# Nonlinear Perturbation Theory for the Large Scale Structure

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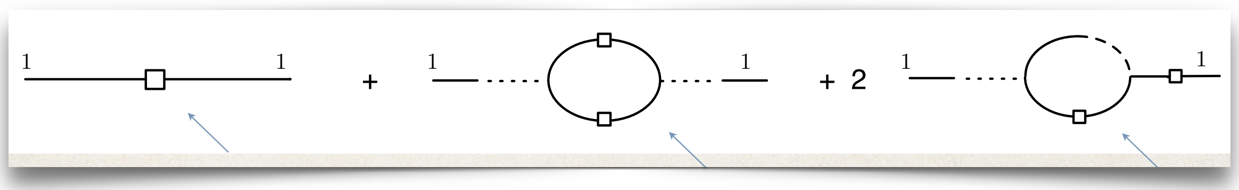
**Lecture 4**

# Outline

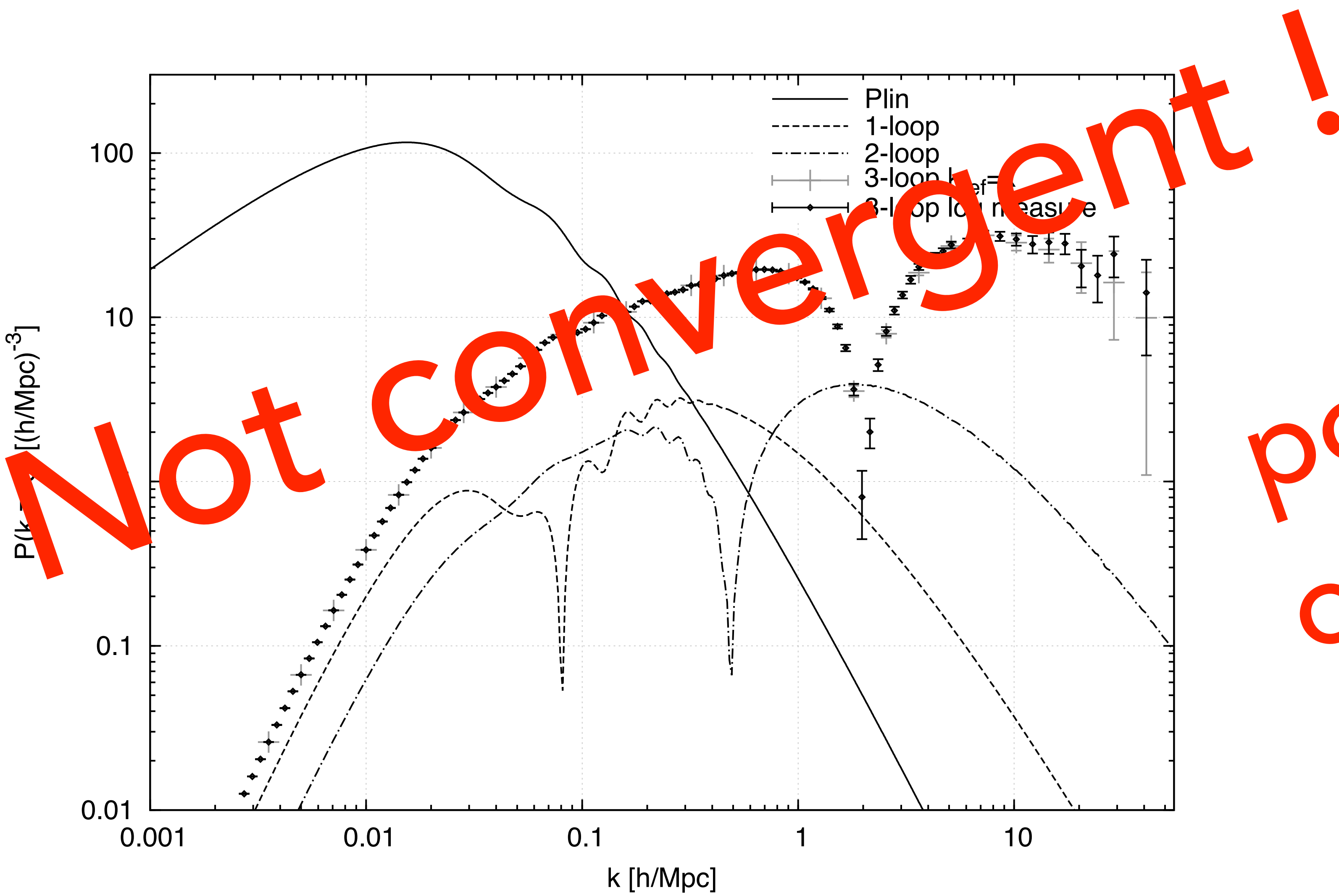
- brief review of statistical field theory
- the setup: Eulerian vs Lagrangian, equations of motion
- structure formation in the LineLand (1+1 dimensions)
- Standard Perturbation Theory
- performance and problems of SPT (response functions)
- IR effects: resummations and BAO’s
- UV behavior: Effective approaches
- From matter to biased tracers
- Redshift space distortions
- Putting all together (state of the art)
- Beyond PT: consistency relations

# Performance of Standard PT

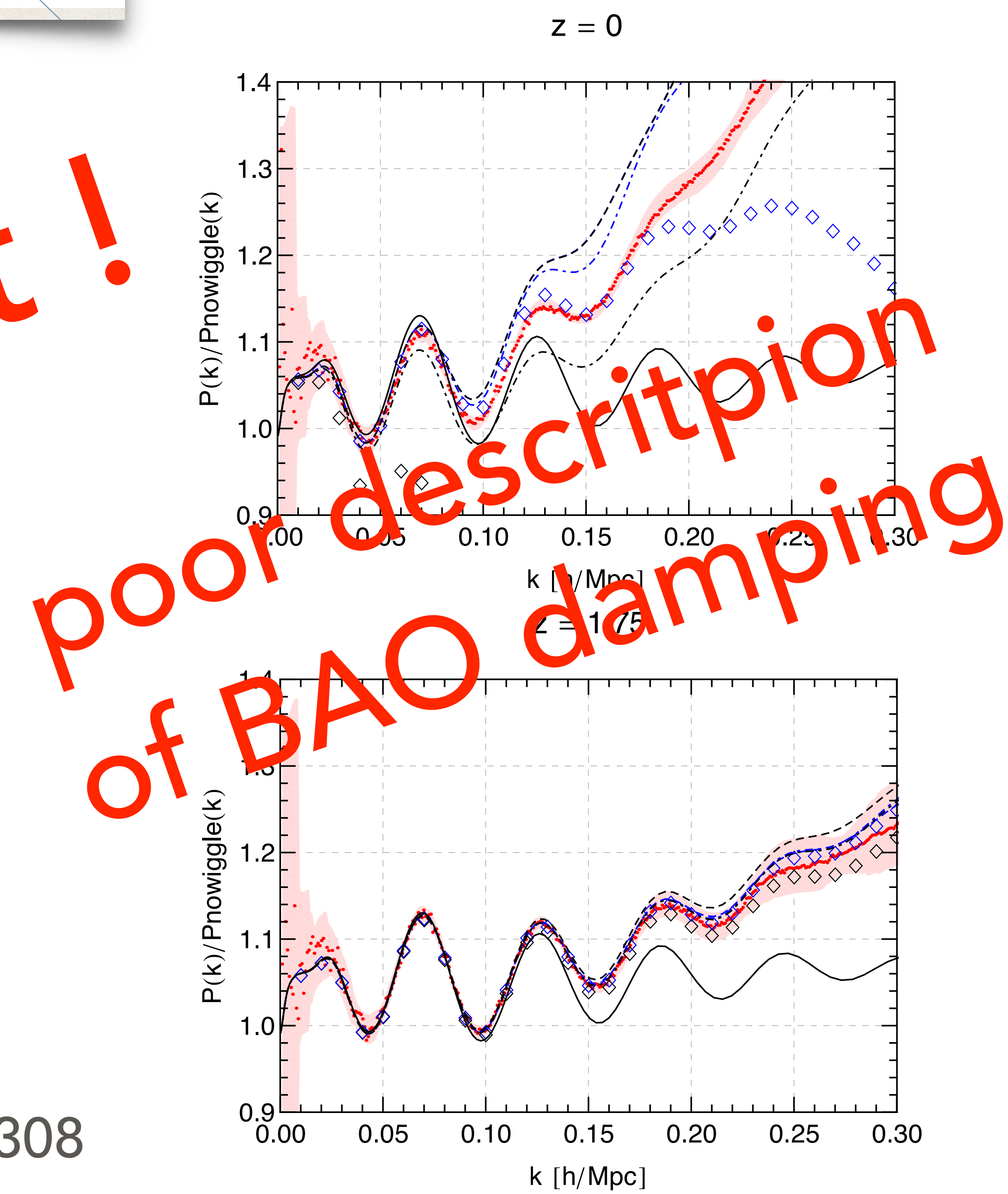
$$P(k,z) = D(z)^2 P^{(1)}(k) + D(z)^4 F^{(1l)}(k) + D(z)^6 F^{(2l)}(k) + \cdots$$



linear 1-loop




Blas et al. 1309.3308



# MODE COUPLING

Linear Response Function

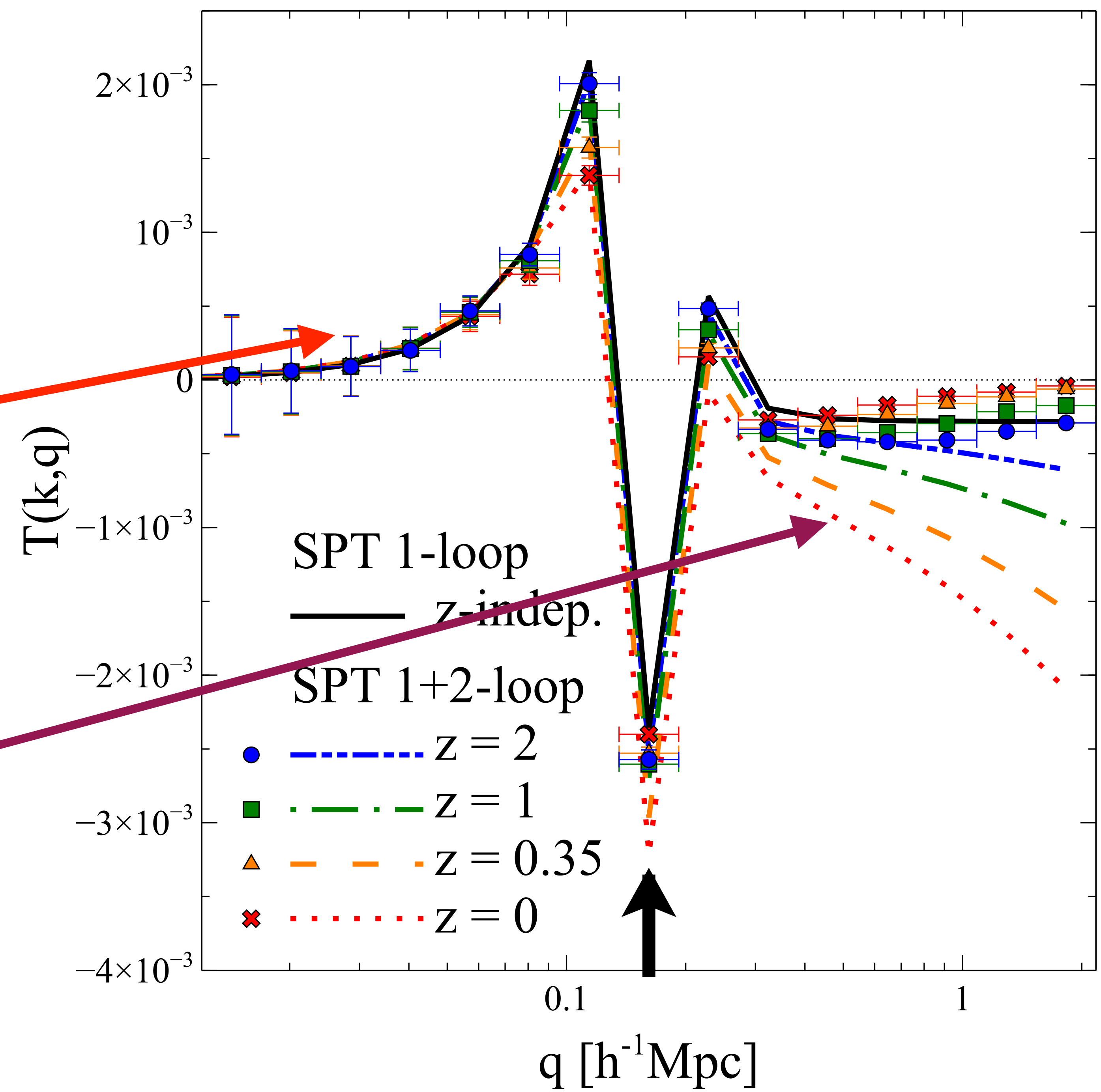
$$K(k,q;z) = q\frac{\delta P^{\rm nl}(k;z)}{\delta P^{\rm lin}(q;z)}$$

IR: "Galilean" invariance (EP)

$$K(k,q;z) \sim q^3$$

UV: SPT over predicts the effect of small scales on intermediate ones

Nishimichi, Bernardeau, Taruya 1411.2970



# Effective approaches to the UV

- Perturbation Theory (even after resummations) fails at short scales due to non-convergent series and multistreaming
- General idea: take the UV physics from N-body simulations and use PT only for the large and intermediate scales

(drop the time dependence)

$$\frac{\partial}{\partial \tau}\delta_R(\mathbf{x}) + \frac{\partial}{\partial x^i}\left[(1+\delta_R(\mathbf{x}))v_R^i(\mathbf{x})\right] = 0$$

**continuity eq.**

$$\frac{\partial}{\partial \tau}v_R^i(\mathbf{x}) + \mathcal{H}v_R^i(\mathbf{x}) + v_R^k(\mathbf{x})\frac{\partial}{\partial x^k}v_R^i(\mathbf{x}) = -\nabla_x^i\phi_R(\mathbf{x}) - J_\sigma^i(\mathbf{x}) - J_1^i(\mathbf{x})$$

**Euler eq.**

**short-distance effects**

$$J_\sigma^i(\mathbf{x}) \equiv \frac{1}{1+\delta_R(\mathbf{x})}\frac{\partial}{\partial x^k}\left((1+\delta_R(\mathbf{x}))\sigma_R^{ki}(\mathbf{x})\right)$$

$$J_1^i(\mathbf{x}) \equiv \frac{1}{1+\delta(\mathbf{x})}\left(\langle(1+\delta)\nabla^i\phi\rangle_R(\mathbf{x}) - (1+\delta_R)(\mathbf{x})\nabla^i\phi_R(\mathbf{x})\right)$$

**To close the system, we must provide information on the short-distance effects**

Buchert, Dominguez, '05, Pueblas Scoccimarro, '09, Baumann et al. '10

M.P., G. Mangano, N. Saviano, M. Viel, 1108.5203, Carrasco, Hertzberg, Senatore,1206.2976 ....

# EXACT TIME-EVOLUTION

$$(\delta_{ab}\partial_\eta + \Omega_{ab})\,\varphi_b^R(\mathbf{k},\eta) = e^\eta I_{\mathbf{k};\mathbf{q}_1,\mathbf{q}_2}\gamma_{abc}(\mathbf{q}_1,\mathbf{q}_2)\varphi_b^R(\mathbf{q}_1,\eta)\varphi_c^R(\mathbf{q}_2,\eta) - h_a^R(\mathbf{k},\eta)$$

$$P_{ab}^R(k) = \langle \varphi_a^R(\mathbf{k})\varphi_b^R(-\mathbf{k})\rangle'$$

$$B_{abc}^R(q_1,q_2,q_3) = \langle \varphi_a^R(\mathbf{q}_1)\varphi_b^R(\mathbf{q}_2)\varphi_c^R(\mathbf{q}_3)\rangle'$$

$$h_a^R(\mathbf{k},\eta) \equiv -i\,\frac{k^i J_R^i(\mathbf{k},\eta)}{\mathcal{H}^2 f^2} e^{-\eta}\delta_{a2}$$

$$\partial_\eta P_{ab}^R(k) = \Big[ -\Omega_{ac}P_{cb}^R(k) \qquad + (a \leftrightarrow b)\Big],$$

Linear PT

single stream

(vorticity treated perturbatively)

fully non-linear, equal-time correlators

need:

1) consistent truncations
2) measurement of UV correlators
3) IR resummation

# UV INFORMATION

Need input on the UV "sources"

$$J^i_\sigma(\mathbf{x}) \equiv \frac{1}{n(\mathbf{x})} \frac{\partial}{\partial x^k} (n(\mathbf{x}) \sigma^{ki}(\mathbf{x}))$$

$$J^i_1(\mathbf{x}) \equiv \frac{1}{n(\mathbf{x})} \left( \langle n_{mic} \nabla^i \phi_{mic} \rangle (\mathbf{x}) - n(\mathbf{x}) \nabla^i \phi(\mathbf{x}) \right)$$

Measure them from N-body simulations

(MP, Mangano, Saviano, Viel 1108.5203, Manzotti, Peloso, MP, Viel, Villaescusa-Navarro 1407.1342)

EFToLSS: Expand in terms of long wavelength fields + power law expansion in momentum, with arbitrary coefficients to be fitted

(Baumann et al. 1004.2488, Carrasco, Hertzberg, Senatore, 1206.2926 …. )

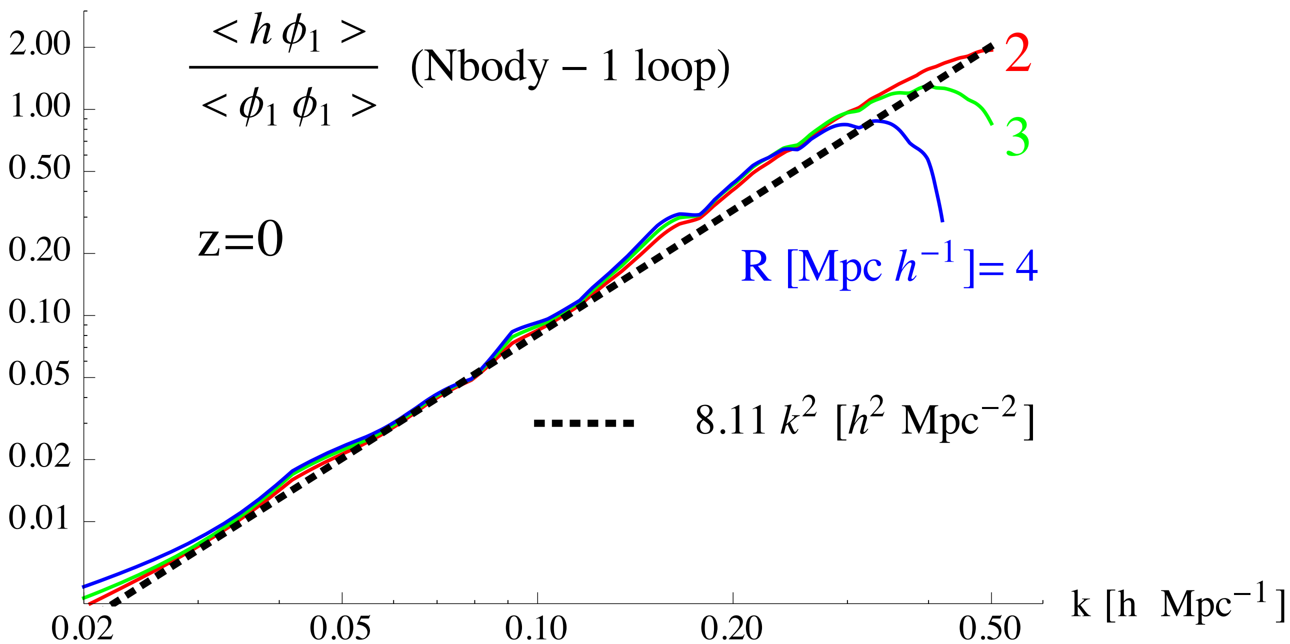
Compute them from first principles. Shell-crossing!

1+1 dim attempts

(Mc Quinn, White, 1502.07389; Taruya, Colombi, 1701.09088; Rampf, Frisch, 1705.08456; McDonald, Vlah, 1709.02834, Pajer, van der Woude, 1710.01736, MP, 1804.09140)

# UV CORRELATORS FROM N-BODY

## scale-dependence

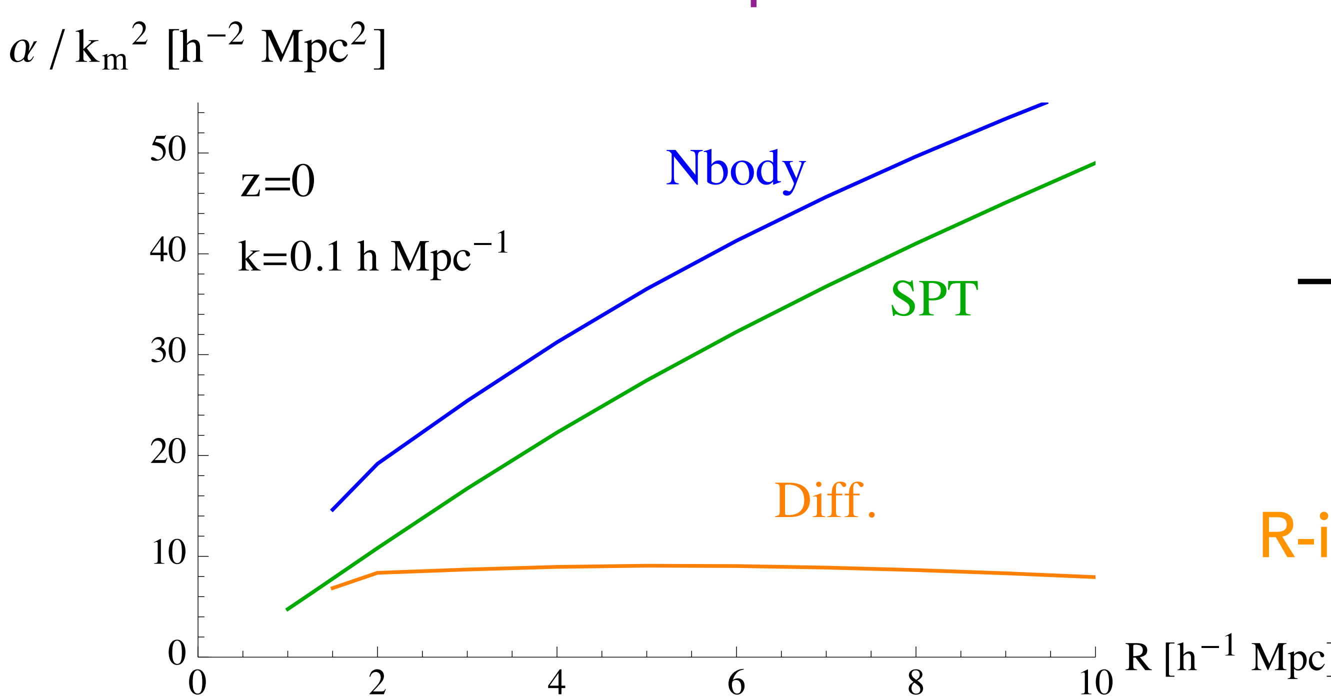


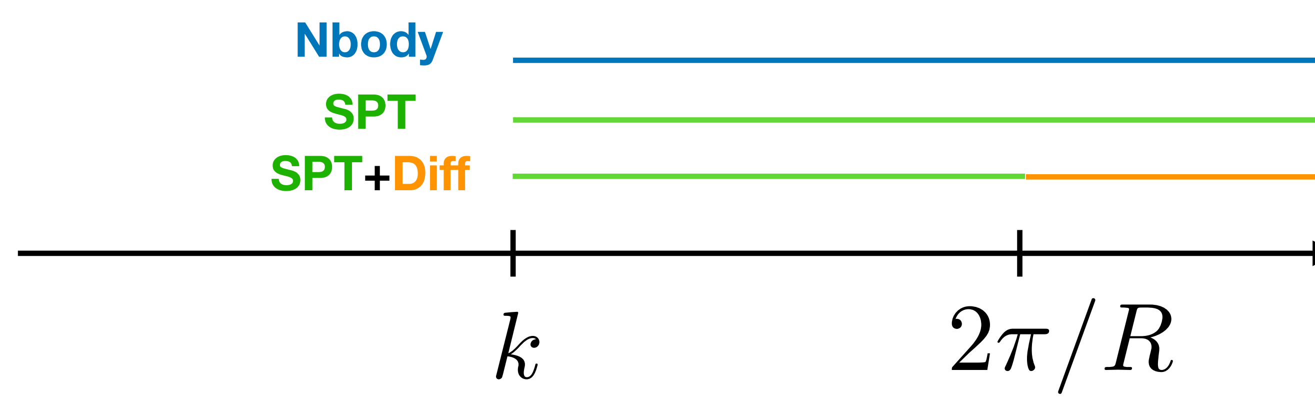

Parameterize the correlator as:

$$\langle h_a^R(\mathbf{k})\varphi_b^R(-\mathbf{k})\rangle' = \alpha^R(\eta)\frac{k^2}{k_m^2}P_{1b}^R(k;\eta)\,\delta_{a2}$$

nonlinear PS

## UV-cutoff dependence




R-independent plateau

# Relation with EFToLSS

Baumann et al 1004.2488
Carrasco et al 1206.2926
...

$$J_1^i + J_\sigma^i$$

$$\dot{\rho}_l + 3H\rho_l + \frac{1}{a}\partial_i(\rho_l v_l^i) = 0 \ ,$$

$$\dot{v}_l^i + H v_l^i + \frac{1}{a} v_l^j \partial_j v_l^i + \frac{1}{a}\partial_i \phi_l = -\frac{1}{a\rho_l}\partial_j \left[\tau^{ij}\right]_\Lambda \ .$$

$$\langle [\tau^{ij}]_\Lambda \rangle_{\delta_l} = p_b \delta^{ij} + \rho_b \left[ c_s^2 \delta_l \delta^{ij} - \frac{c_{bv}^2}{Ha}\delta^{ij}\partial_k v_l^k - \frac{3}{4}\frac{c_{sv}^2}{Ha}\left(\partial^j v_l^i + \partial^i v_l^j - \frac{2}{3}\delta^{ij}\partial_k v_l^k\right)\right] + \Delta\tau^{ij} + \ldots \ .$$

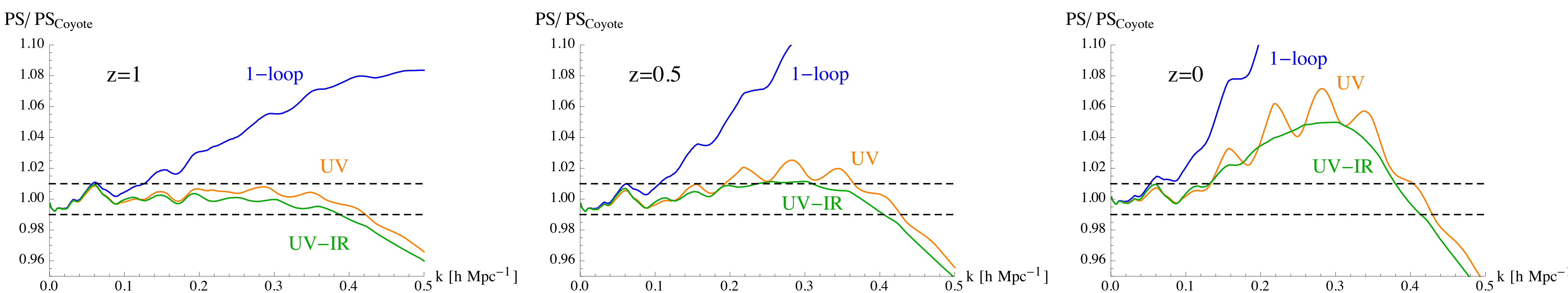
derivative expansion, or expansion in k/k_nl

coefficients should be scale independent, nice results for simple power law linear PS

# “MINIMAL” SETTING AND PERFORMANCE

$P^{nw}(k)$ 1-loop SPT + UV source

$P^{w}(k)$ 1-loop SPT + IR resummation+ UV source




**Noda, Peloso, M.P. 1705.01475**

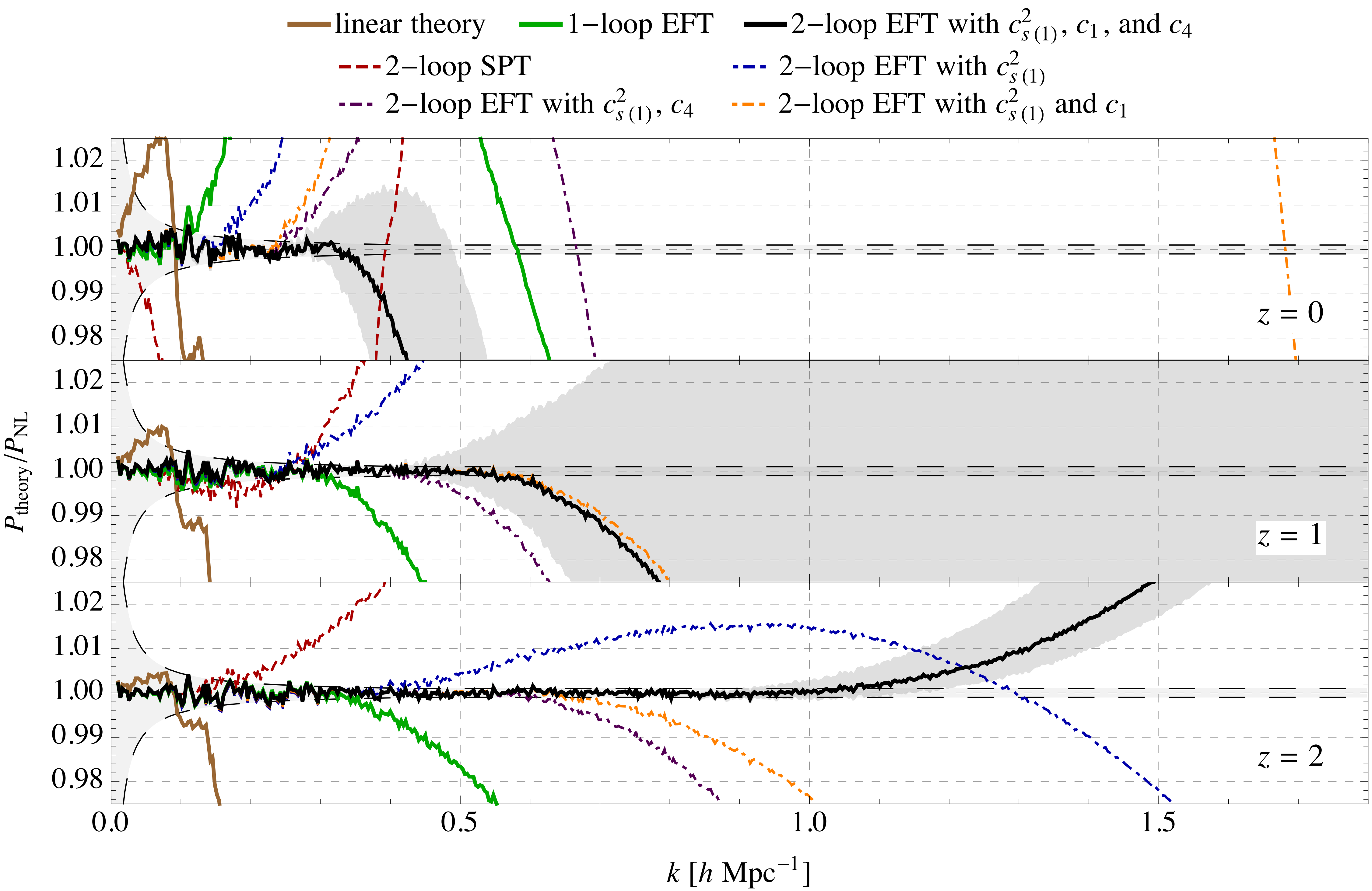
Broad band: $k_{max}$ ~ 0.4 h/Mpc @ z=1 —> ~0.1 @ z=0 (go to 2-loop…)

no fitting on the PS!! (results comparable to EFToLSS @ 1-loop)

BAO residuals: ok at all redshifts

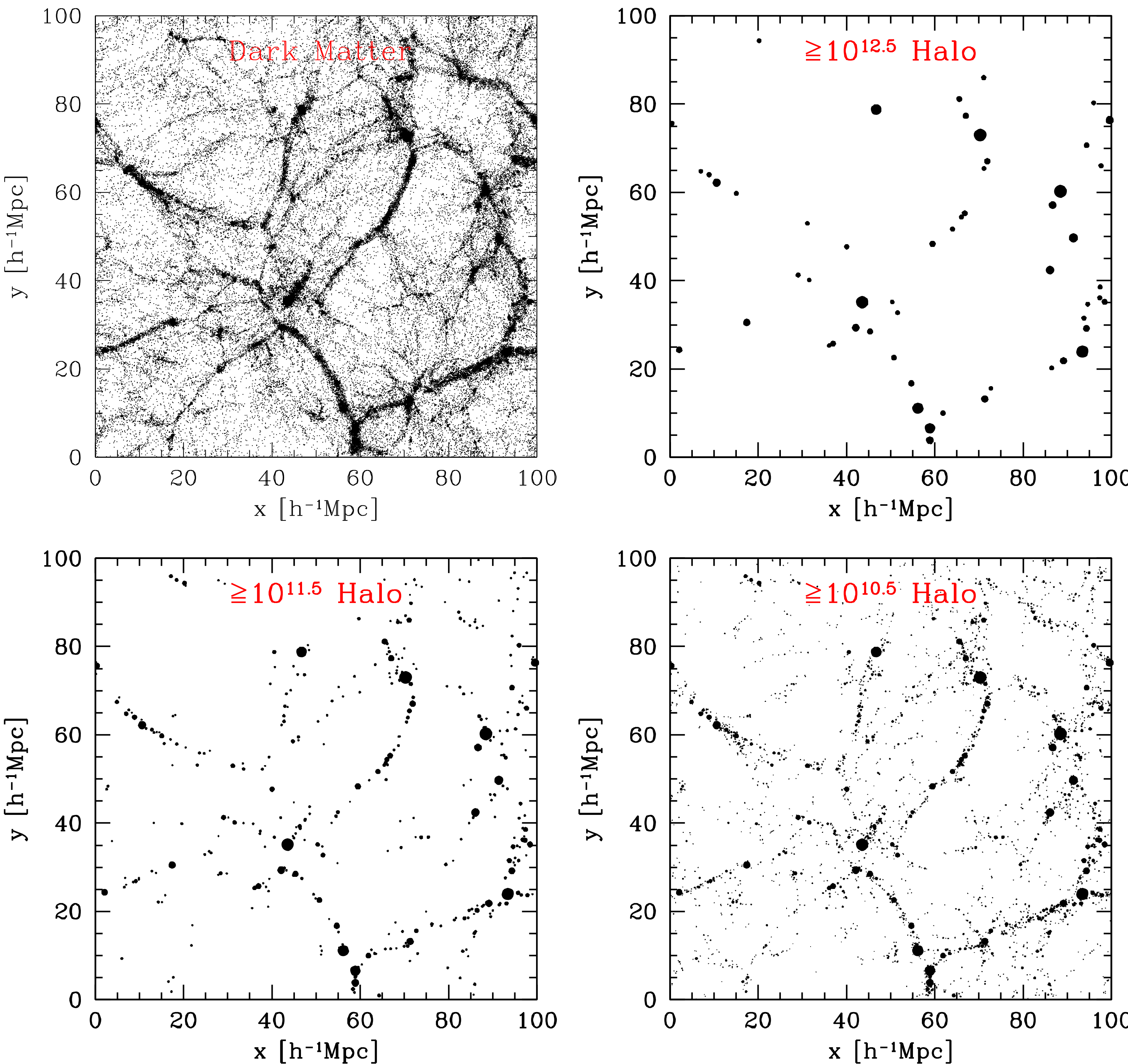
next order: 2-loop PT + $\langle J\, \delta\delta \rangle$ correlators

# PERFORMANCE OF THE EFT OF LSS




**Foreman, Perrier, Senatore, 1507.0532**

# Bias




BIAS: distribution of Galaxy and DM Halos
is a nonlinear and non local function of the DM one.

$$\delta_g = \mathcal{F}[\delta_{DM}, (\nabla_i \nabla_j \Phi)^2, \cdots]$$

Zhang, Y. et al, ApJ 706, 747, (09)

Review by Desjacques, Jeong, Schmidt, 1611.09787

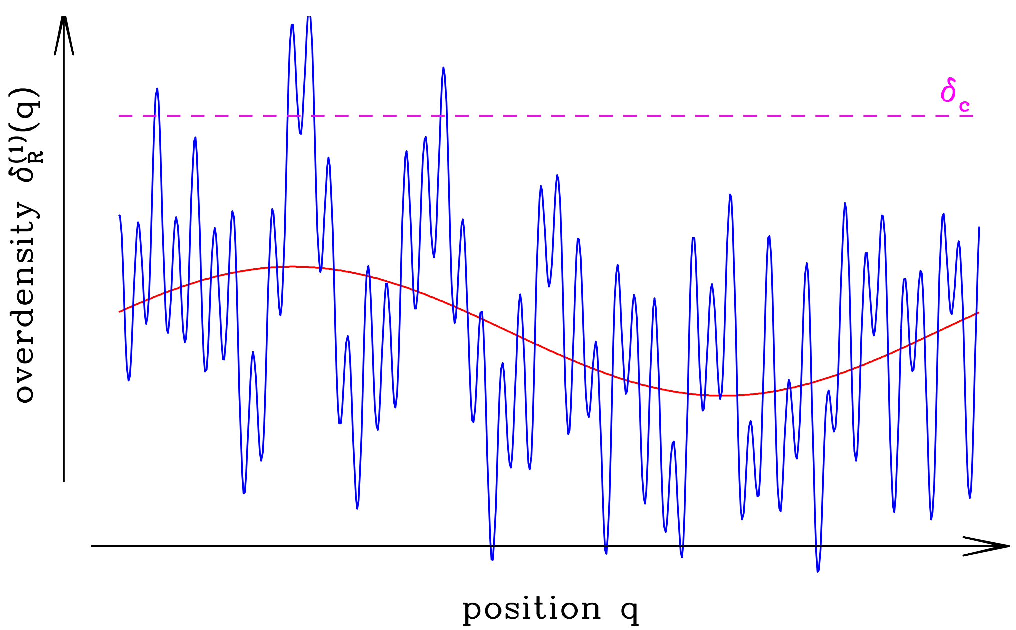
# The Perturbative Bias Expansion

$$\delta_g(\mathbf{x},\tau) = \mathcal{F}[\delta; (\partial_i\partial_j\Phi)^2; \epsilon; \cdots] \longrightarrow \sum_{\mathcal{O}} b_{\mathcal{O}}(\tau)\,\mathcal{O}(\mathbf{x},\tau)$$

non-linear, nonlocal, stochastic

bias parameters

statistical fields describing the galaxies' environment




**Effect of a long-wavelength perturbation on the density of local tracers (galaxies, halos…)**

$$\delta_g = b_1[\delta] + b_{\nabla^2\delta}[\nabla^2\delta] + [\varepsilon]$$
$$+ \frac{1}{2}b_2[\delta^2] + b_{K^2}[(K_{ij})^2] + [\varepsilon_\delta\delta]$$
$$+ \dots$$

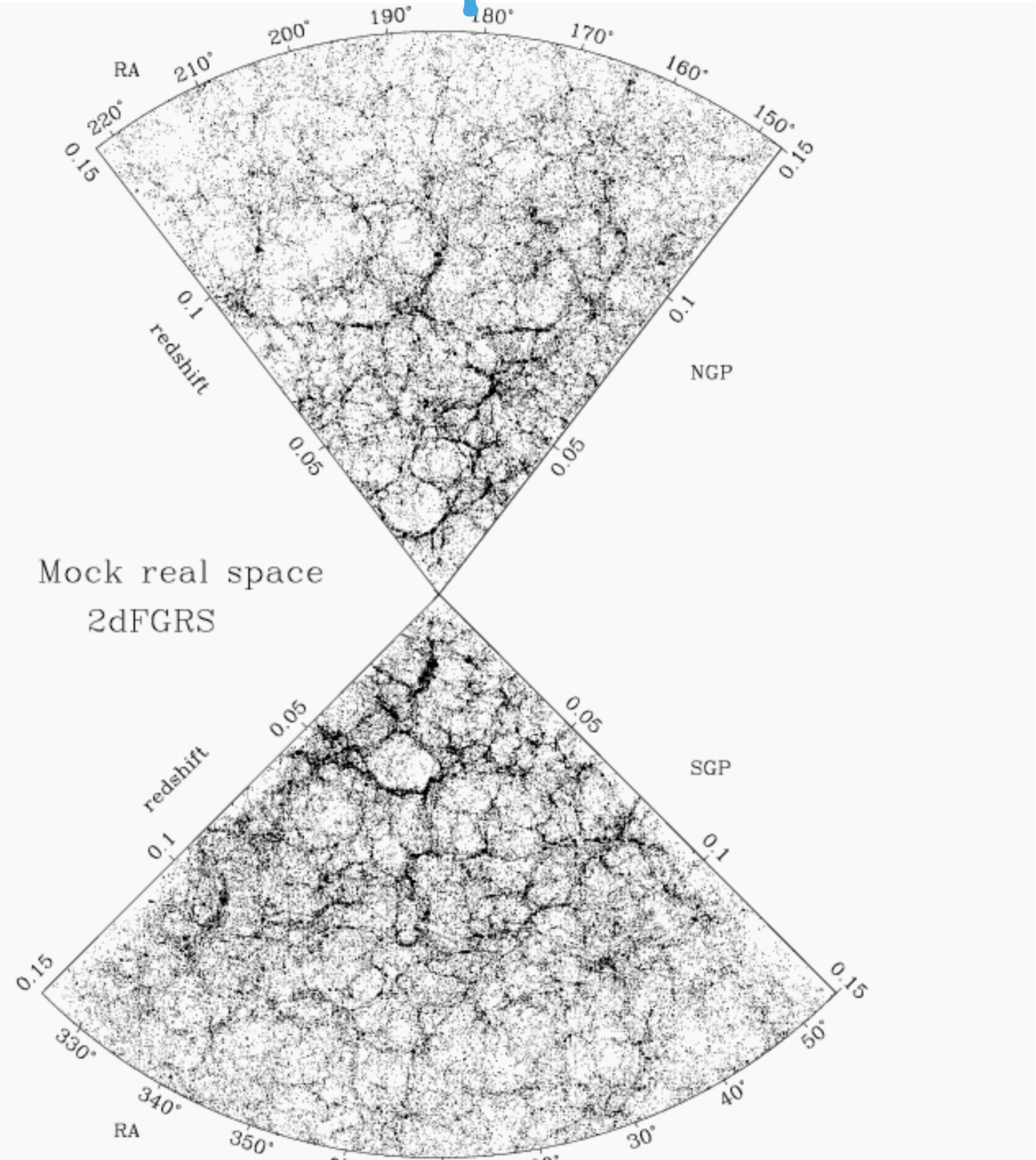
**Local: 2 derivatives of $\Phi$**

**Higher derivative**

**Stochastic**

$K_{ij} = (\partial_i\partial_j/\nabla^2 - \delta_{ij}/3)\delta$ **Tidal field**

# real-space

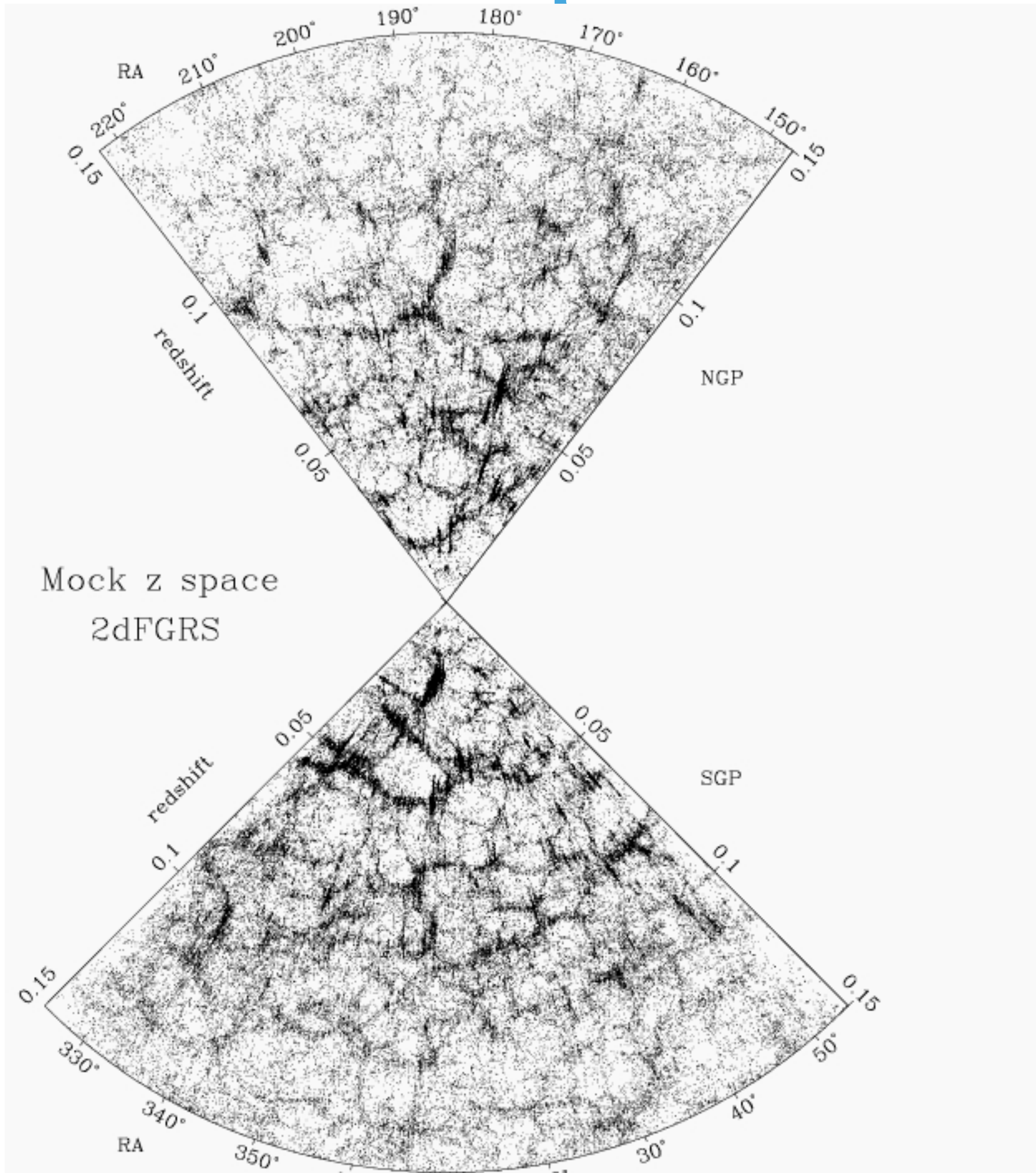


RA
220°
210°
200°
190°
180°
170°
160°
150°
0.15
0.1
0.05
redshift
NGP
Mock real space
2dFGRS
SGP
0.05
0.1
0.15
330°
340°
350°
30°
40°
50°
RA

# redshift-space

large scale: Kaiser

small scale: FoG



# IR-UV mixing in redshift space

## Real to redshift space mapping:

$$\vec{x}_n \to \vec{s}_n = \vec{x}_n + \frac{p_n^z}{a\mathcal{H}m}\hat{z}$$ (plane parallel approx.)

$$\delta_D(\vec{k}) + \delta_s(\vec{k}) = \int \frac{d^3\vec{x}}{(2\pi)^3} e^{i\vec{k}\cdot\vec{x}} \left[1 + \delta(\vec{x})\right] \exp\left[ik_z v_z(\vec{x})/\mathcal{H}\right]$$

see Scoccimarro '04,

$\langle\delta_s(\vec{x})\,\delta_s(\vec{y})\rangle$ gets contributions from terms like $\langle\delta(\vec{x})\,\delta(\vec{y})v_z^2(\vec{y})\rangle \sim \langle\delta(\vec{x})\,\delta(\vec{y})\rangle\langle v_z^2\rangle$

**even at large** $|\vec{x} - \vec{y}|$

**short scale effect**

# Large scales feel short ones!!

Problems for PT even at very large scales

# Models for RSD

$$\mu = \hat{k} \cdot \hat{z}$$

L-Kaiser: $P_g^{\rm S}(k,\mu) = (1+f\mu^2)^2 P_g(k)$

NL-Kaiser:

$$P_g^{\rm S}(k,\mu) = P_{g,\delta\delta}(k) + 2f\mu^2 P_{g,\delta\theta}(k) + f^2\mu^4 P_{\theta\theta}(k)$$

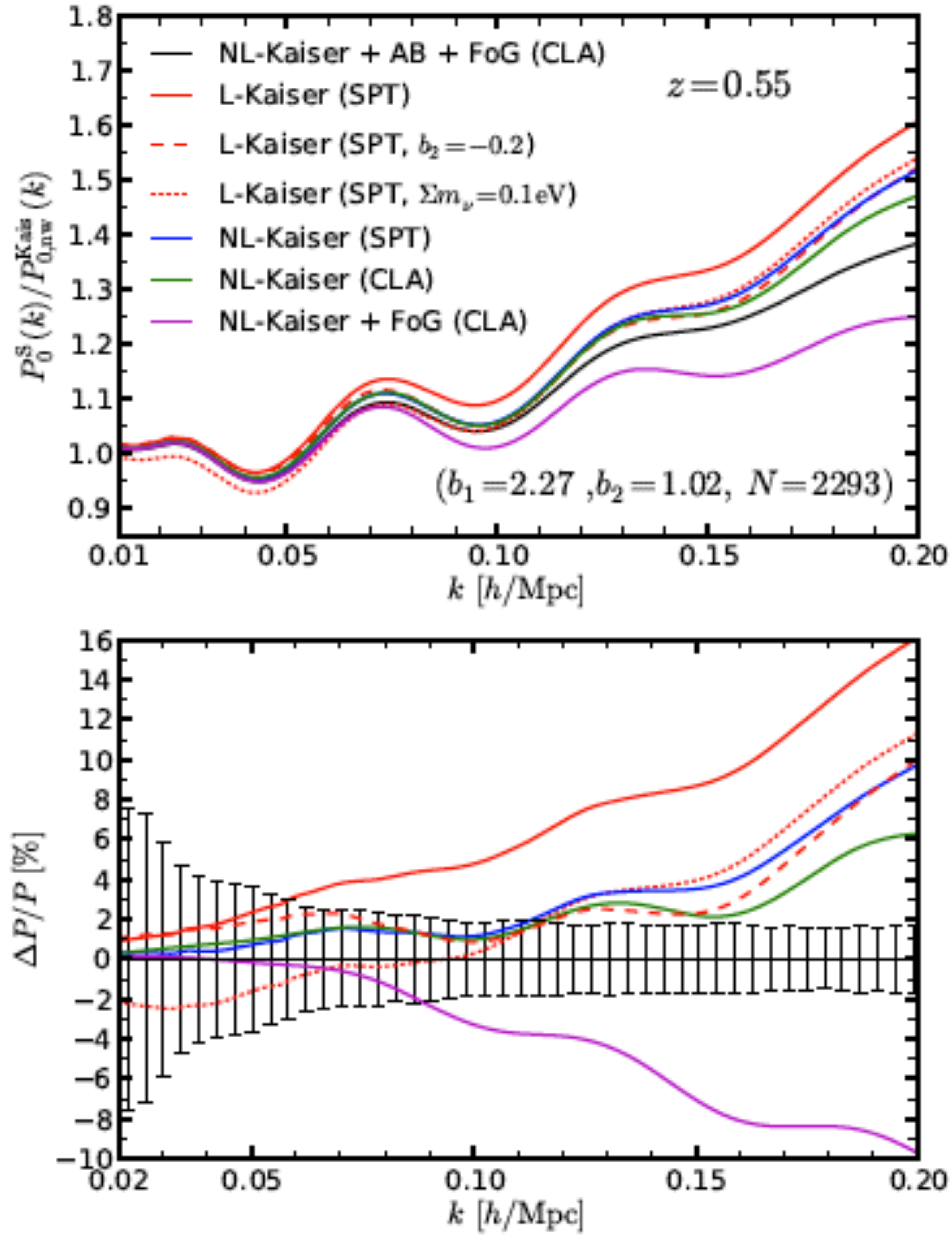
NL-Kaiser +FoG:

$$P_g^{\rm S}(k,\mu) = \exp\left(-f^2\sigma_{\rm V}^2 k^2\mu^2\right) \times \left[P_{g,\delta\delta}(k) + 2f\mu^2 P_{g,\delta\theta}(k) + f^2\mu^4 P_{\theta\theta}(k)\right]$$

NL-Kaiser +AB+FoG:

(Taruya, Nishimichi, Saito 1006.0699)

$$P_g^{\rm S}(k,\mu) = \exp\left(-f^2\sigma_{\rm V}^2 k^2\mu^2\right) \times \left[P_{g,\delta\delta}(k) + 2f\mu^2 P_{g,\delta\theta}(k) + f^2\mu^4 P_{\theta\theta}(k) + b_1^3 A(k,\mu;\beta) + b_1^4 B(k,\mu;\beta)\right],$$




Zhao et al (BOSS) 1211.3741

# Putting all together…

D'Amico et al. 1909.05271
Ivanov et al. 1909.05277
Colas et al. 1909.07951

$$P_{g,\ell}(k) = P^{\rm tree}_{g,\ell}(k) + P^{\rm 1-loop}_{g,\ell}(k) + P^{\rm noise}_{g,\ell}(k) + P^{\rm ctr}_{g,\ell}(k)$$

$$P^{\rm tree}_{g}(k,\mu) = (b_1 + f\mu^2)^2 P_{\rm lin}(k)$$

**RSD linear (Kaiser)**

$$\delta_g = b_1\delta + \frac{b_2}{2}\delta^2 + b_{\mathcal{G}_2}\mathcal{G}_2$$

$$P^{\rm noise}_{g,0}(k) = P_{\rm shot}\ , \qquad P^{\rm noise}_{g,2}(k) = 0$$

$$P^{\rm ctr,LO}_{\ell}(k) \equiv -2\, c_\ell^2\, k^2\, P_{\rm lin}(k)\ , \qquad \ell = 0,2 \qquad P^{\rm ctr,NLO}(k,\mu) \equiv \tilde{c}\, k^4\, \mu^4\, f^4\, (b_1 + f\mu^2)^2 P_{\rm lin}(k)$$

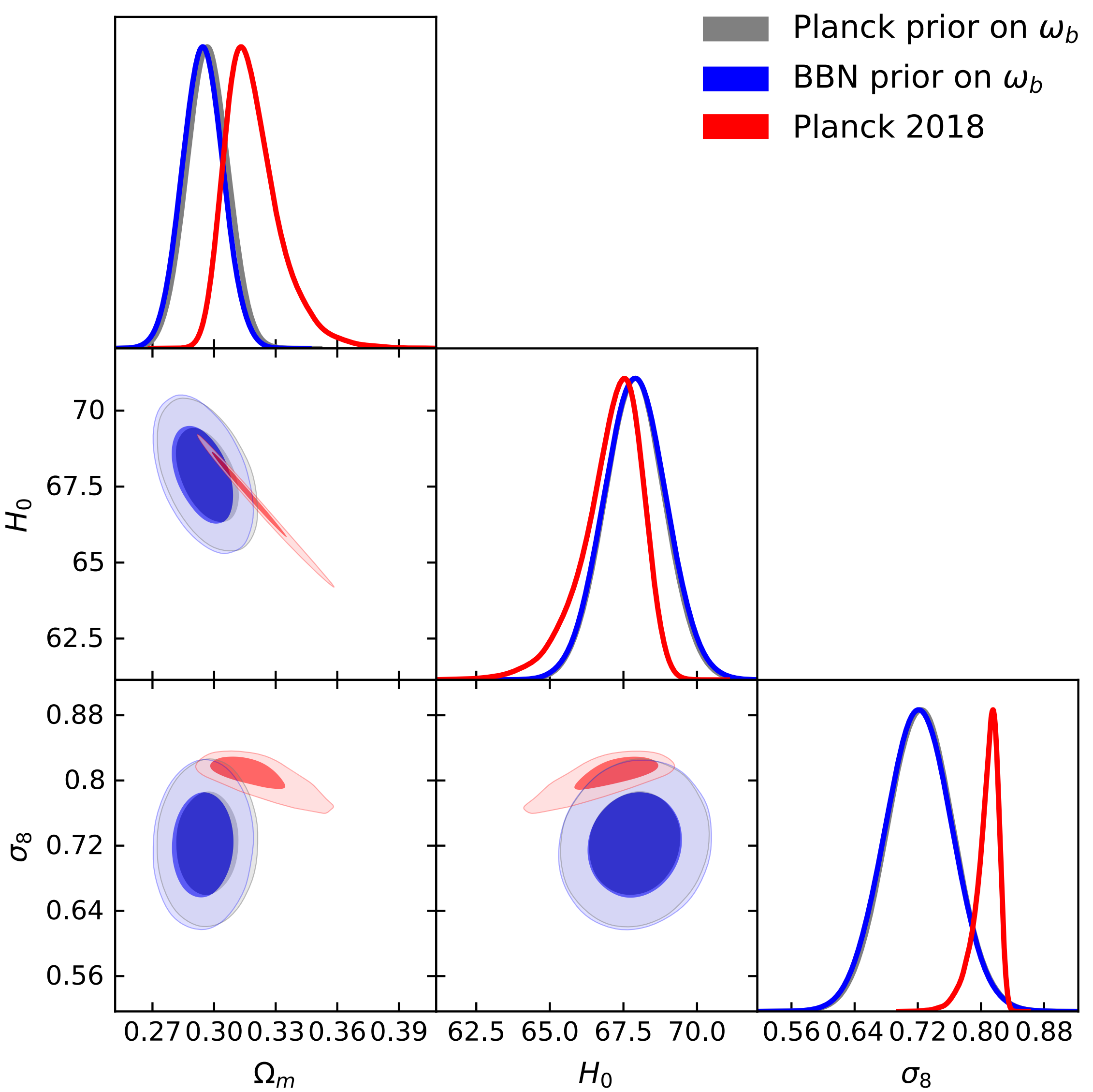
**EFT counterterms in redshift space**

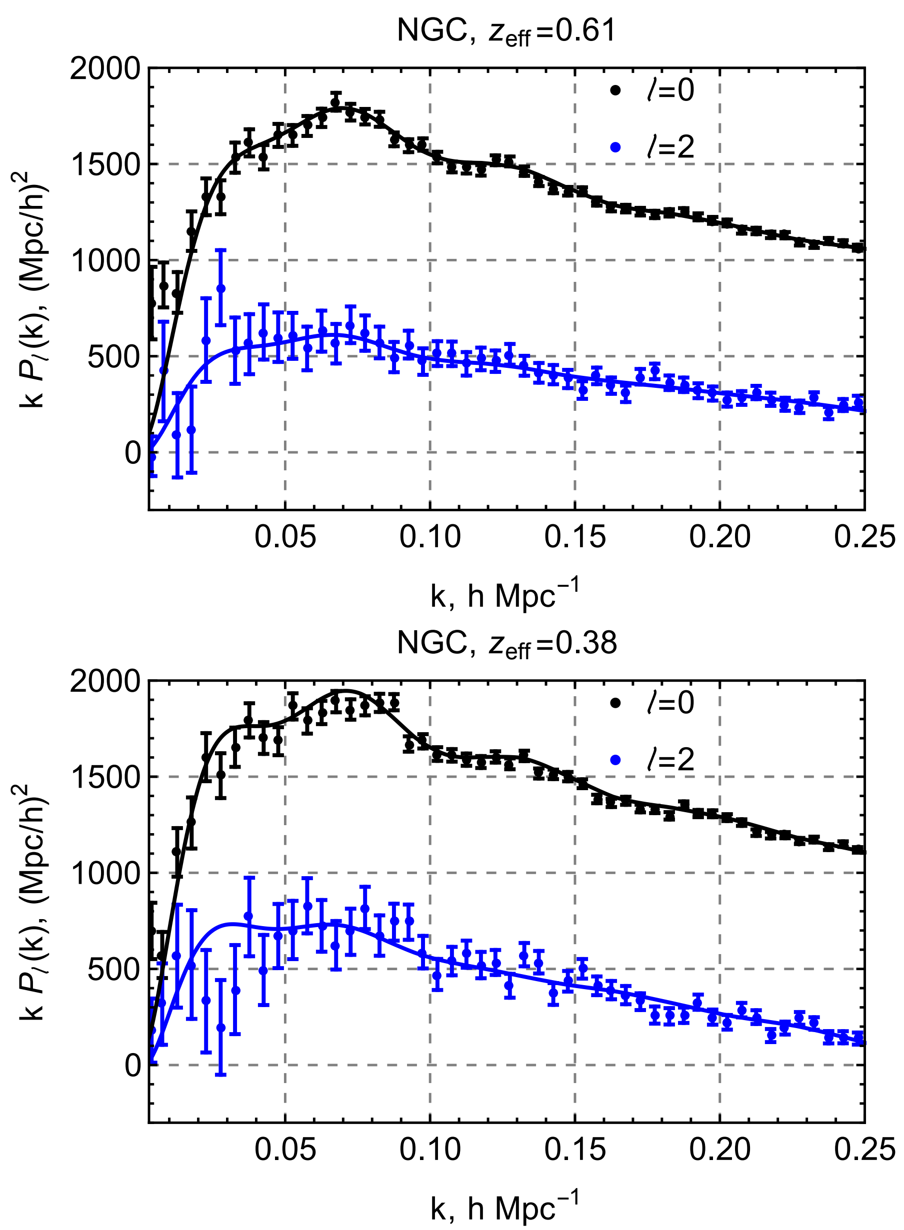
**RSD beyond kaiser**

**+ IR resummation**

# Putting all together…

**D'Amico et al. 1909.05271**
**Ivanov et al. 1909.05277**
**Colas et al. 1909.07951**



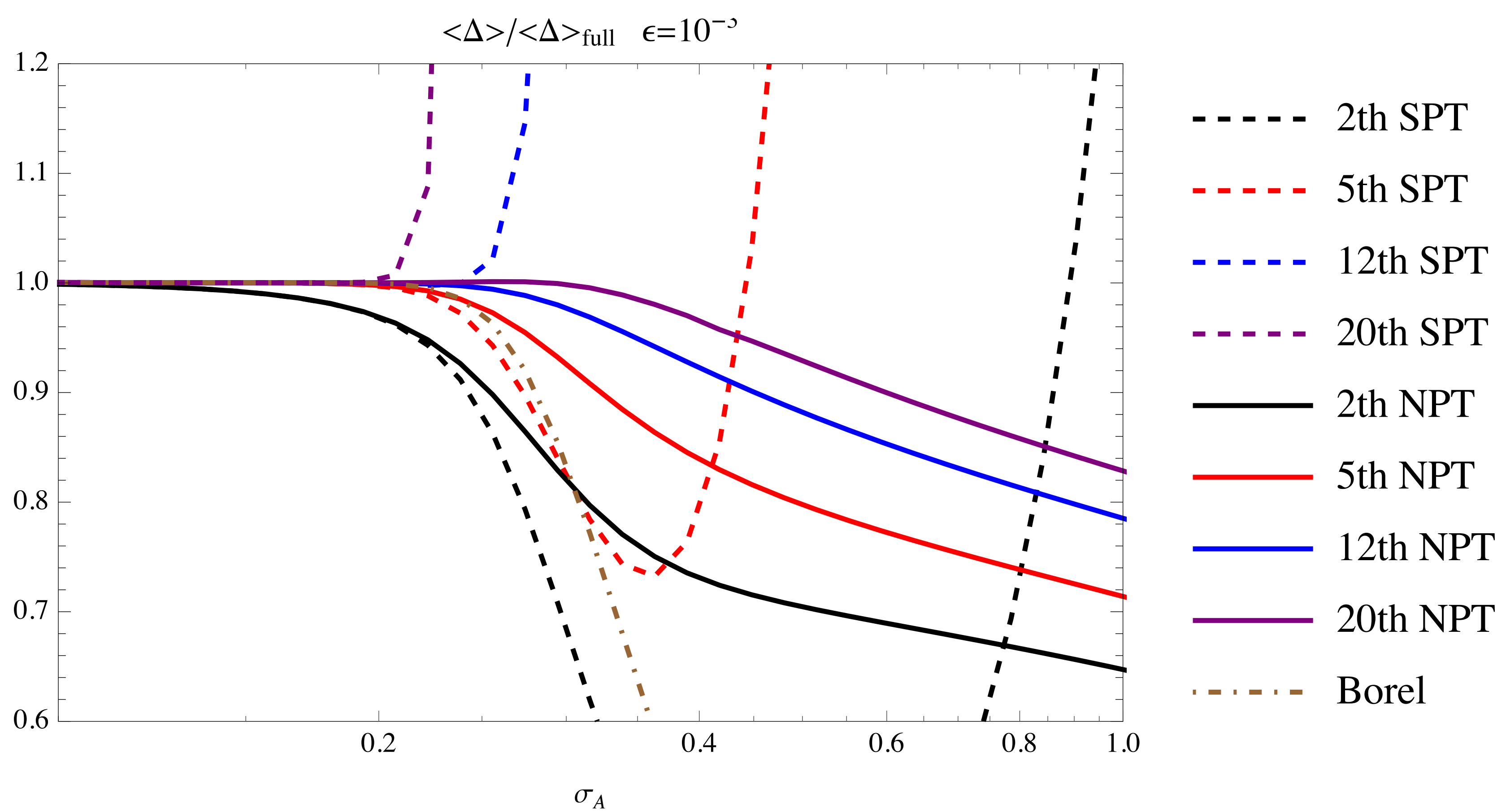

**Constraints on (some) cosmological parameters already comparable with Planck**

MP 1804.09140

# Beyond the perturbative expansion

$$\langle \Delta(\bar{x}=0, A; \epsilon) \rangle_{\sigma_A} \simeq \sum_{n=1}^{N_{\max}} c_{2n}(\epsilon) \sigma_A^{2n}$$

**SPT**



# Consistency relations

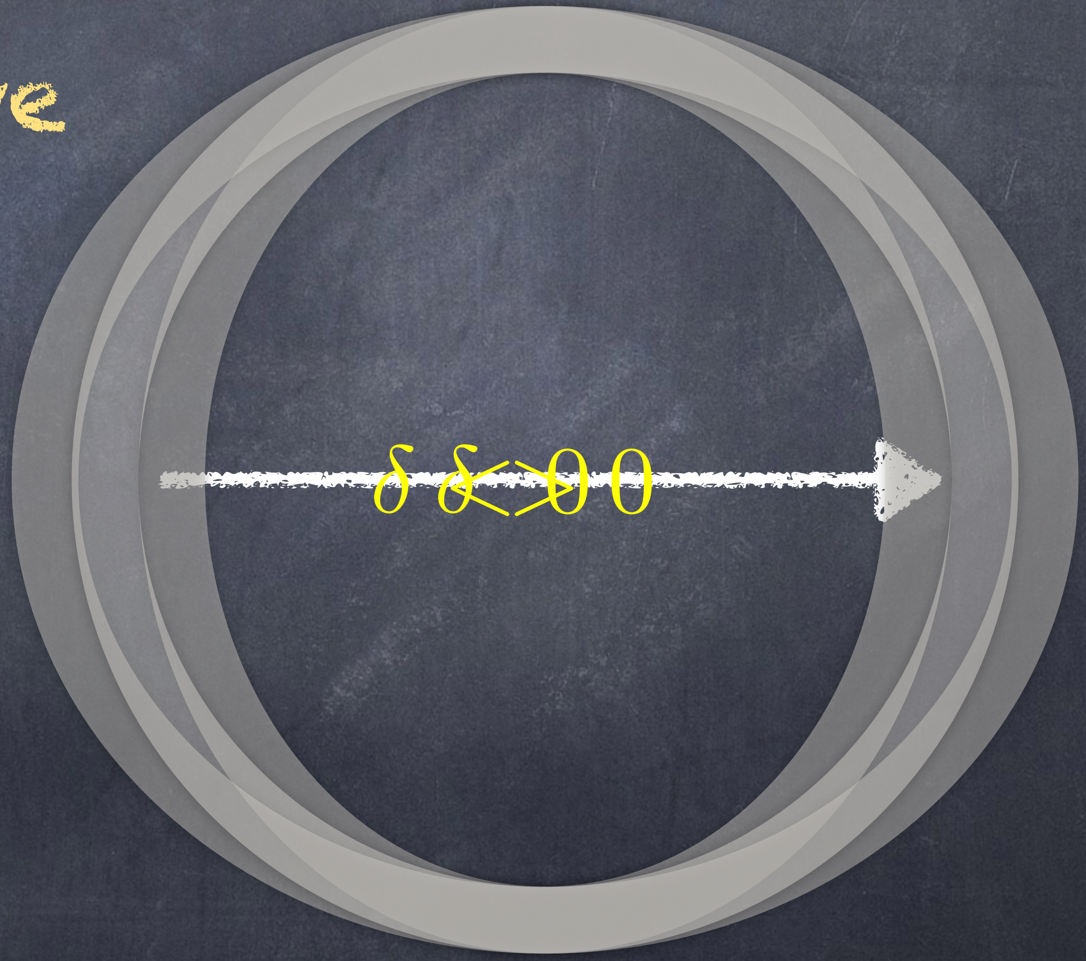
# Constant gradient displacement: O(q°) contributions

also for $\mathbf{d}_\beta = \mathbf{d}_\gamma$

angular dependence: $\propto \mu^2$

- equal times OK
- depends on the derivative of the corr. function

$$\propto \mu^2 \frac{d \log \xi(r)}{d \log r}$$



see also Baldauf et al. '15

# Equal-time sqeezed limit

$$\lim_{q/k\to 0} \frac{B_{\alpha\alpha\alpha}(q,k_+,k_-)}{P_{\alpha\alpha}(q)P_{\alpha\alpha}(k)} = -\frac{\mu^2}{b_\alpha(q)}\frac{d\log P_{\alpha\alpha}(k)}{d\log k} + O\left(\left(\frac{q}{k}\right)^0\right)$$

unchanged by nonlinearities:

bias: $b_\alpha(q;\tau_\alpha) \equiv \dfrac{P_{\alpha\alpha}(q;\tau_\alpha,\tau_\alpha)}{P_{\alpha m}(q;\tau_\alpha,\tau_\alpha)}$

let's check it

# In redshift space

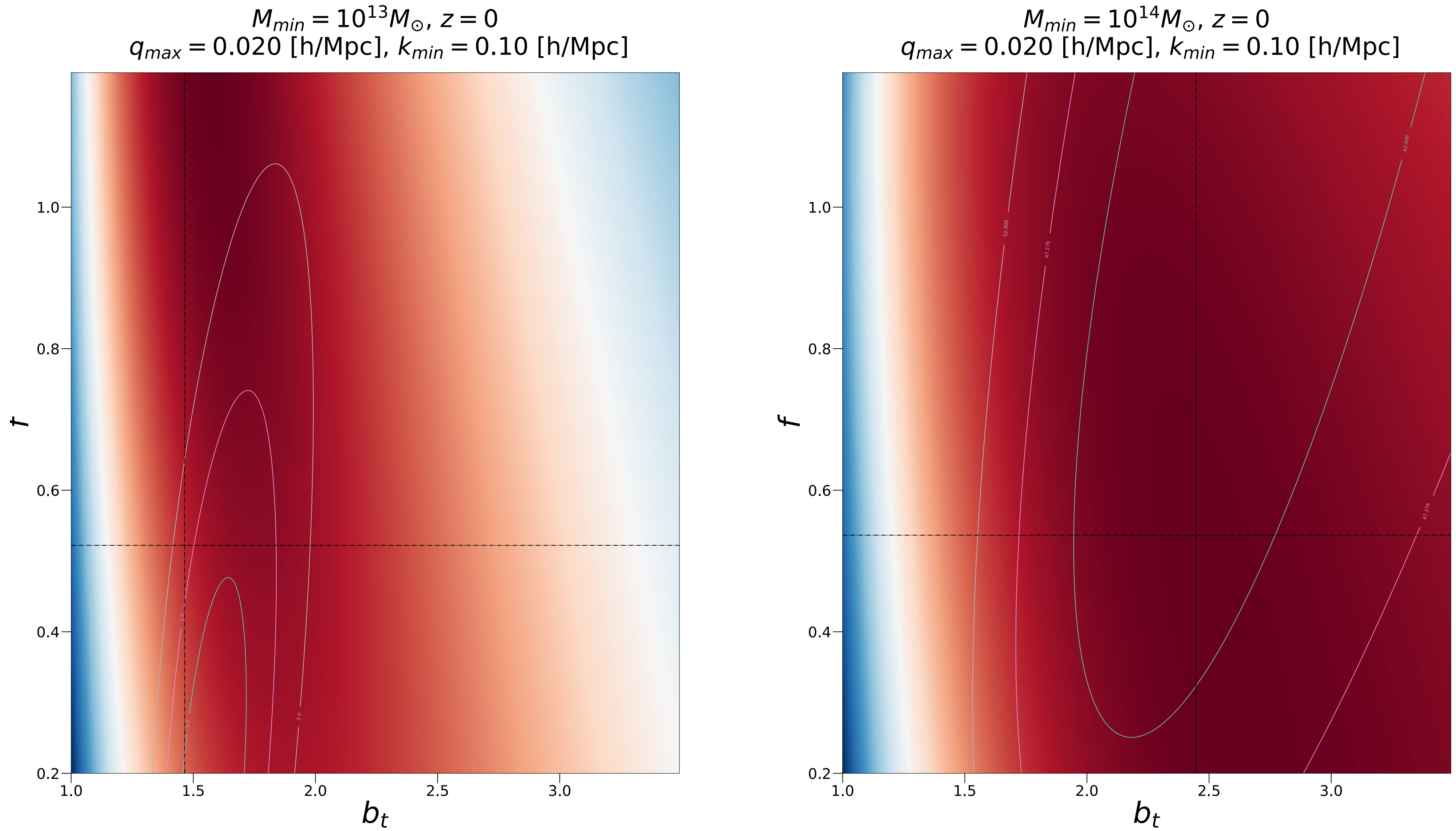
$$\frac{B^{(l=0)}(q,k)}{P^{(l_q=0)}(q)P^{(l_k=0)}(k)} = -\left[\frac{1}{3b_t} + \frac{b_t-1}{9\,b_t}\beta_t\frac{1+\frac{3}{5}\beta_t}{1+\frac{2}{3}\beta_t+\frac{1}{5}\beta_t^2}\right]\frac{d\ln P^{(l_k=0)}(k)}{d\ln k}$$
$$-\ \frac{2\beta_t[2+b_t(5+3\beta_t)]}{225\,b_t\left(1+\frac{2}{3}\beta_t+\frac{1}{5}\beta_t^2\right)}\frac{P^{(l_k=2)}(k)}{P^{(l_k=0)}(k)}\frac{d\ln P^{(l_k=2)}(k)}{d\ln k}$$

**on large scales:**

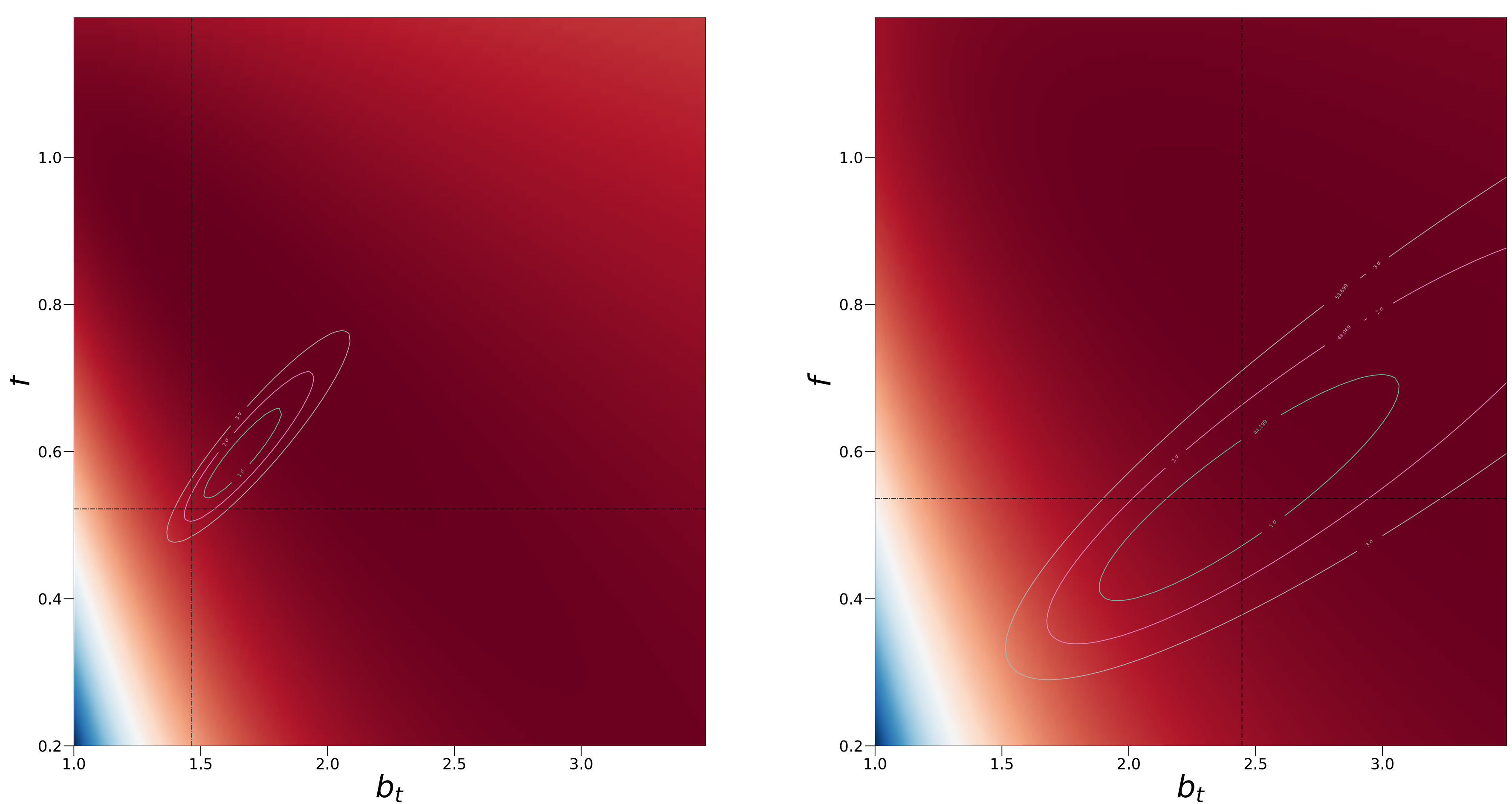
$$\frac{P_t^{(2)}(q)}{P_t^{(0)}(q)} = \frac{20(7\beta_t+3\beta_t^2)}{7(15+10\beta_t+3\beta_t^2)}$$

$$\beta_t = f/b_t$$

## Consistency relation




## Consistency relation + PS(quad)/PS(monopole)



# Consistency relations

**Ask Marco!!**


Thank YOU!!