frac{a}{a_0} = -\log(1+z)$$

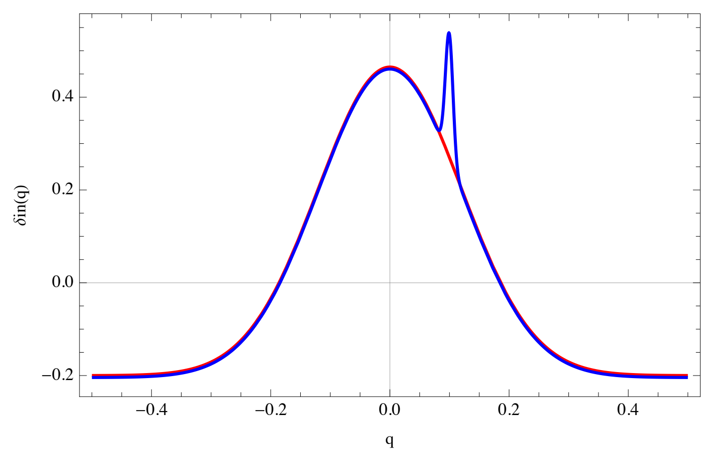
initial conditions on the linear growing mode: $\Psi(q,\eta_{\rm in}) = \chi(q,\eta_{\rm in}) = \frac{v(q,\eta_{\rm in})}{\mathcal{H}(\eta_{\rm in})}$

Algorithm:

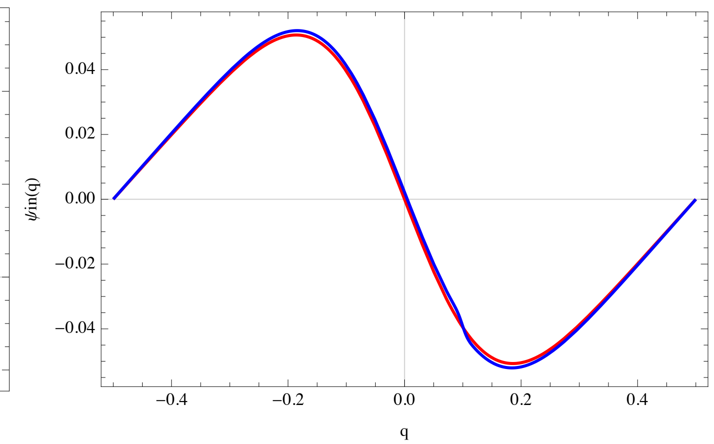
1) For each x, find the set of all q's such that $q + \Psi(q,\eta) = x$
2) Compute the force in x, valid for all q's in the corresponding set;
3) Increment $\chi(q,\eta)$ and $\Psi(q,\eta)$
4) Go to 1)

Gaussian overdensity

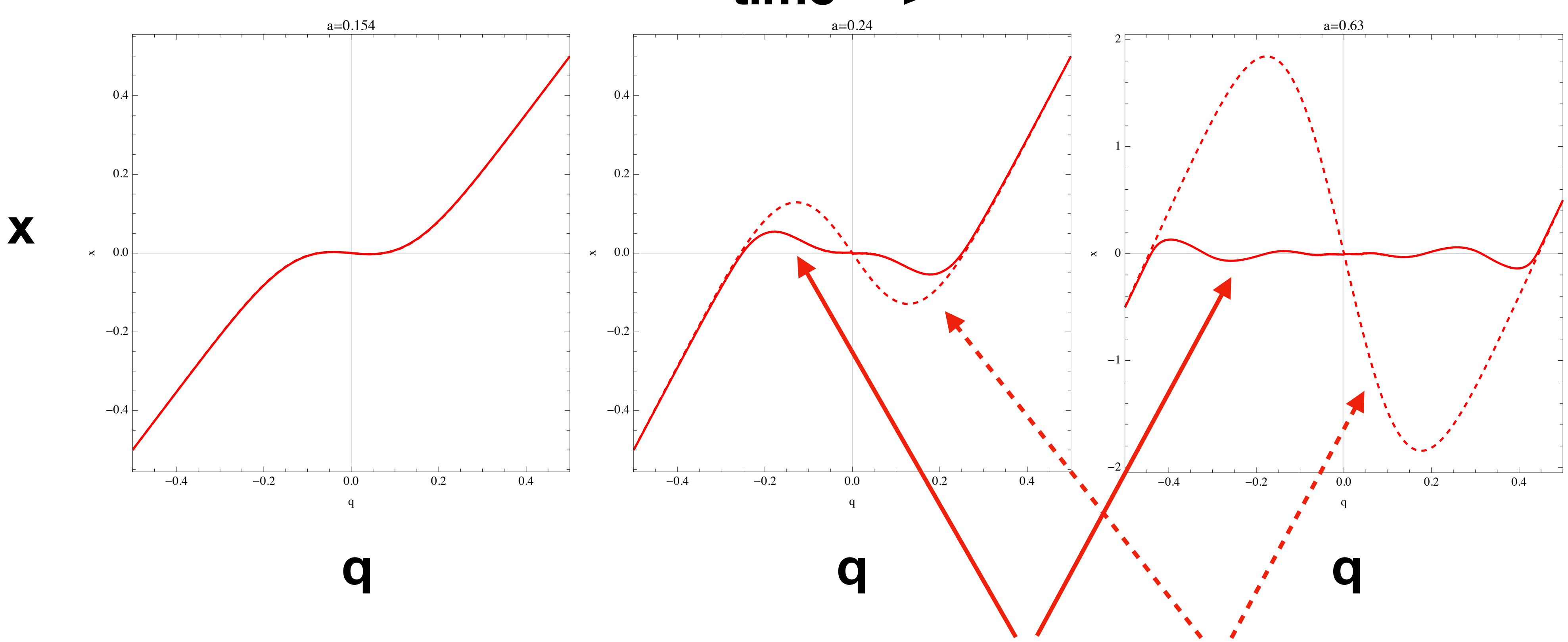
$\delta(q, \eta_{in})$



$\psi(q, \eta_{in})$



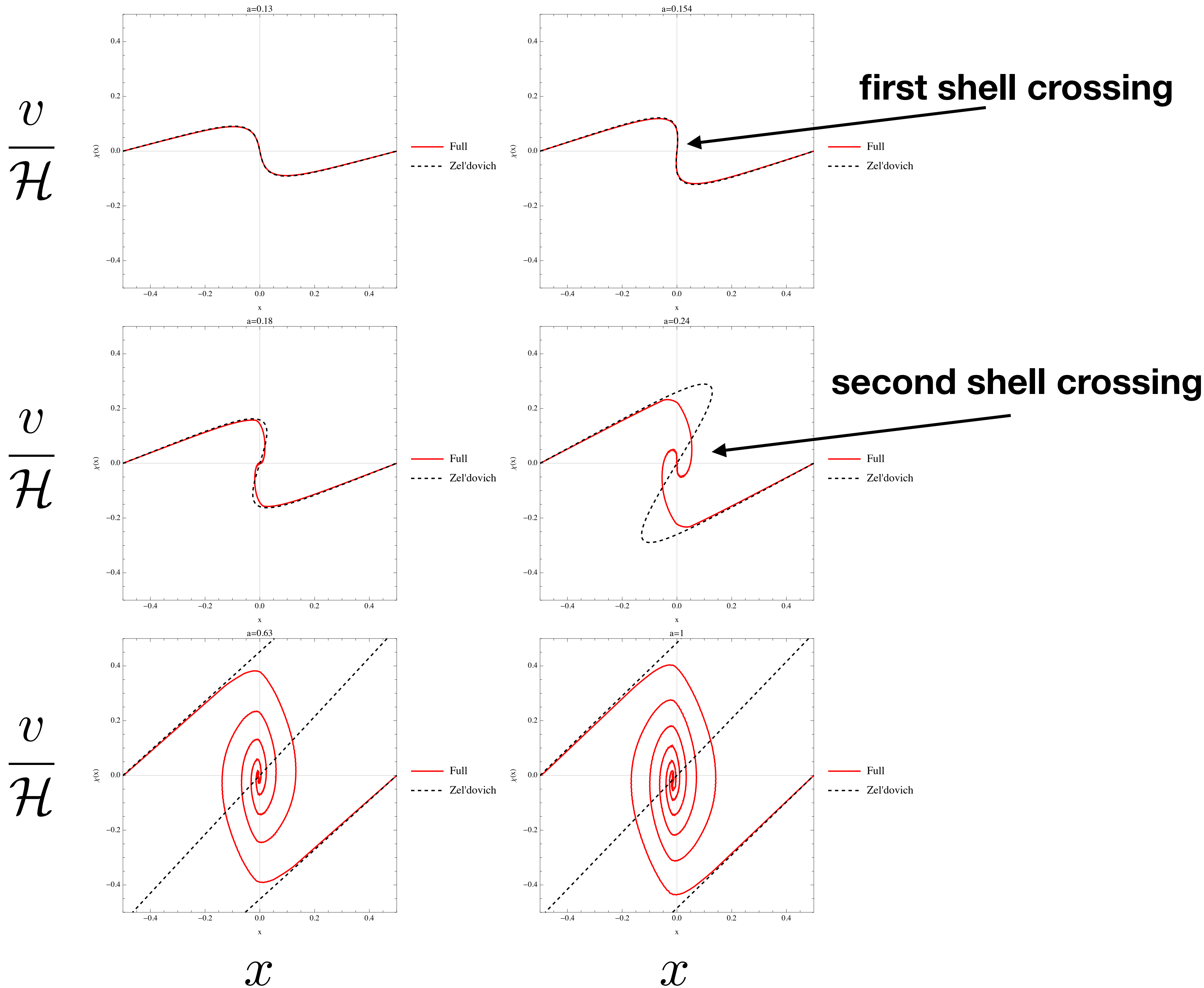
time —>



first shell crossing

exact **Zel'dovich**

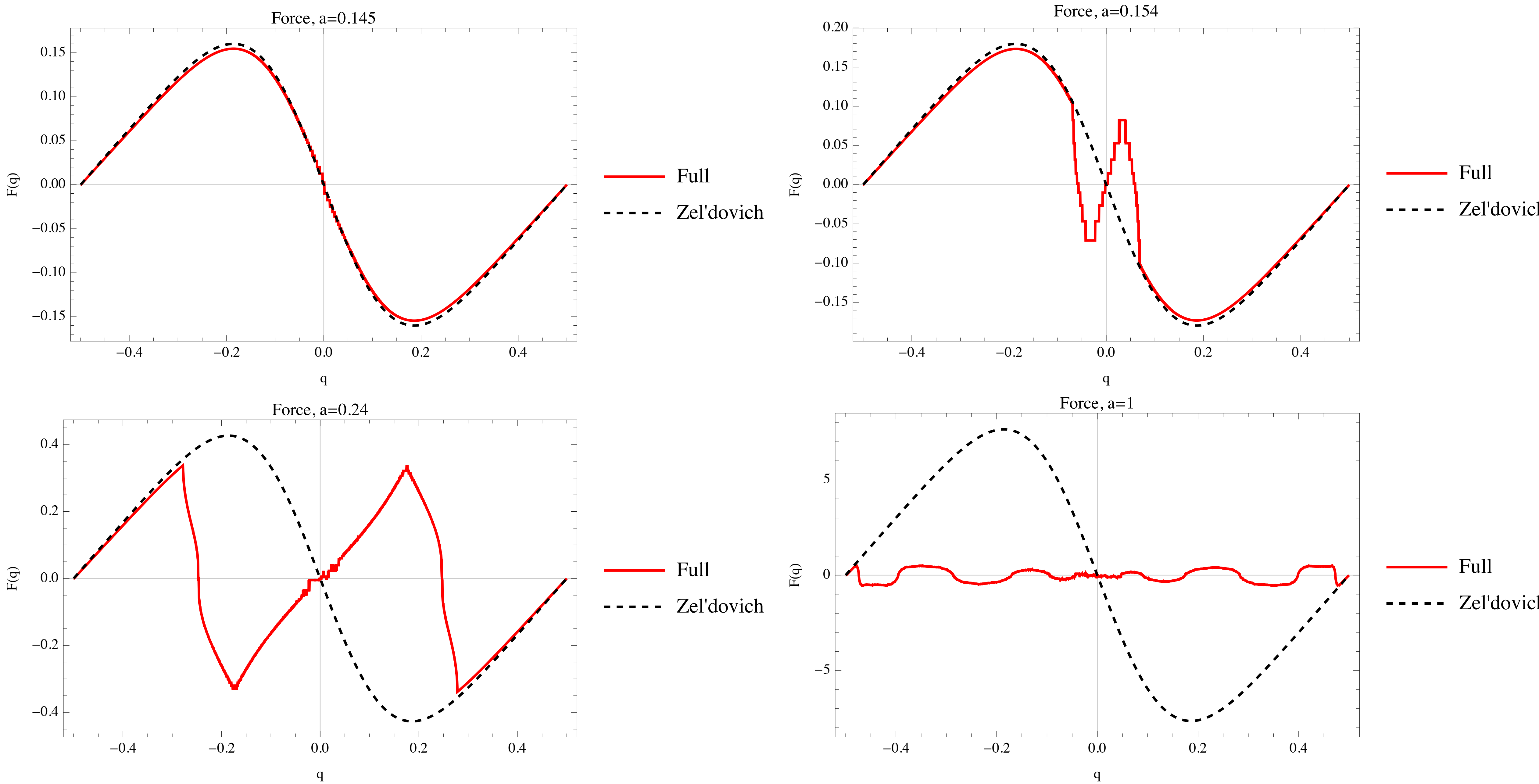
Phase space



a=0.13
a=0.154
a=0.18
a=0.24
a=0.63
a=1
Full
Zel'dovich
first shell crossing
second shell crossing
$\frac{v}{\mathcal{H}}$
x

Force

first shell crossing

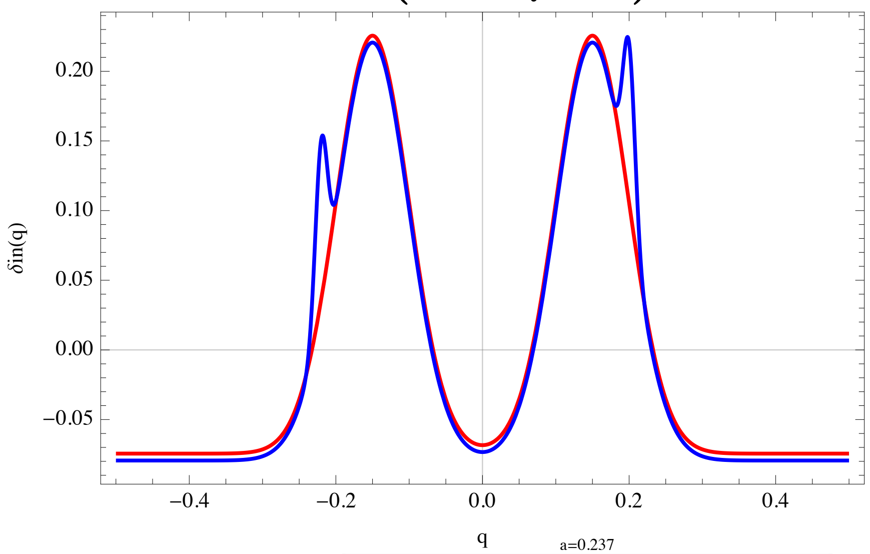


q **q**

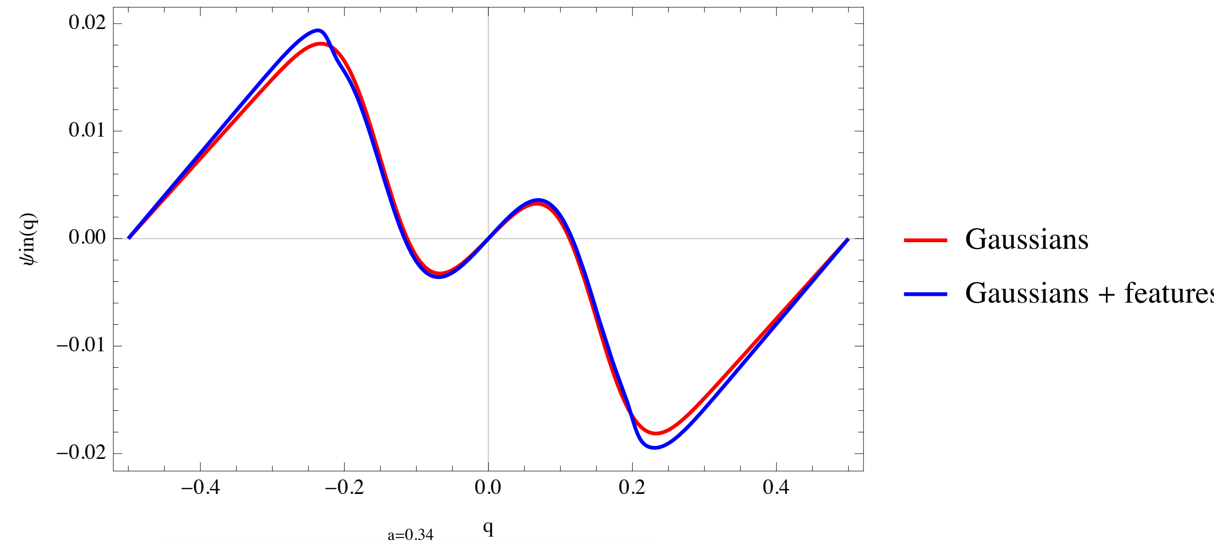
Zel’dovich fails at O(1) soon after the first shell-crossing!

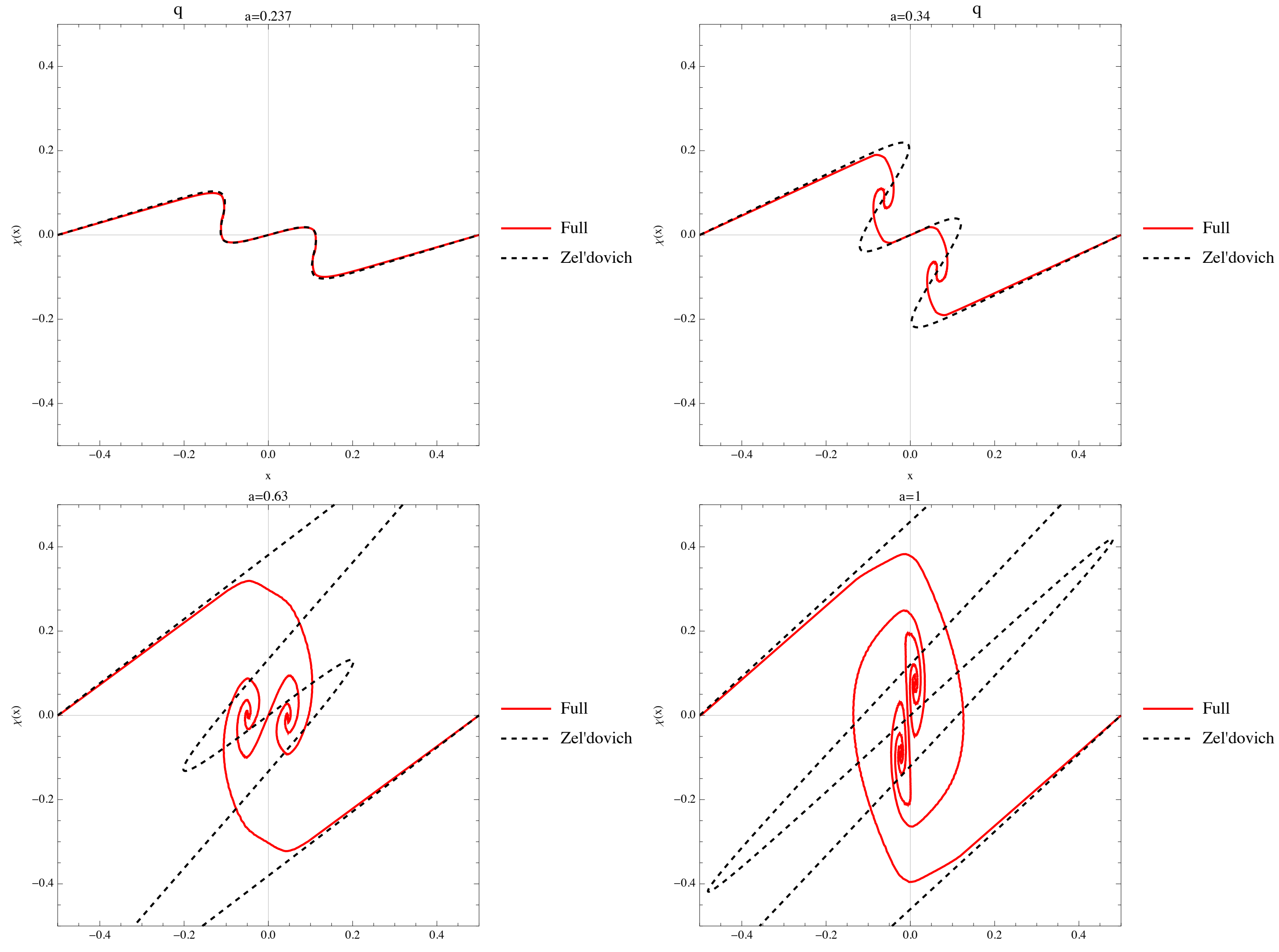
Two gaussians

$\delta(q, \eta_{in})$



$\psi(q, \eta_{in})$



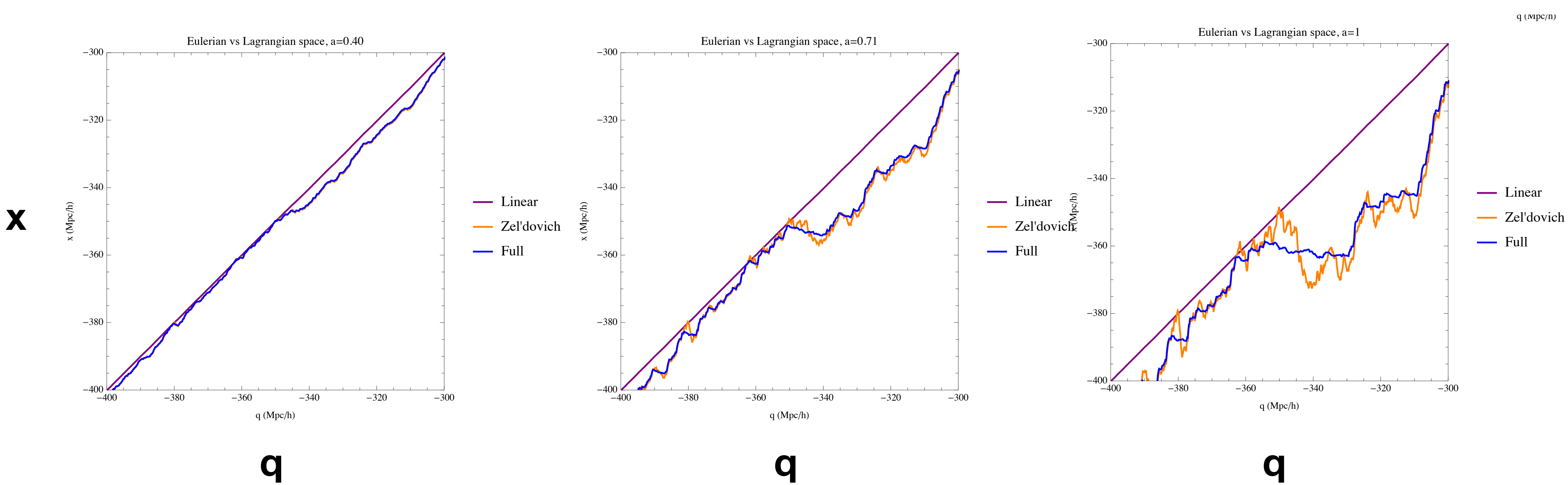


Phase space

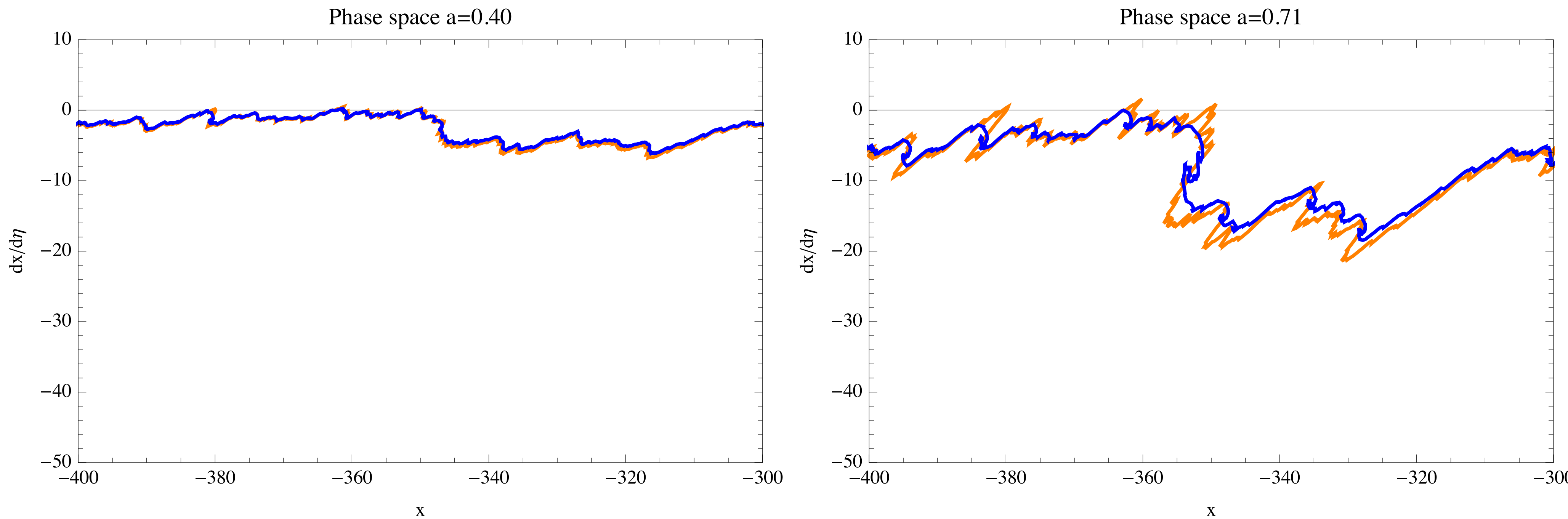
1+1 Universe

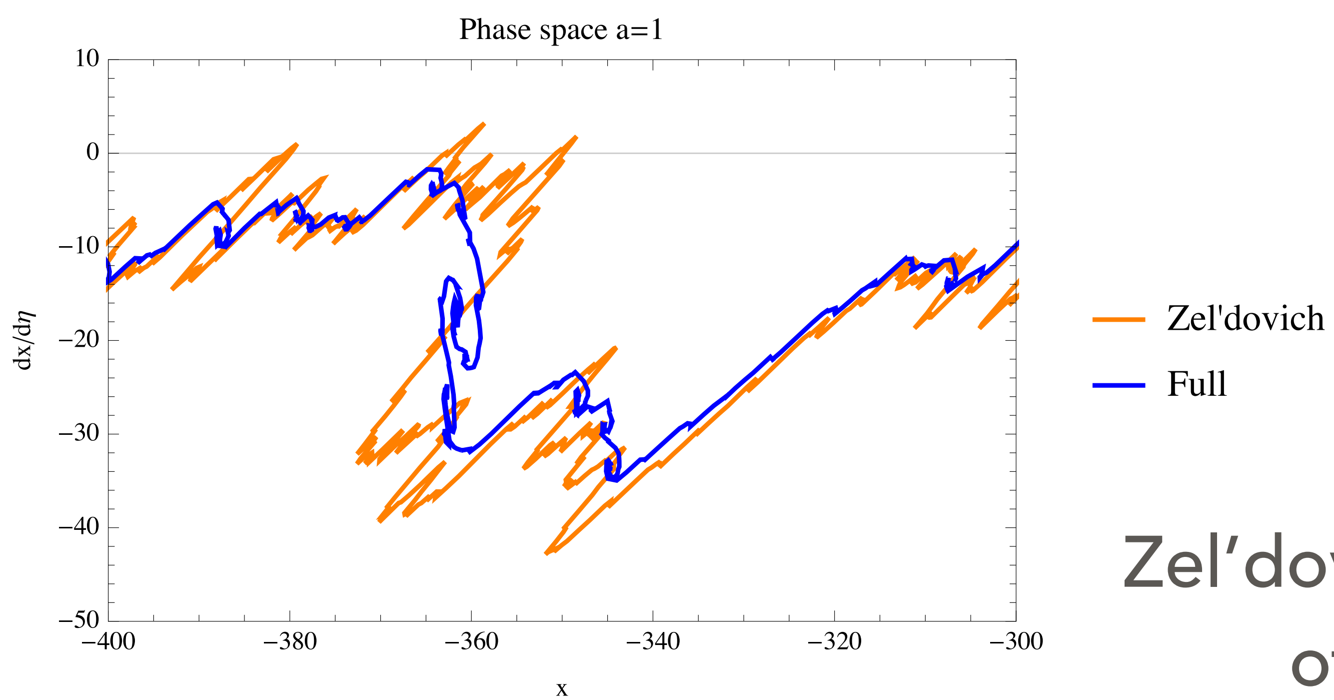
$$\Psi(q,\eta_{in}) = \chi(q,\eta_{in}) = \frac{1}{L}\sum_{m=1}^{N_p} c_m \cos\left(q\, p_m + \phi_m\right)$$

$c_m's$ from a Rayleigh distribution with random phases $\qquad \sigma_m = \sqrt{\frac{L P_{1D}(p_m)}{2 p_m^2}}$



Phase space





Zel’dovich overestimates the extension of the multistreaming regions

Lessons from the LineLand

1) **The perturbative expansion fails in voids and in shell-crossing (multistreaming) regions;**
2) **In voids, the perturbative series can be analytically continued to the exact solution, in multistreaming regions this is not possible;**
3) **The dynamics after shell-crossing becomes *nonlocal* in lagrangian space;**
4) **Any extension of the Zel'dovich approximation beyond shell-crossing is meaningless: the force gets O(1) corrections.**