Exclusive channels in high-energy hadronic reactions and F. Low's theorem

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1 Introduction

Low's theorem (F. E. Low, PR 110 (1958) 974) relates the amplitude for a process to that for the process with the emission of an additional photon in the limit of zero photon energy. It is a strict consequence of QFT (gauge invariance!).

$$a + b \longrightarrow c_1 + \cdots + c_n ,$$

$$a + b \longrightarrow c_1 + \cdots + c_n + \gamma(k),$$

$$k^0 \equiv \omega \longrightarrow 0 .$$

But experimentalists wanting to check Low's theorem cannot measure at $\omega = 0$. The question is then: what are the $k \neq 0$ where Low's result applies to a given accuracy?

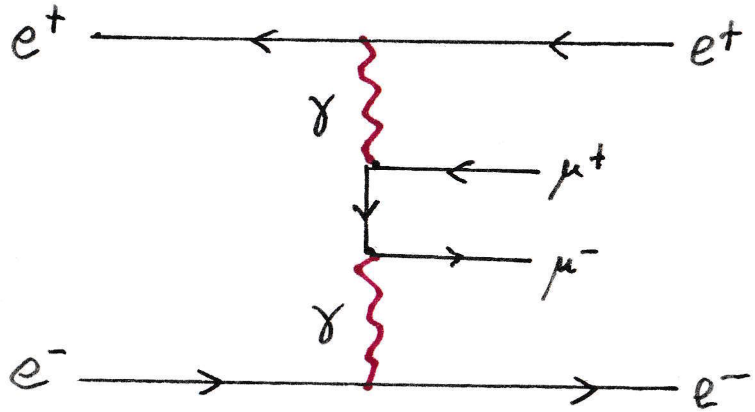
As a motto of our investigations I would like to cite a word of my teacher Walter Thirring who used to say:
"If we have an approximate result we do not know where it applies, unless we have the exact result. But then the approximate result is no longer useful."

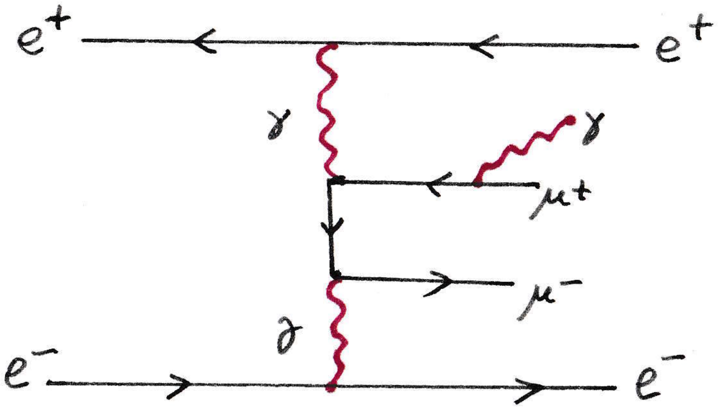
Let us first consider in this spirit purely leptonic reactions, e.g.,

$$e^+ + e^- \to e^+ + \mu^+ + \mu^- + e^- ,$$

$$e^+ + e^- \to e^+ + \mu^+ + \mu^- + e^- + \gamma .$$

Typical diagrams are as follows:





These are QED processes. We know the interaction and we can calculate in perturbation theory (nearly) everything. Emission of an additional photon is part of the radiative corrections to the basic process. We can compare the exact result with any approximation based on Low's theorem.

It may be worthwhile to invite an expert on such radiative correction calculations and ask him to talk on such comparisons.

Next we consider hadronic reactions, e.g.,

$$p + p \longrightarrow \text{many hadrons at low } p_T ,$$

$$p + p \longrightarrow \text{many hadrons at low } p_T + \gamma(k).$$

We have a theory for such reactions, QCD, but we cannot (yet) calculate the amplitudes for such processes from first principles. The same is true for <u>exclusive processes</u> at large c.m. energy $\sqrt{s}$ and small momentum transfers $|\vec{p}_T|$. But for these reactions we have developed a <u>model</u>, based on considerations in QCD, which allows us to calculate the amplitudes for the cases of no photon and one additional photon. We can then compare the "exact" model result with approximations based on Low's theorem.

2 Soft high-energy exclusive hadronic processes and the tensor-pomeron model

We are interested here in exclusive processes at large c.m. energy $\sqrt{s}$ but small momentum transfers $\sqrt{|t|}$.

Examples:

$$\left.\begin{aligned} p + p &\longrightarrow p + p, \\ \pi^{\pm} + p &\longrightarrow \pi^{\pm} + p, \end{aligned}\right\} \text{elastic scattering}$$

$$p + p \longrightarrow p + X + p, \quad \text{central exclusive production, CEP.}$$

On the same footing we have photon induced processes:

$$\gamma^{(*)} + p \rightarrow V + p, \quad V = \rho^0, \omega, \phi, J/\psi,$$

$$\gamma^{(*)} + p \longrightarrow \gamma^{(*)} + p, \quad \text{DIS structure functions.}$$

These reactions can be described with the help of

<u>exchange objects</u>:

$\mathbb{P}$ $(C = +1)$ pomeron, should dominate at large energies,

$\mathbb{O}$ $(C = -1)$ odderon (?)

$$\mathbb{R}: \quad \left.\begin{array}{l} f_{2R},\ a_{2R} \quad (C = +1) \\ \omega_R,\ \rho_R \quad (C = -1) \end{array}\right\} \text{ reggeons}$$

γ $(C = -1)$ photon

Now I could say many words about the development of various views of the pomeron. I shall not do this but only present the model for the pomeron and the other hadronic exchanges which we proposed in 2014.

C. Ewerz, M. Maniatis, O. N., Ann. Phys. 342 (2014) 31

The model is based on earlier investigations of soft high-energy reactions in QCD using functional integral techniques,

O. N., Ann. Phys. 209 (1991) 436.

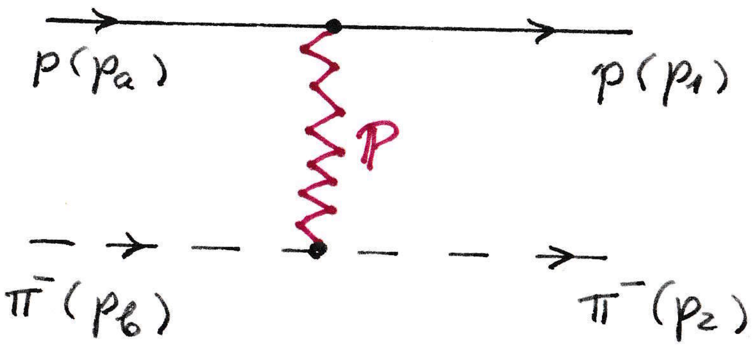
In our present tensor-pomeron model we describe:

$\mathbb{P}$ and f_{2R}, a_{2R} ($C=+1$ exchanges) as effective rank 2 symmetric tensor exchanges,

$\mathbb{O}$ and ω_R, ρ_R ($C=-1$ exchanges) as effective vector exchanges.

Example: high energy $\pi^- p$ elastic scattering.

Leading exchange: $\mathbb{P}$



$\mathbb{P}$ field:

$$\mathbb{P}_{\mu\nu}(x) = \mathbb{P}_{\nu\mu}(x),$$

$$\mathbb{P}_{\mu\nu}(x)\, g^{\mu\nu} = 0.$$

Coupling Lagrangians $\mathbb{P}pp$ and $\mathbb{P}\pi\pi$:

$$\mathcal{L}'_{\mathbb{P}pp}(x) = -3\beta_{\mathbb{P}NN}\, \mathbb{P}_{\mu\nu}(x)\, \frac{i}{2}\, \bar{\psi}_p(x)\left[\gamma^\mu \overleftrightarrow{\partial}^\nu + \gamma^\nu \overleftrightarrow{\partial}^\mu - \frac{1}{2} g^{\mu\nu} \gamma^\lambda \overleftrightarrow{\partial}_\lambda\right]\psi_p(x),$$

$$\mathcal{L}'_{\mathbb{P}\pi\pi}(x) = 2\beta_{\mathbb{P}\pi\pi}\, \mathbb{P}_{\varkappa\lambda}(x)\left(g^{\varkappa\mu} g^{\lambda\nu} - \frac{1}{4} g^{\varkappa\lambda} g^{\mu\nu}\right)$$
$$\left[\pi^-(x)\,\partial_\mu\partial_\nu\pi^+(x) + \left(\partial_\mu\partial_\nu\pi^-(x)\right)\pi^+(x) - \left(\partial_\mu\pi^-(x)\right)\left(\partial_\nu\pi^+(x)\right) - \left(\partial_\nu\pi^-(x)\right)\left(\partial_\mu\pi^+(x)\right)\right].$$

Here $\beta_{\mathbb{P}NN}$ and $\beta_{\mathbb{P}\pi\pi}$ are coupling constants with values known from phenomenology:

$$\beta_{\mathbb{P}NN} = 1.87 \text{ GeV}^{-1} \quad , \quad \beta_{\mathbb{P}\pi\pi} = 1.76 \text{ GeV}^{-1} .$$

From the coupling Lagrangians we get in the standard way the vertices where we include in addition suitable form factors.

The effective $\mathbb{P}$ propagator is chosen such as to give the observed Regge-behaviour of the amplitudes:

$$i\Delta^{(\mathbb{P})}_{\mu\nu,\kappa\lambda}(s,t) = \frac{1}{4s}\left(g_{\mu\kappa}g_{\nu\lambda} + g_{\mu\lambda}g_{\nu\kappa} - \frac{1}{2}g_{\mu\nu}g_{\kappa\lambda}\right)\left(-is\alpha'_{\mathbb{P}}\right)^{\alpha_{\mathbb{P}}(t)-1} ,$$

$$\alpha_{\mathbb{P}}(t) = 1.0808 + \alpha'_{\mathbb{P}}t \quad , \quad \alpha'_{\mathbb{P}} = 0.25 \text{ GeV}^{-2} .$$

All these numbers come from the Donnachie-Landshoff (DL) fits to the pp and πp total cross sections; see e.g.

A. Donnachie, H.G. Dosch, P.V. Landshoff, O.N., "Pomeron Physics and QCD", CUP, 2002.

The DL approach assumes, however, a vector coupling of the pomeron to protons and pions. But this view presents serious problems, as it gives, taken at face value, opposite sign for pp and $\bar{p}p$ total cross sections, which is, of course, unphysical. Our tensor pomeron gives automatically the same result for the pomeron parts of the pp and $\bar{p}p$ total cross sections, as it should be.

Finally I can mention that also in the holographic approach to QCD a tensor character for the pomeron is preferred. See e.g.

Brower, Polchinski, Strassler, Tan, JHEP 12 (2007) 005,

Domokos, Harvey, Mann, PRD 80 (2009) 126015,

Iatrakis, Ramamurti, Shuryak, PRD 94 (2016) 045005.

3 Applications of the tensor-pomeron model

By now we have made quite a number of applications of our tensor-pomeron model.

- $\gamma p \to \pi^+ \pi^- p$ Bolz, Ewerz, Maniatis, O.N., Sauter, Schöning, JHEP 01 (2015) 151.

- $p p \to p p$, spin dependence. Ewerz, Lebiedowicz, O.N., Szczurek, PL B 763 (2016) 382.

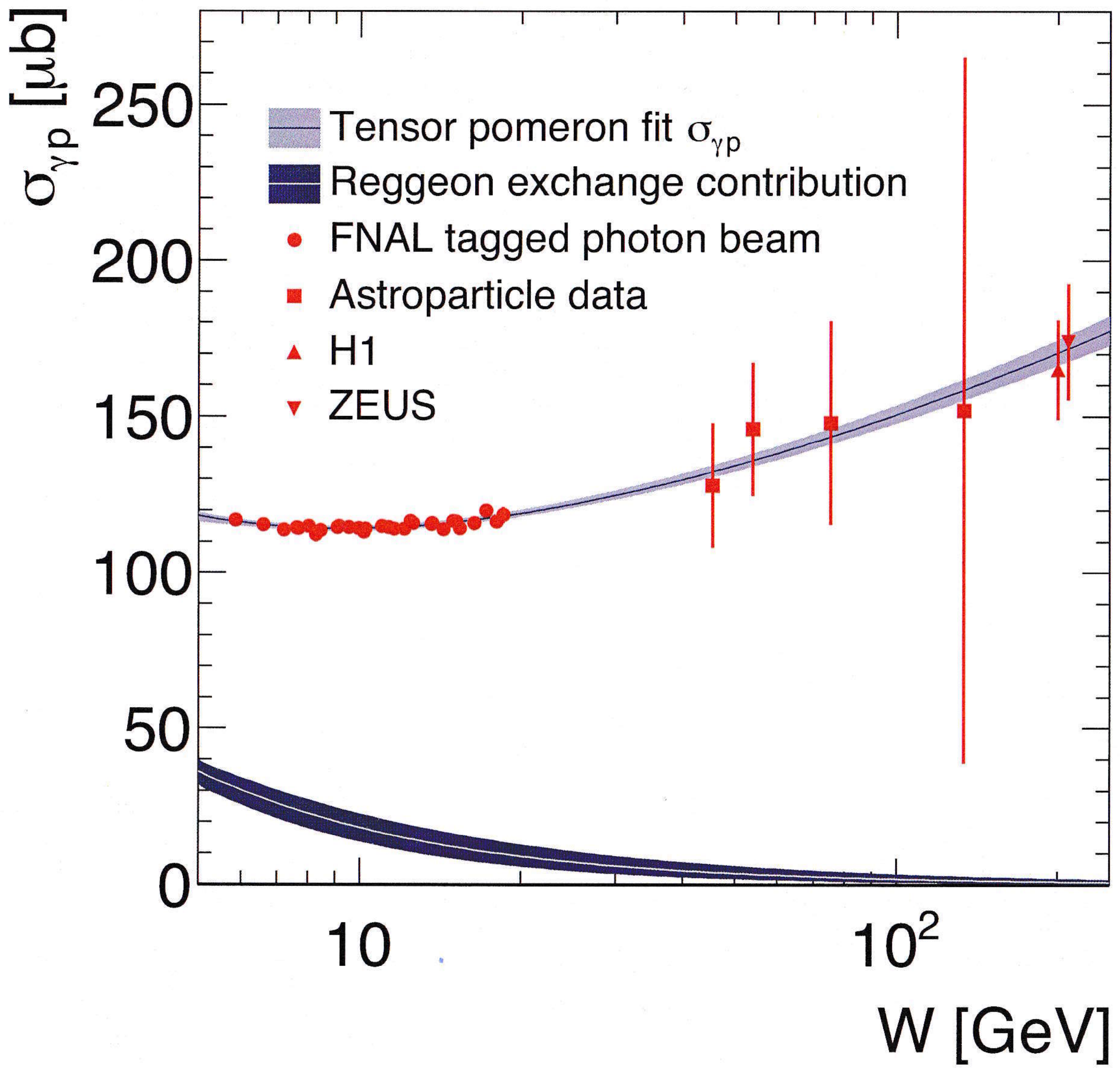
In this paper we could show that the data on the single spin flip in pp elastic scattering from the STAR experiment at $\sqrt{s} = 200$ GeV exclude a scalar character of the pomeron, but are perfectly compatible with our tensor-pomeron ansatz.

- photo production and low x DIS,

  Britzger, Ewerz, Glazov, O.N., Schmitt, PRD 100 (2019) 114007.

In this paper we could show that a vector pomeron decouples completely in the total photoabsorption cross section and in the structure functions of DIS. In contrast, with the tensor pomeron we get excellent fits. I show this for the case of $\sigma_{tot}(\gamma p)$ for real photons.

Fit results: photoproduction



- $pp \longrightarrow pXp$ CEP reactions

X	
η, η', f_0	Lebiedowicz, O.N., Szczurek, Ann. Phys. 344 (2014) 301
ρ^0	– " – , PRD 91 (2015) 074023
$\pi^+\pi^-, f_0, f_2$	– " – , PRD 93 (2016) 054015
$\pi^+\pi^-\pi^+\pi^-$	– " – , PRD 94 (2016) 034017
ρ^0 with proton diss.	– " – , PRD 95 (2017) 034036
$p\bar{p}$	– " – , PRD 97 (2018) 094027
K^+K^-	– " – , PRD 98 (2018) 014001
$K^+K^-K^+K^-$ via $\phi\phi$	– " – , PRD 99 (2019) 094034
$f_2 \longrightarrow \pi^+\pi^-$	– " – , PRD 101 (2020) 034008
$\phi \longrightarrow K^+K^-, \mu^+\mu^-$	– " – , PRD 101 (2020) 094012
$f_1(1285), f_1(1420)$	Lebiedowicz, Leutgeb, O.N., Rebhan, Szczurek, PRD 102 (2020) 114003

- CEP of $p\bar{p}$ in ultraperipheral heavy ion collisions,
  Klusek-Gawenda, Lebiedowicz, O.N., Szczurek, PRD 96 (2017) 094029

Many thanks go to all colleagues with whom I had the pleasure to collaborate on these projects.

In my opinion CEP reactions deserve detailed studies with the new detector, which is studied here, for the LHC. Let me just mention the good possibilities to search for odderon effects in CEP of single ϕ and double ϕ final states X. The ϕ's come out at low p_T and their K^+, K^- decay products are then also at low p_T. If I understand it correctly, this is the region where the new detector will have high sensitivities.

4 Soft photons in exclusive processes

I hope to have convinced you that we have a decent model for exclusive high-energy reactions at large c.m. energy $\sqrt{s}$ and small momentum transfers $\sqrt{|t|}$. The interactions, $\mathbb{P}pp$, $\mathbb{P}\pi\pi$, etc., are described by coupling Lagrangians.

It is easy to include photons: we follow the rules of minimal substitution: Whenever we have a derivative in the Lagrangian we replace it by the corresponding covariant derivative. Thus:

$$\partial_\mu \pi^\pm(x) \longrightarrow (\partial_\mu \pm ie A_\mu(x))\, \pi^\pm(x),$$

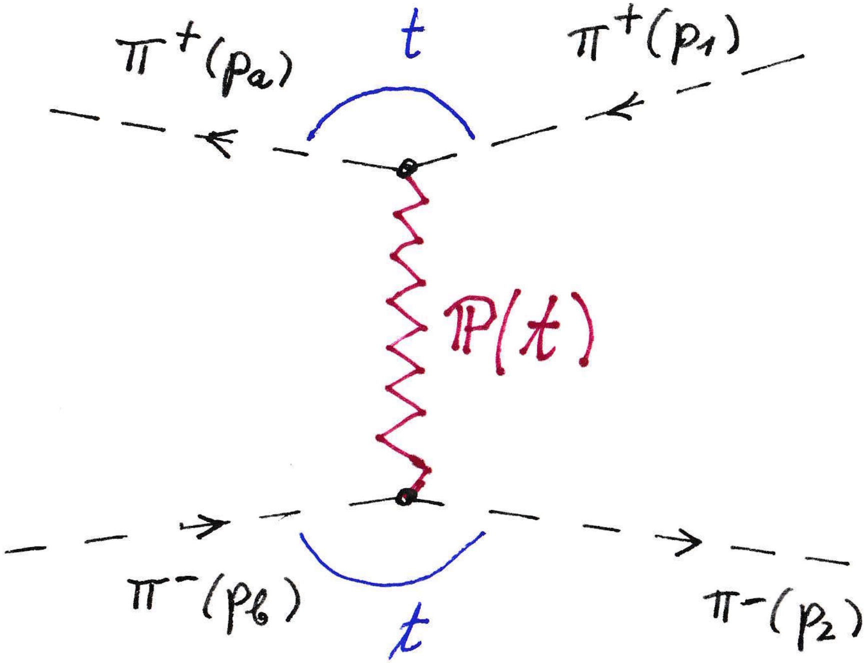
$$\partial_\mu \psi_p(x) \longrightarrow (\partial_\mu + ie A_\mu(x))\, \psi_p(x).$$

This gives without new free parameters, in a well defined way, from the $\mathbb{P}\pi\pi$ the $\mathbb{P}\gamma\pi\pi$ vertex, from $\mathbb{P}pp$ the $\mathbb{P}\gamma pp$ vertex, etc.

Now Piotr Lebiedowicz, Antoni Szczurek and myself started the project to calculate photon emission for some exclusive reactions in our tensor pomeron model. The idea is to compare our "exact" model results with approximations based on Low's theorem. We started to consider a simple, but somewhat academic, case: high energy $\pi\pi$ elastic scattering without and with photon radiation.

$$\pi^+(p_a) + \pi^-(p_b) \longrightarrow \pi^+(p_1) + \pi^-(p_2) \,,$$

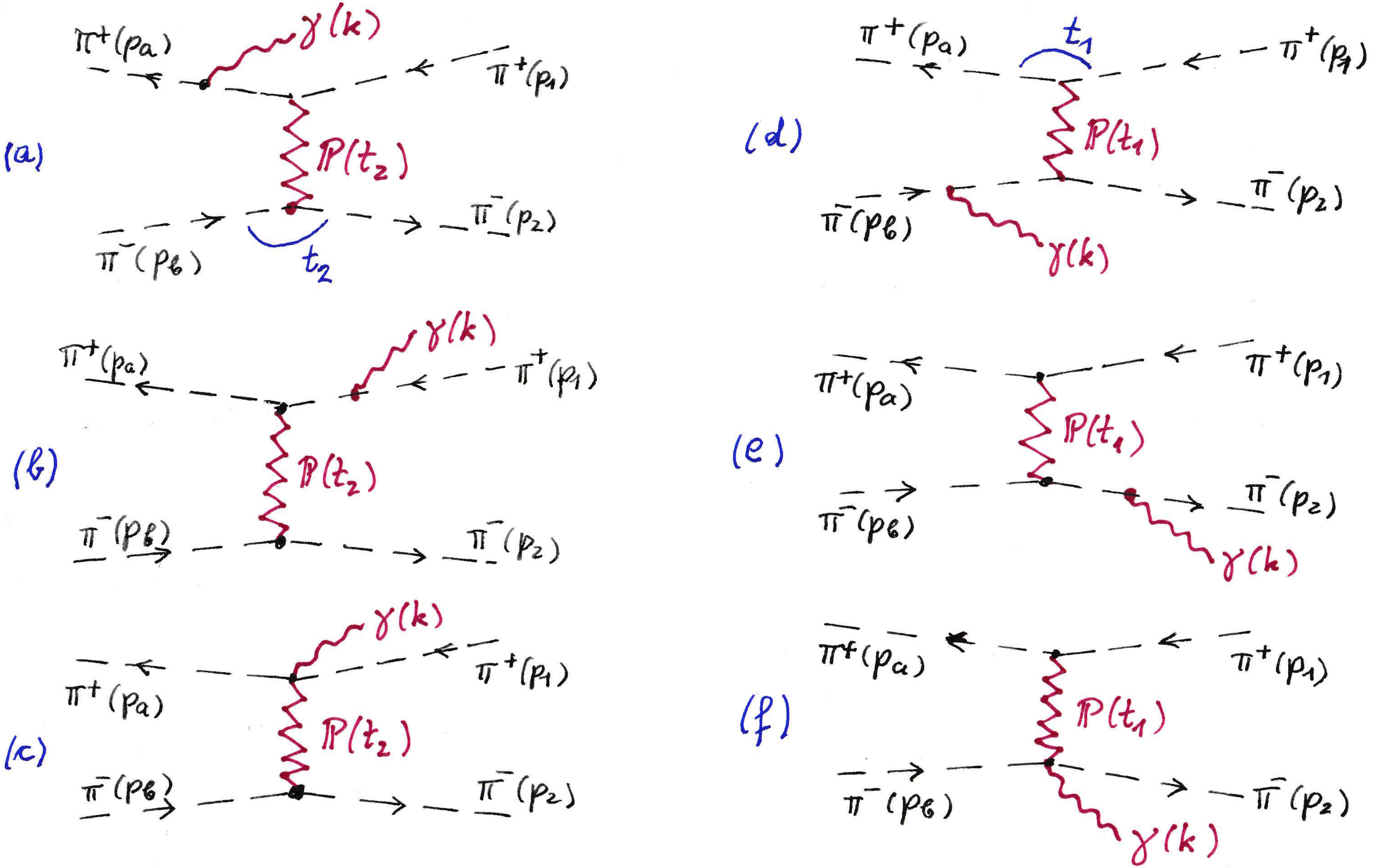
$$\pi^+(p_a) + \pi^-(p_b) \longrightarrow \pi^+(p_1) + \pi^-(p_2) + \gamma(k) \,.$$



$$p_a + p_b = p_1 + p_2$$

$$s = (p_a + p_b)^2 = (p_1 + p_2)^2$$

$$t = (p_a - p_1)^2 = (p_b - p_2)^2$$



$$s = (p_a + p_b)^2, \quad s' = (p_1+p_2)^2 \neq s \quad ; \quad t_1 = (p_a - p_1)^2, \quad t_2 = (p_b - p_2)^2 = (p_a - p_1 - k)^2 \neq t_1$$

The analytic expressions for our diagrams are as follows:

$$\langle \pi^+(p_1), \pi^-(p_2) | T | \pi^+(p_a), \pi^-(p_b) \rangle \equiv \mathcal{M}^{(0)}(s,t)$$

$$= i \left[2\beta_{\mathbb{P}\pi\pi} F_M(t) \right]^2 \frac{1}{4s} (-is\alpha'_{\mathbb{P}})^{\alpha_{\mathbb{P}}(t)-1}$$

$$\left[2(p_a+p_1, p_b+p_2)^2 - \frac{1}{2}(p_a+p_1)^2 (p_b+p_2)^2 \right]$$

$$= 2is \left[2\beta_{\mathbb{P}\pi\pi} F_M(t) \right]^2 (-is\alpha'_{\mathbb{P}})^{\alpha_{\mathbb{P}}(t)-1}$$

$$\left[1 - \frac{4m_\pi^2 - t}{s} + \frac{3}{16s^2}(4m_\pi^2 - t)^2 \right],$$

$$F_M(t) = (1 - t/m_0^2)^{-1}, \quad m_0^2 = 0.5 \text{ GeV}^2.$$

$$\mathcal{M}_\lambda^{(a)} = ie\left[2\beta_{\mathbb{P}\pi\pi}F_M(t_2)\right]^2 \frac{1}{4s}\left(-is\alpha'_{\mathbb{P}}\right)^{\alpha_{\mathbb{P}}(t_2)-1}$$

$$\left[2(p_a+p_1-k,\, p_b+p_2)^2 - \frac{1}{2}(p_a+p_1-k)^2(p_b+p_2)^2\right]\frac{(2p_a-k)_\lambda}{(p_a-k)^2-m_\pi^2},$$

$$\mathcal{M}_\lambda^{(d)} = -ie\left[2\beta_{\mathbb{P}\pi\pi}F_M(t_1)\right]^2 \frac{1}{4s}\left(-is\alpha'_{\mathbb{P}}\right)^{\alpha_{\mathbb{P}}(t_1)-1}$$

$$\left[2(p_a+p_1,\, p_b+p_2-k)^2 - \frac{1}{2}(p_a+p_1)^2(p_b+p_2-k)^2\right]\frac{(2p_b-k)_\lambda}{(p_b-k)^2-m_\pi^2},$$

Gauge invariance is fulfilled:

$$\left(\mathcal{M}_\lambda^{(a)} + \mathcal{M}_\lambda^{(b)} + \mathcal{M}_\lambda^{(c)}\right)k^\lambda = 0,$$

$$\left(\mathcal{M}_\lambda^{(d)} + \mathcal{M}_\lambda^{(e)} + \mathcal{M}_\lambda^{(f)}\right)k^\lambda = 0.$$

Approximations based on Low's result for $\omega = k^0 \to 0$:

$$\mathcal{M}_\lambda^{\mathrm{Low}\,1} = e\,\mathcal{M}^{(0)}(s,t_2)\left[-\frac{p_{a\lambda}}{(p_a\cdot k)} + \frac{p_{1\lambda}}{(p_1\cdot k)}\right]$$

$$+\,e\,\mathcal{M}^{(0)}(s,t_1)\left[\frac{p_{b\lambda}}{(p_b\cdot k)} - \frac{p_{2\lambda}}{(p_2\cdot k)}\right],$$

$$\mathcal{M}_\lambda^{\mathrm{Low}\,2} = e\,\mathcal{M}^{(0)}(s,t')\left[-\frac{p_{a\lambda}}{(p_a\cdot k)} + \frac{p_{1\lambda}}{(p_1\cdot k)} + \frac{p_{b\lambda}}{(p_b\cdot k)} - \frac{p_{2\lambda}}{(p_2\cdot k)}\right],$$

$$t' = \min(t_1, t_2).$$

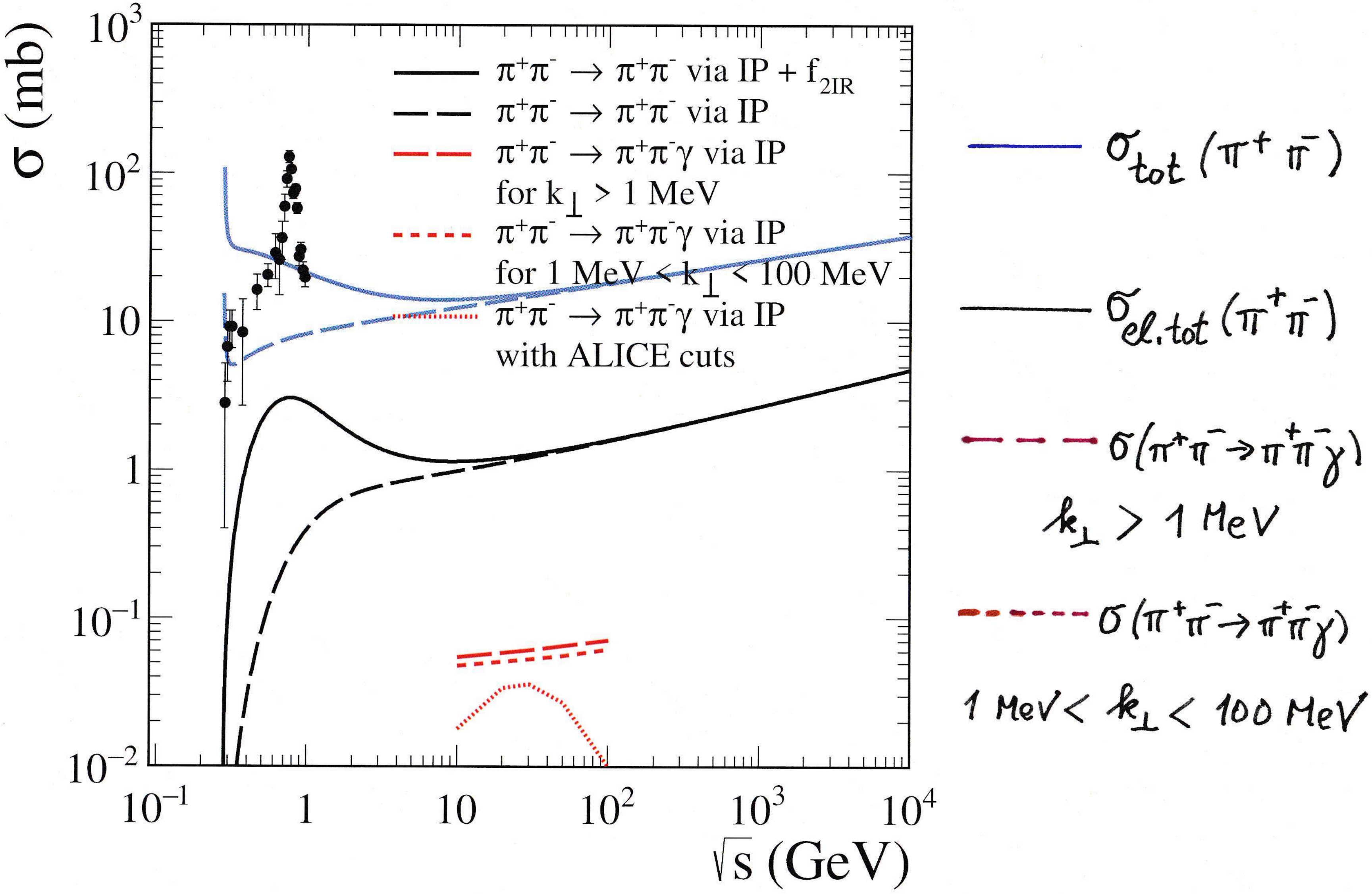
For $k \to 0$ we have $\mathcal{M}_\lambda^{\mathrm{Low}\,1} \to \mathcal{M}_\lambda^{\mathrm{Low}\,2}$ and both go to the exact result $\mathcal{M}_\lambda$. But for finite k there is no unique "Low result".

5 Numerical results for $\pi\pi$ scattering

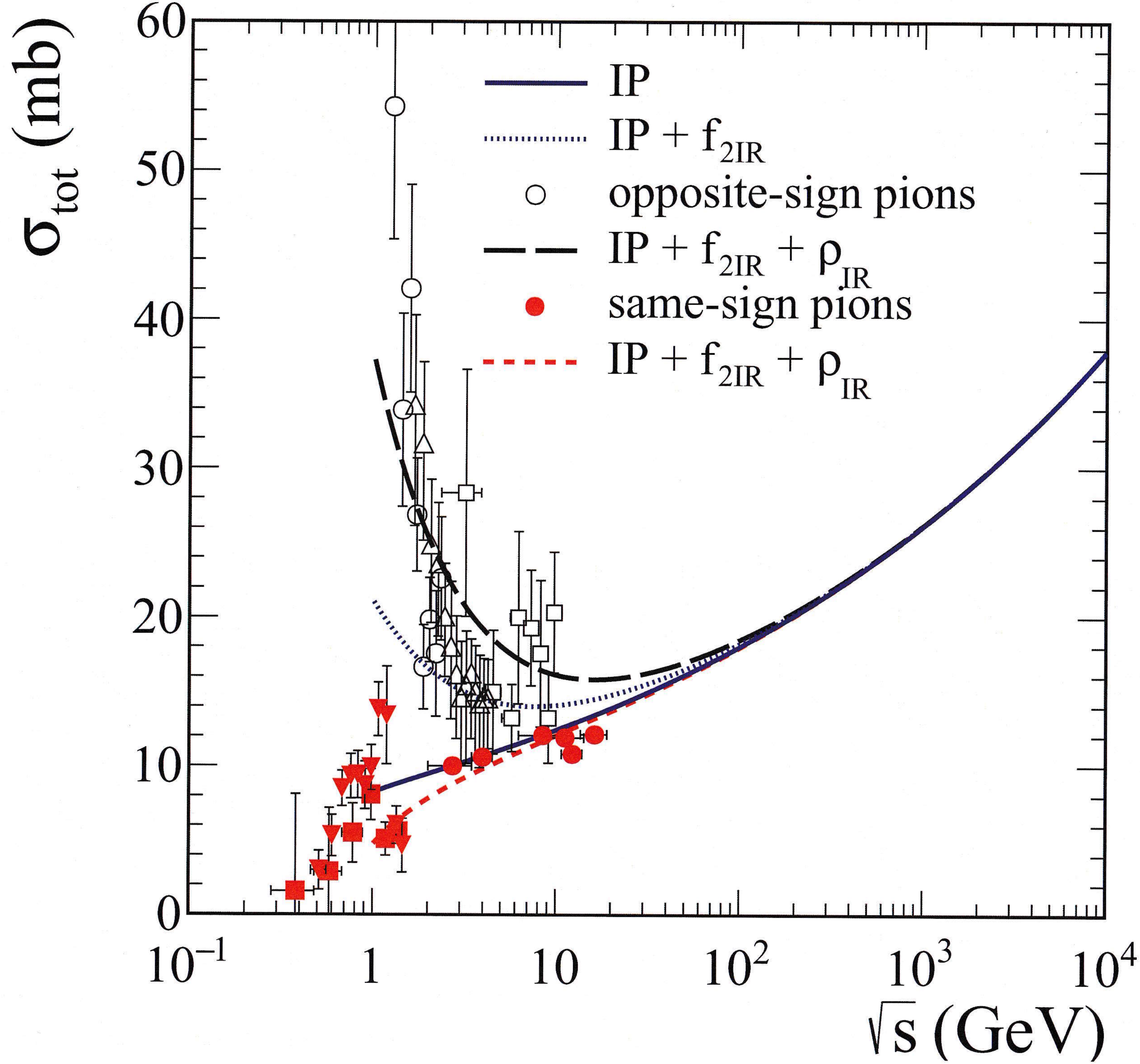
I am very grateful to Piotr for producing in a short time nice results which I can now show you.

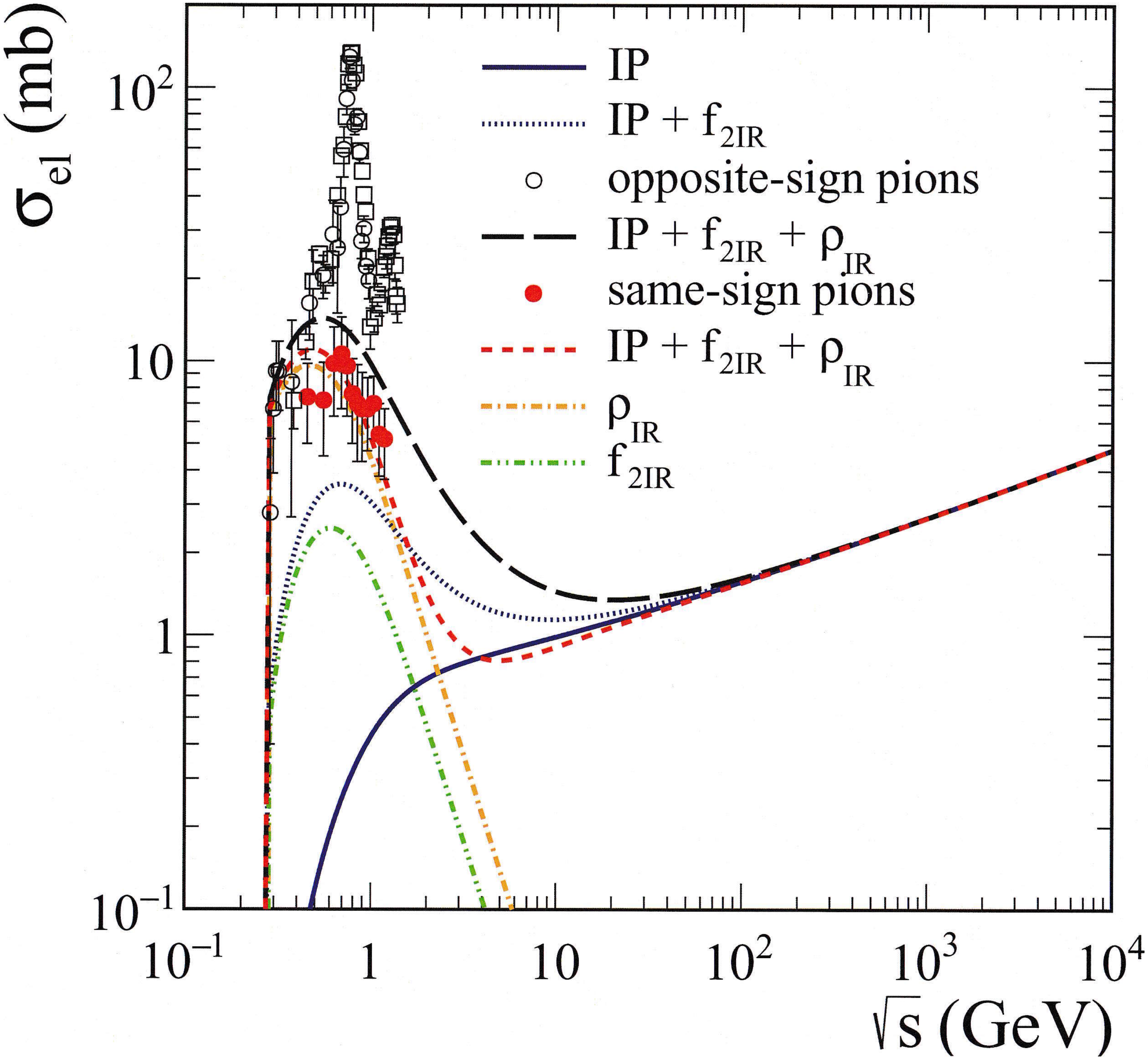
- Total and total elastic cross sections for $\pi\pi$ scattering
- Rapidity distribution of γ in $\pi^+\pi^-\gamma$.
- Distributions in ω and $k_\perp$.
- Acoplanarity distribution.
- Twodimensional distribution of

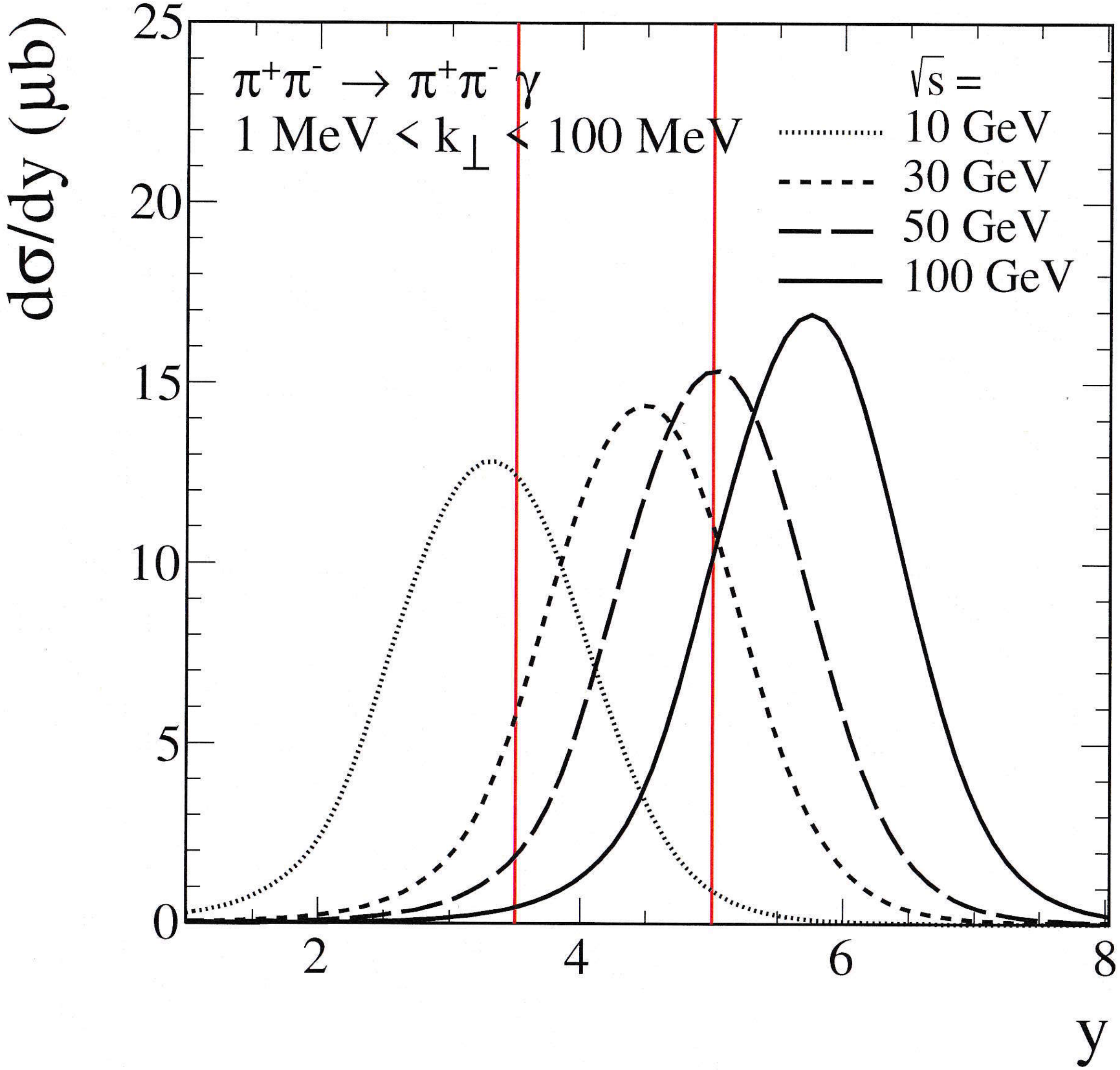
$$R(\omega, k_\perp) = \sigma^{LOW}(\omega, k_\perp) / \sigma(\omega, k_\perp).$$

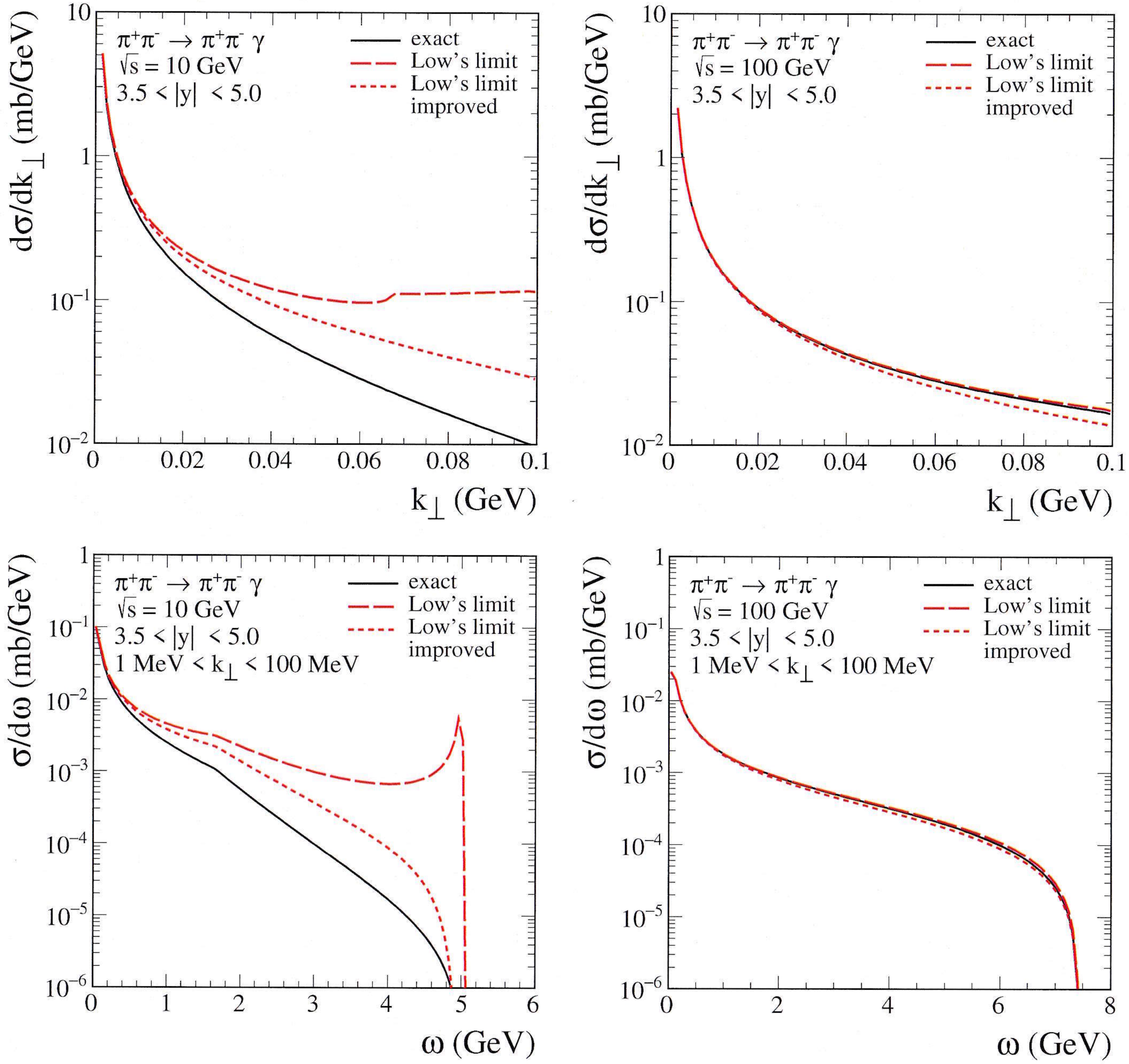


e $\pi^+\pi^-$ scattering cross sections without and with photon radiation as a function o
nass energy $\sqrt{s}$. Our results for elastic scattering are compared with the experime


IP
IP + f2IR
opposite-sign pions
IP + f2IR + ρIR
same-sign pions
IP + f2IR + ρIR
σtot (mb)
0
10
20
30
40
50
60
10−1
1
10
10²
10³
10⁴
√s (GeV)


σ_{el} (mb)
10^2
10
1
10^{-1}
10^{-1}
1
10
10^2
10^3
10^4
$\sqrt{s}$ (GeV)
IP
IP + f_{2IR}
opposite-sign pions
IP + f_{2IR} + ρ_{IR}
same-sign pions
IP + f_{2IR} + ρ_{IR}
ρ_{IR}
f_{2IR}


dσ/dy (μb)
25
20
15
10
5
0
2
4
6
8
y
π+π- → π+π- γ
1 MeV < k⊥ < 100 MeV
√s =
10 GeV
30 GeV
50 GeV
100 GeV



Low 1
Low 2

FIG. 3. The distributions in transverse momentum of the photon and in the energy of the photon. for the $\pi^+\pi^- \to \pi^+\pi^-\gamma$ reaction calculated for $\sqrt{s} = 10$ GeV (left) and for $\sqrt{s} = 100$ GeV (right). The calculations were done for (1.1). The black solid line corresponds to exact model, the red long-dashed line corresponds to Eq. (1.2), the red short-dashed line corresponds to Eq. (1.3).

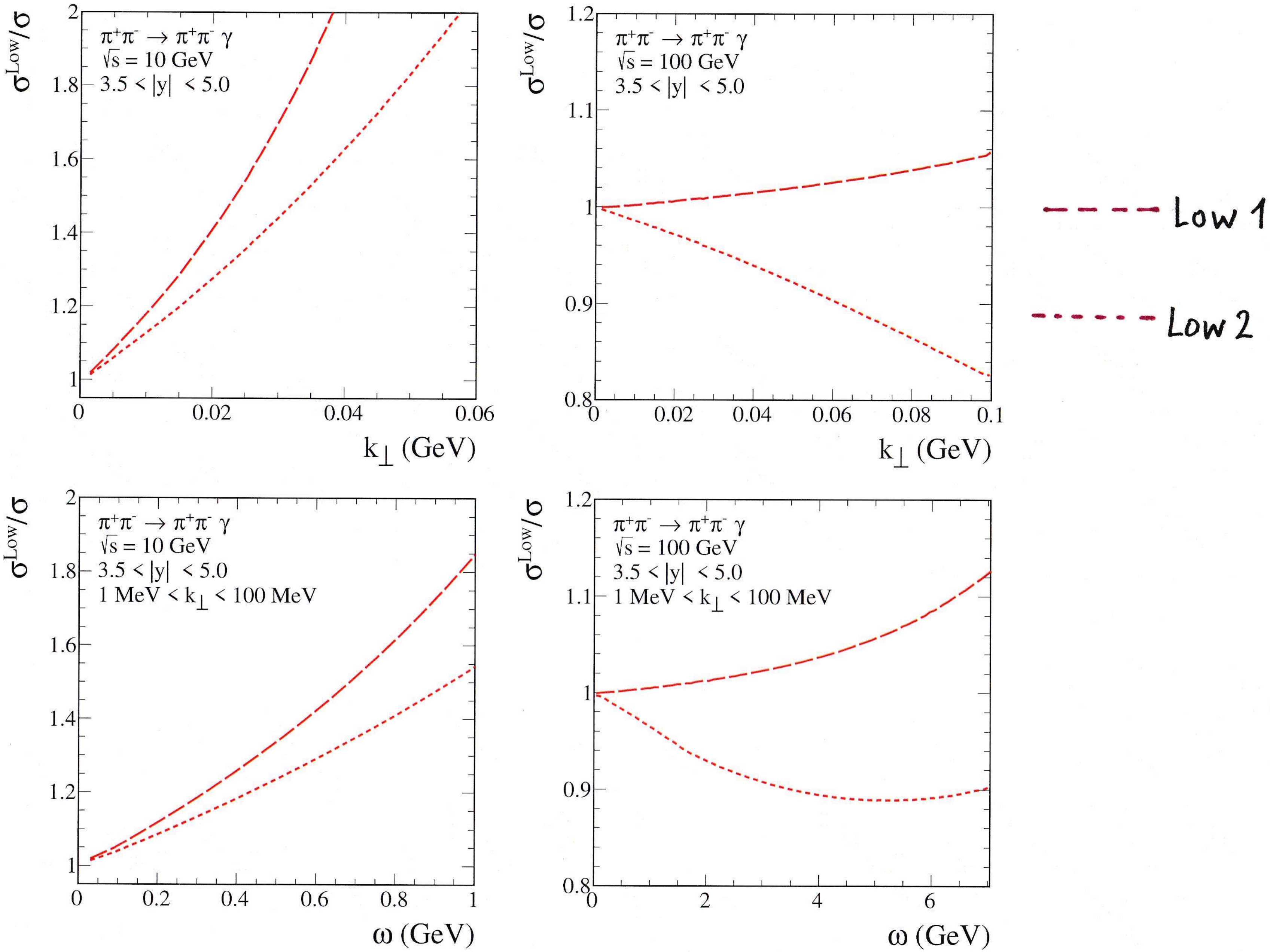
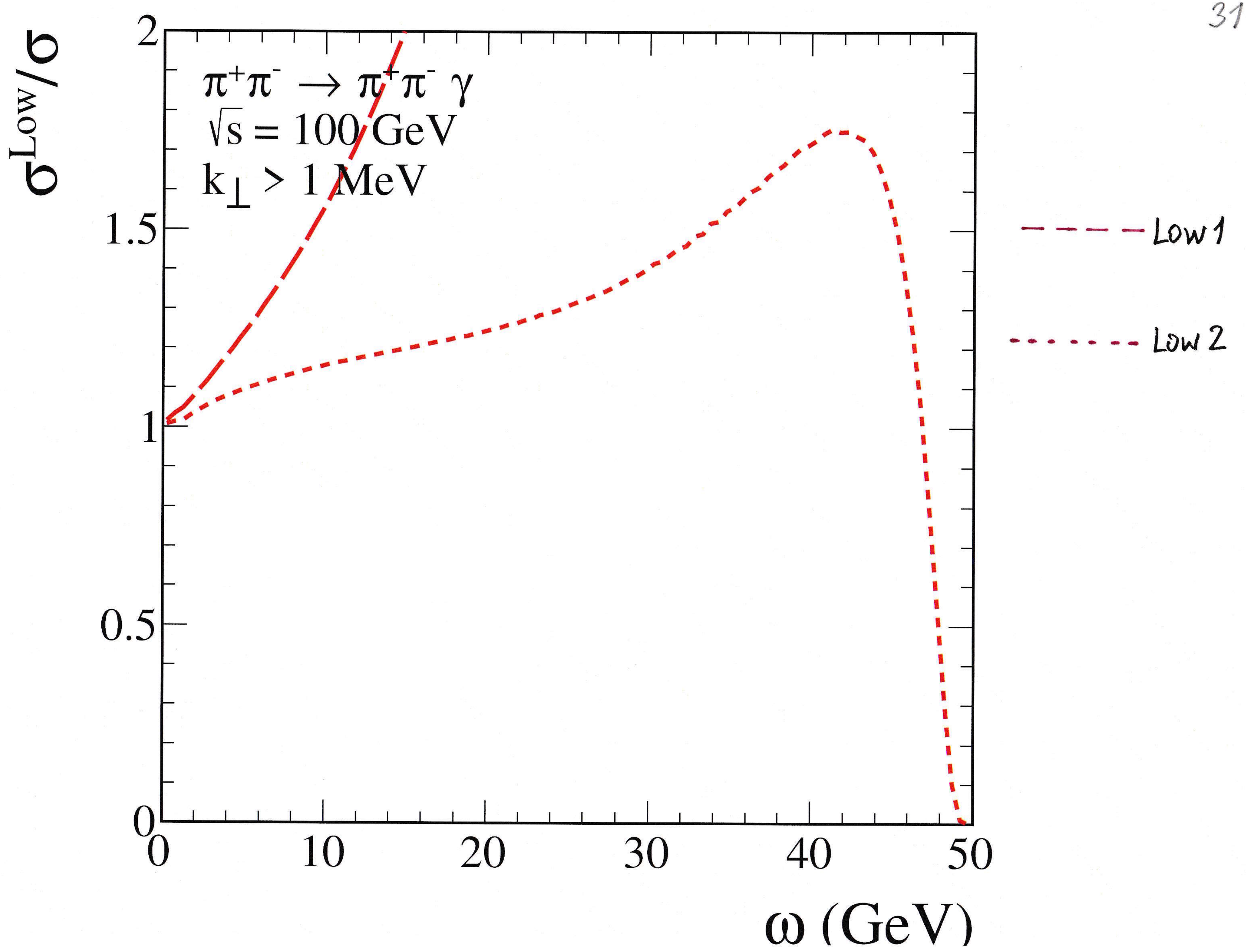
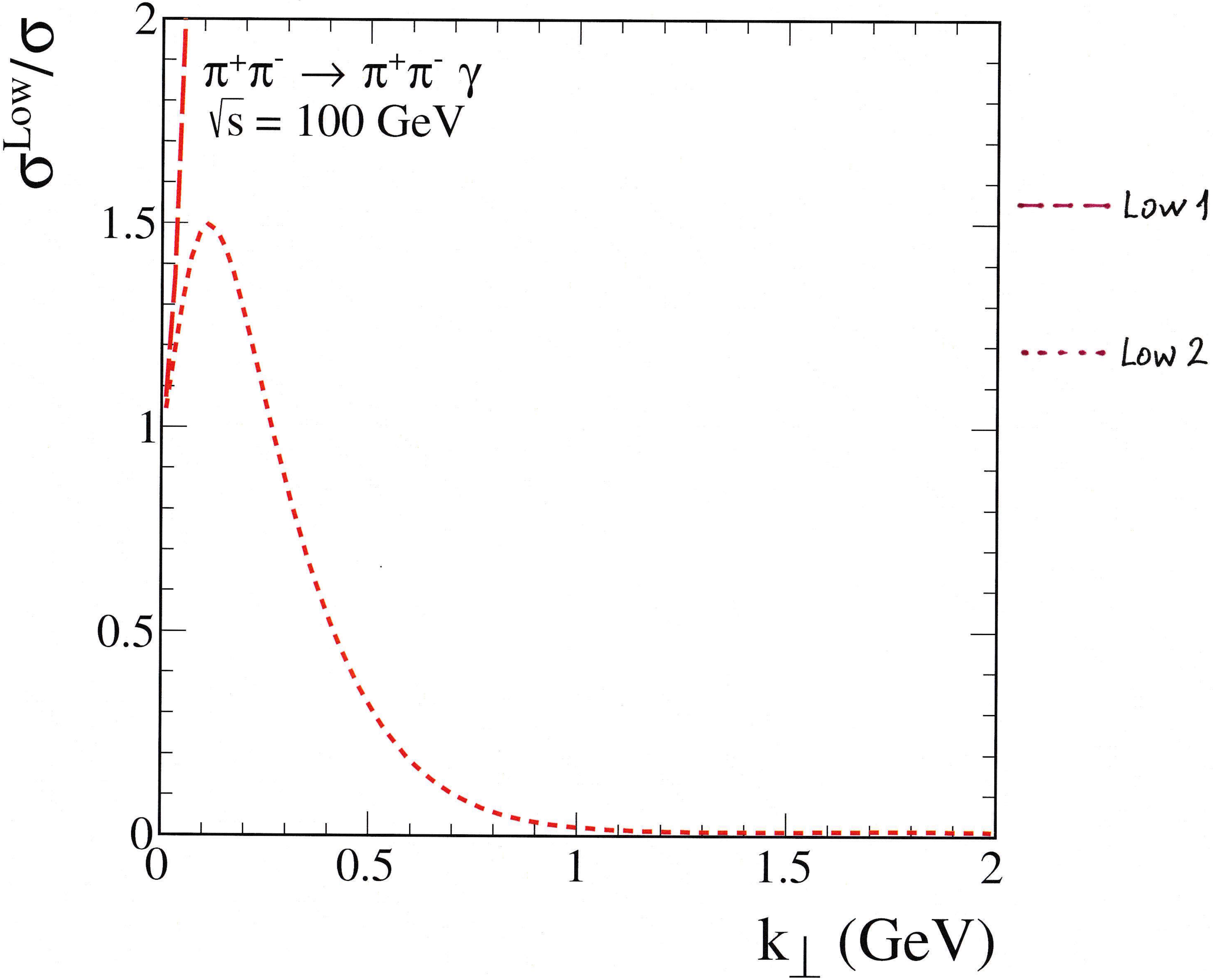


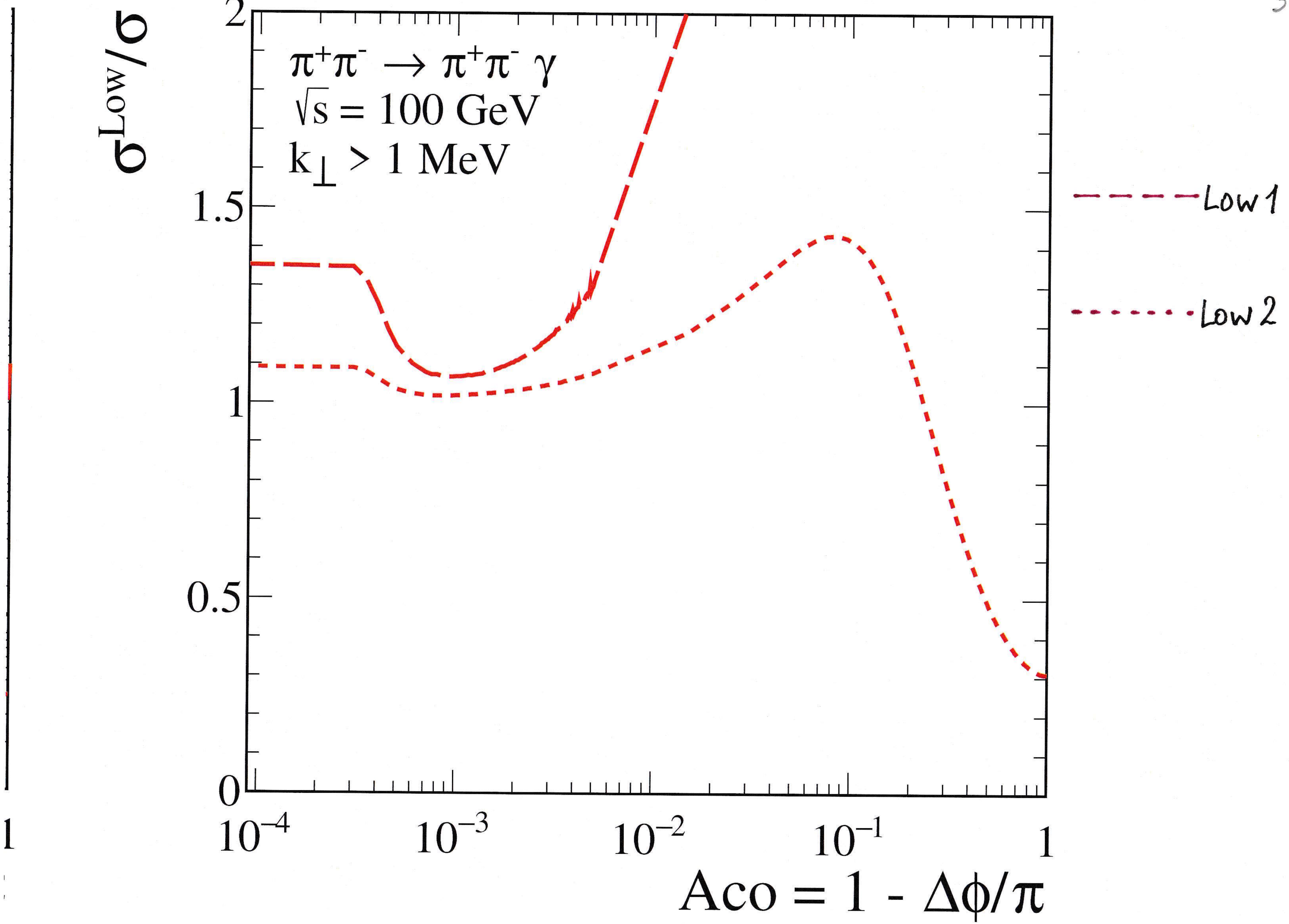
FIG. 4. The ratios $\sigma^{\rm Low}/\sigma$ for (1.1). The meaning of the lines is the same as in Fig. 3. The red long-dashed line corresponds to Eq. (1.2), the red short-dashed line corresponds to Eq. (1.3).



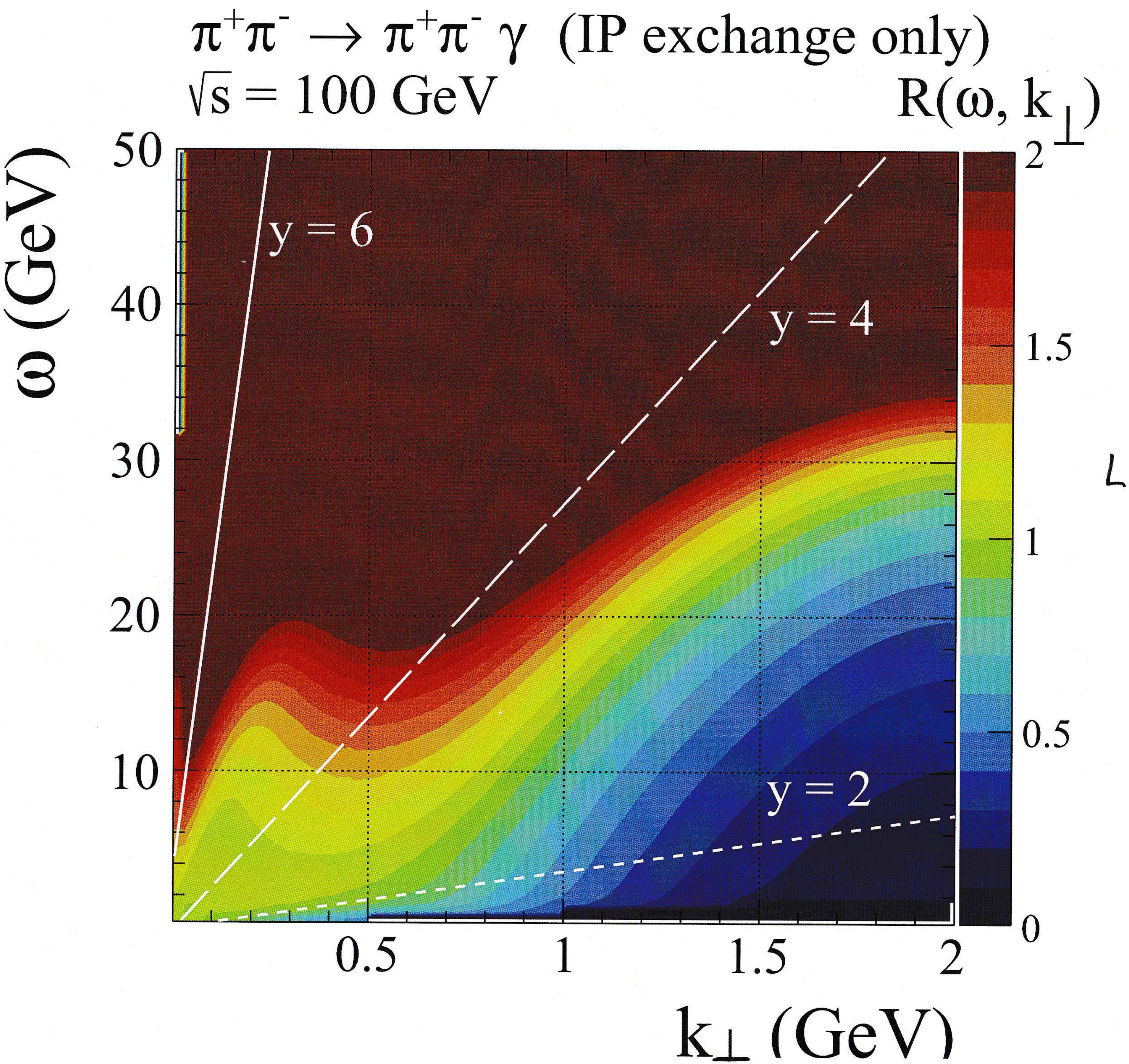
σ^{Low}/σ
$\pi^+\pi^- \to \pi^+\pi^- \gamma$
$\sqrt{s} = 100$ GeV
$k_\perp > 1$ MeV
2
1.5
1
0.5
0
0
10
20
30
40
50
ω (GeV)
Low 1
Low 2

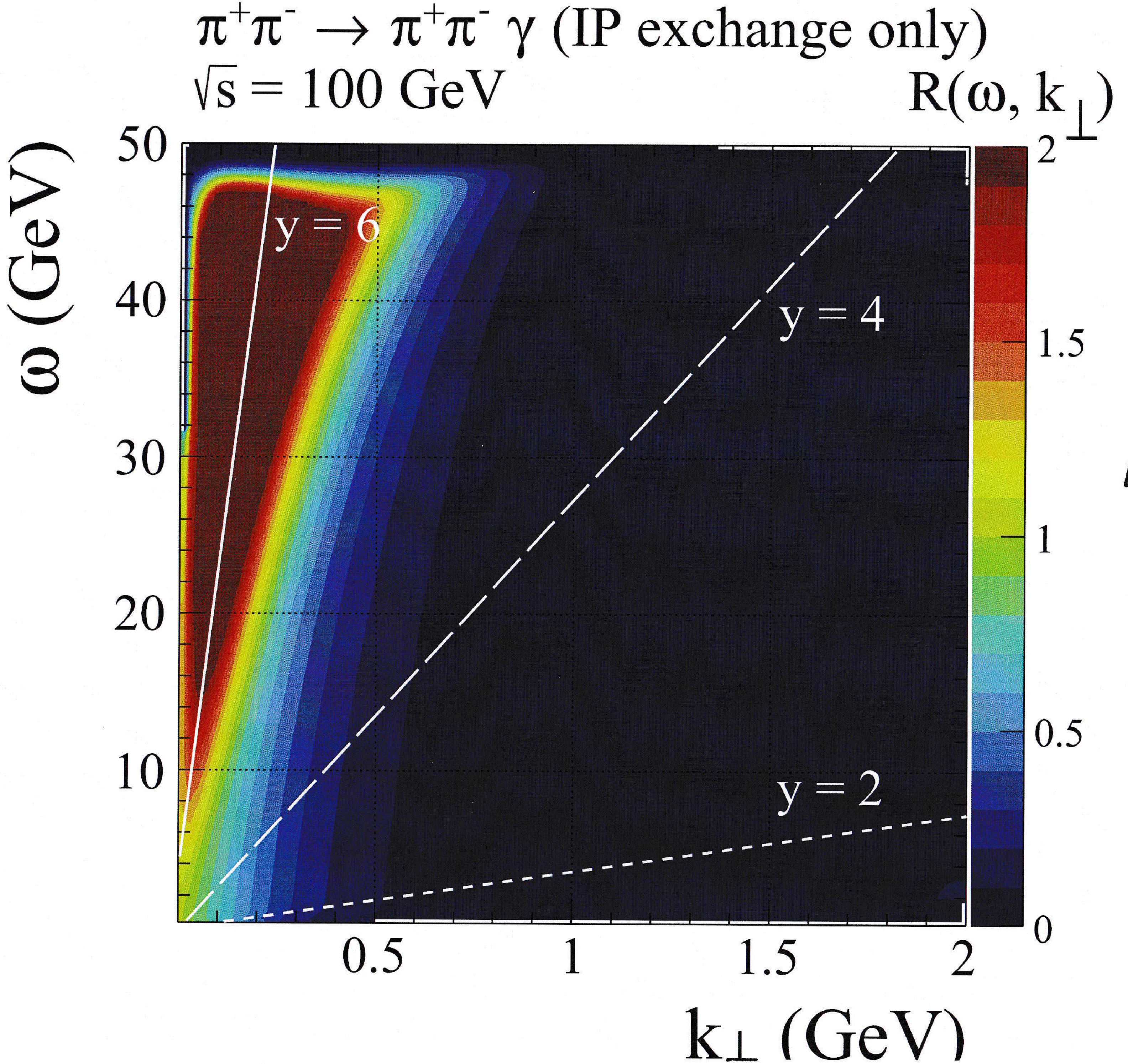


σ^{Low}/σ
$\pi^+\pi^- \to \pi^+\pi^- \gamma$
$\sqrt{s}$ = 100 GeV
2
1.5
1
0.5
0
0
0.5
1
1.5
2
$k_\perp$ (GeV)
Low 1
Low 2



σ^{Low}/σ
$\pi^+\pi^- \to \pi^+\pi^- \gamma$
$\sqrt{s}$ = 100 GeV
$k_\perp$ > 1 MeV
Low 1
Low 2
0
0.5
1
1.5
2
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
Aco = 1 - $\Delta\phi/\pi$


$\pi^+\pi^- \to \pi^+\pi^- \gamma$ (IP exchange only)
$\sqrt{s}$ = 100 GeV
$R(\omega, k_\perp)$
ω (GeV)
$k_\perp$ (GeV)
y = 6
y = 4
y = 2
0.5
1
1.5
2
10
20
30
40
50
0
Low 1


π⁺π⁻ → π⁺π⁻ γ (IP exchange only)
√s = 100 GeV
R(ω, k⊥)
ω (GeV)
k⊥ (GeV)
y = 6
y = 4
y = 2
50
40
30
20
10
0.5
1
1.5
2
0

Low 2

6 Conclusions and outlook

- We have explained the tensor-pomeron model for high-energy soft hadronic reactions. The model allows us to calculate photon emission in such processes.
- We have studied the reactions $\pi^+\pi^- \to \pi^+\pi^-$ and $\pi^+\pi^- \longrightarrow \pi^+\pi^-\gamma$ in detail and have compared the "exact" model results with approximations based on Low's theorem.
- For photon momenta $k \neq 0$ there is no unique "Low result".
- It should be straightforward to study in the same way the processes $pp \to pp$ and $pp \to pp\gamma$.
- To study in this way processes like $pp \to p\pi^+\pi^- p$ and $pp \longrightarrow p\pi^+\pi^- p\gamma$ is in principle possible but much more difficult.

Thank you for
your attention!