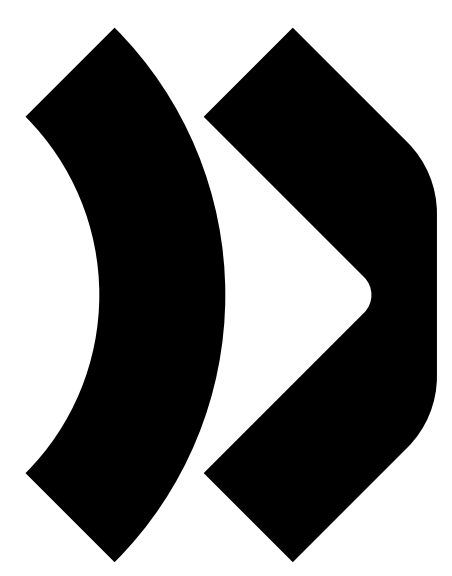
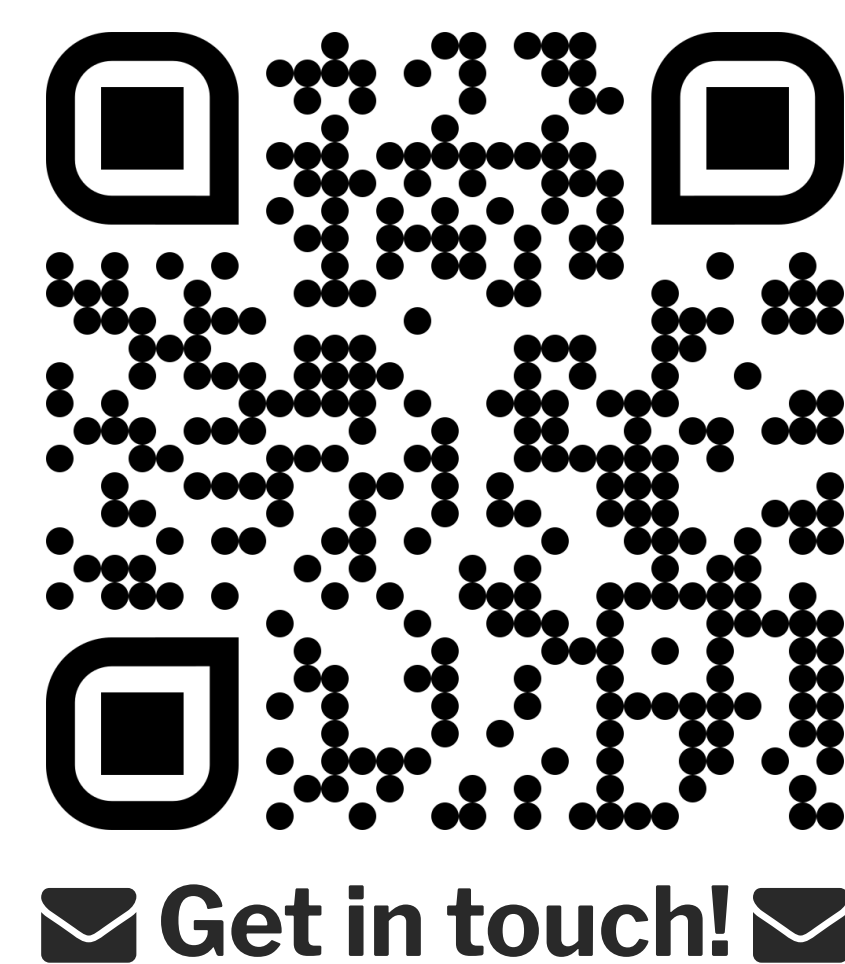




Low-cost Adaptation of a Geometry Foundation Model for Bayesian Shape Inference

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Inferring geometry from sparse, indirect, noisy measurements

Three-dimensional shapes are potentially infinitely complex, and hence inferring them from sparse, indirect, and/or noisy measurements is a **severely ill-posed inverse problem**. This motivates us to formulate this problem in a **Bayesian** manner to establish principled **uncertainties over the unknown geometry**. However, doing so in practice **requires** two ingredients:

- ▶ a **geometric parameterisation** that is sufficiently flexible and well conditioned, and
- ▶ a **strong prior** enforcing domain knowledge of plausible shapes.

Geometric inference problems like these occur throughout the physical (e.g., geophysical sub-surface modelling) and biological sciences (e.g., medical imaging). As a particular **example**, here we consider **asteroid lightcurve inversion** [1] on sparse photometry from *Gaia*, determining the shape of asteroids from only a few dozen measurements of their brightness.

Building a solution on a geometry foundation model

We show that the **latent space of a pretrained geometry foundation model**, in our case one originally built for engineering optimisation, can supply the geometric parametrisation. On top of this, we construct a **lightweight prior** using only $\mathcal{O}(100)$ **example geometries without retraining or fine-tuning** the foundation model. Which is then embedded in to an **end-to-end differentiable likelihood or simulator** to facilitate a range of efficient inference strategies for both MAP point estimates and full posteriors.

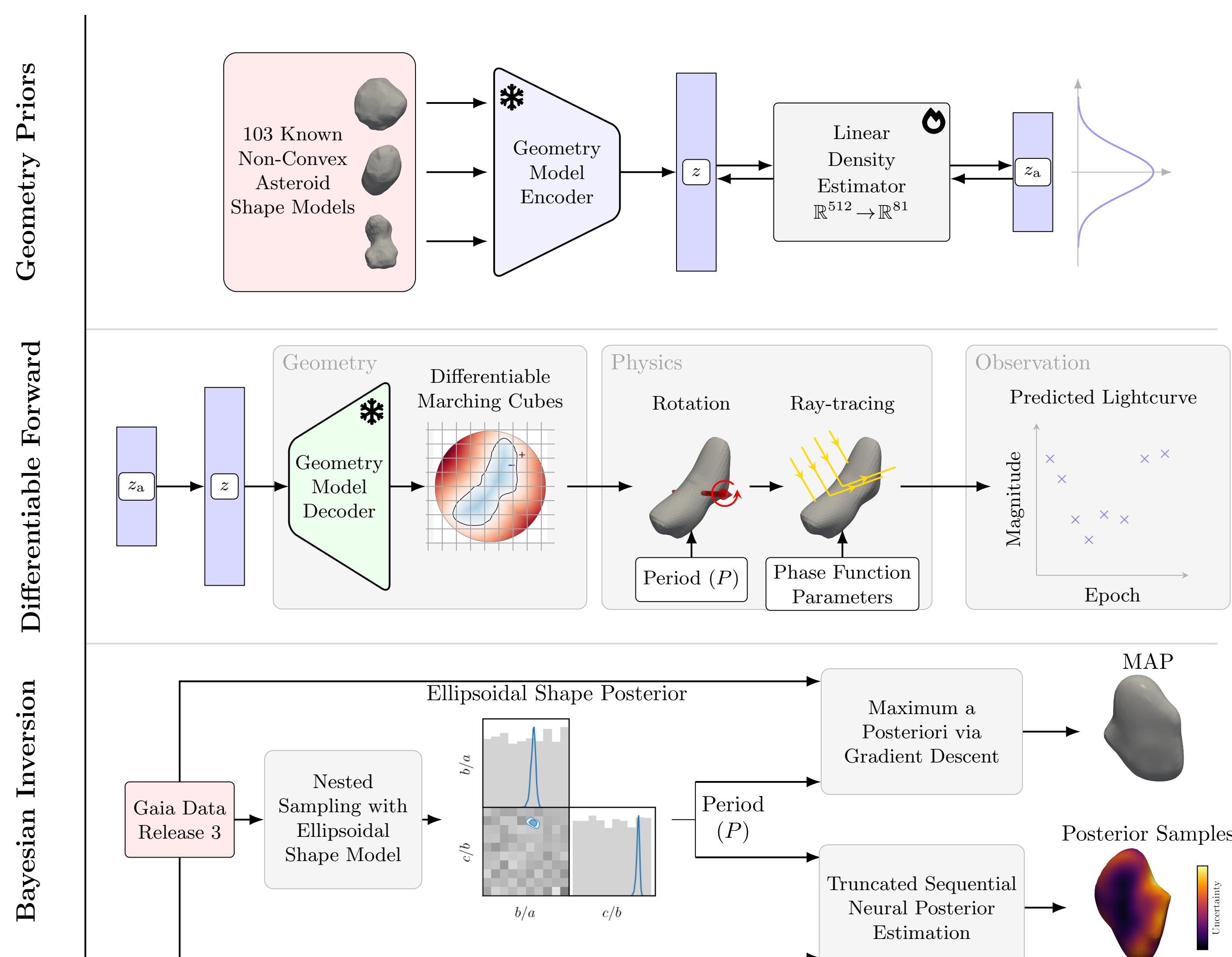


Figure 1. **Overview of the asteroid shape inference pipeline.** A prior $\pi(z_a)$ over non-convex asteroid geometries is fit to the latent encodings of 103 example asteroid shapes. The foundation-model decoder, differentiable marching cubes, and ray tracing then form an end-to-end differentiable forward model mapping asteroid shape, rotation period, and phase parameters to a lightcurve. We apply this to Bayesian lightcurve inversion of *Gaia* Data Release 3 [2] photometry, adopting a two-stage approach, first using a nested-sampling pass with an ellipsoidal shape model to determine the asteroid's period, before gradient-based MAP or TSNPE yields the final non-convex shape point estimate or posterior.

The geometry foundation model

We use a pre-existing general geometry foundation model, originally built for gradient-based optimisation, and trained on 25 million meshes. It is a variational autoencoder with a **DiffusionNet** [3] encoder, a **512-dimensional latent** $z \in \mathbb{R}^{512}$, and a **modulated residual network** [4] decoder that outputs a signed distance field.

Constructing a prior in latent space

We fit a Gaussian prior in a linear subspace of the latent space. Crucially, we derive the prior that **maximises the likelihood of the decoded geometries rather than their latent codes**, by exploiting the interpretation of the foundation model encoder as a (variational) posterior over latents given a geometry. This avoids conflating weakly-constrained latent directions with genuine shape variation.

Structurally the resultant prior resembles that of probabilistic PCA,

$$\pi(z) = \mathcal{N}(b, AA^T), \quad z = Az_a + b, \quad \pi(z_a) = \mathcal{N}(0, I_{z_a}).$$

However, the definitions of b and A now factor in both the encoder mean and covariance for each example shape.

An end-to-end differentiable forward model

The decoder is embedded in a fully differentiable forward model. The decoded signed distance field is converted to a mesh with **differentiable marching cubes** [5], then **ray traced** for self-shadowing and occlusion. For asteroids it maps latent shape, period, and phase parameters to a lightcurve via Lommel-Seeliger scattering, marginalising nuisance parameters analytically.

Inference strategies

The well-conditioned, low-dimensional parameter space our method constructs plus the differentiable forward opens up the geometric inverse problem to a **range of strategies**:

- ▶ **Gradient-based MAP**: Exploiting the differentiability for a best-fit shape point estimate in an average of 9.0 min per asteroid.
- ▶ **Truncated sequential neural posterior estimation (TSNPE)** [6, 7]: Full Bayesian posteriors in an average of 19.1 h per asteroid.

Results for asteroid lightcurve inversion

Applied to the sparse photometry of **Gaia Data Release 3** [2], gradient-based MAP allows us to infer shapes for **3,236 asteroids** without the convexity assumption standard to the field, representing a **more than an order of magnitude increase** in the number of asteroids with non-convex geometries. Furthermore, for 11 asteroids, we use TSNPE to derive the **first Bayesian uncertainties of non-convex asteroid geometries**.

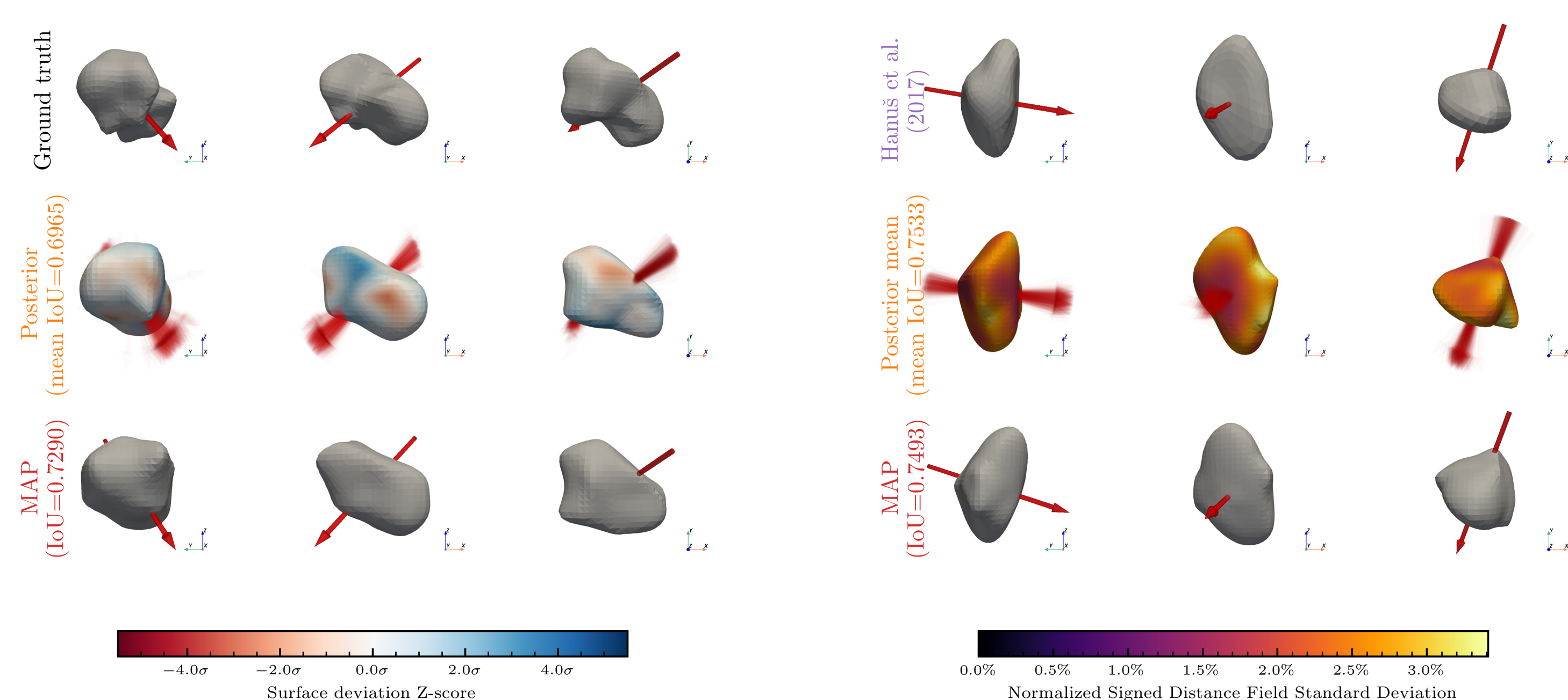


Figure 2. Shape comparison at orthogonal views: ground truth or literature model (top), our posterior-mean shape (middle), and MAP shape (bottom). **Left: Synthetic benchmark asteroid** with the poorest ground-truth agreement. Surface colour encodes the deviation Z-score from the known ground truth. **Right: Results for the asteroid 43 Ariadne.** Surface colour encodes signed-distance field standard deviation, i.e., uncertainty, as a percentage of the asteroid's longest axis. Our inferences agree well with the literature model [8] derived from combined adaptive optics imaging and dense photometry.

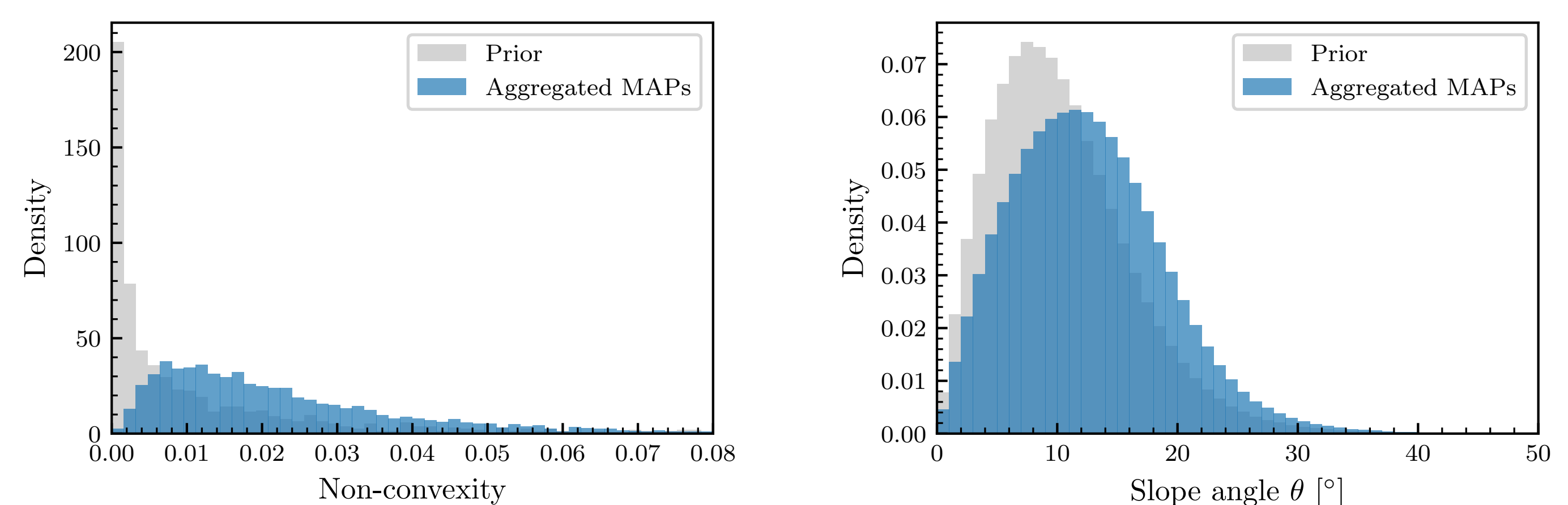


Figure 3. **Left: non-convexity distribution** for the aggregated MAP geometries compared to the prior. **Right: surface slope-angle distributions**, showing a cut-off near 30° consistent with asteroids having a cohesionless rubble-pile structure.

Deriving non-convex **geometries at scale** also lets us recover the theoretically **expected anti-correlation between asteroid size and non-convexity**. The effect is weak but statistically significant with a partial Spearman $\rho = -0.14^{+0.04}_{-0.03}$, 95% CI, when controlling for SNR.

An adaptable and general approach

Notably, our method **does not require any further training**, with the prior fit analytically. This allows **easy exploration of alternative physics modelling** in the forward model, e.g., alternative rotation models or photometry parameterisations.

Because, our approach requires only an amenable pre-existing foundation model and only a handful of domain example geometries, the recipe should **transfer directly to other geometric inverse problems**, as well as Bayesian inference in **other domains** where generative models with well-conditioned latent spaces exist. Providing a general method for principled Bayesian inference without large domain-specific training datasets.

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Paper submitted to MNRAS and hopefully appearing on arXiv soon!

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