ENTANGLEMENT BEYOND QUBITS

HIGH DIMENSIONAL ENTANGLEMENT FOR QUANTUM COMMUNICATION

QUIT Physics group & friends
Entanglement in Quantum Fields 2021

INFORMATION-FROM CLASSICAL TO QUANTUM

Classical information- bit strings

$$0010100100011101010001001010100010$$

Quantum information- encoding into quantum degrees of freedom

$$|0010100100011101010001001010100010\rangle$$

Superposition and entanglement

$$|\psi\rangle = a|0010100100011101010001001010100010\rangle + b|0010101100001101010101010010100010\rangle$$

ENCODING-FROM (QU)BITS TO (QU)DITS

Classical	Quantum
$00 \equiv \tilde{0}$	$\lvert 00\rangle \equiv \lvert\tilde{0}\rangle$
$01 \equiv \tilde{1}$	$\lvert 01\rangle \equiv \lvert\tilde{1}\rangle$
$10 \equiv \tilde{2}$	$\lvert 10\rangle \equiv \lvert\tilde{2}\rangle$
$11 \equiv \tilde{3}$	$\lvert 11\rangle \equiv \lvert\tilde{3}\rangle$

Ideally the same (apart from finite size effects)

ENCODING-FROM ABSTRACT TO PHYSICAL

Classical

- Redudant
- Naturally noise resistant
- Easy to copy

e.g.
Magnetisation (HDD)
Frequency or amplitude of many photons

Quantum

- Encoded in fundamental degrees of freedom
- Limited by ability to control and isolate
- No-cloning

e.g.
Single photon polarisation $|H\rangle$ or $|V\rangle$
Orbital angular momentum of single photons
Relative time of arrival (time bins)

ENTANGLEMENT (BIPARTITE)

$$|\psi\rangle = \sum_{i,j=0}^{d-1} c_{ij}|ij\rangle \xrightarrow{\text{Schmidt decomposition}} |\psi\rangle = \sum_{i=0}^{r-1} \lambda_i|\tilde{\imath}\tilde{\imath}\rangle$$

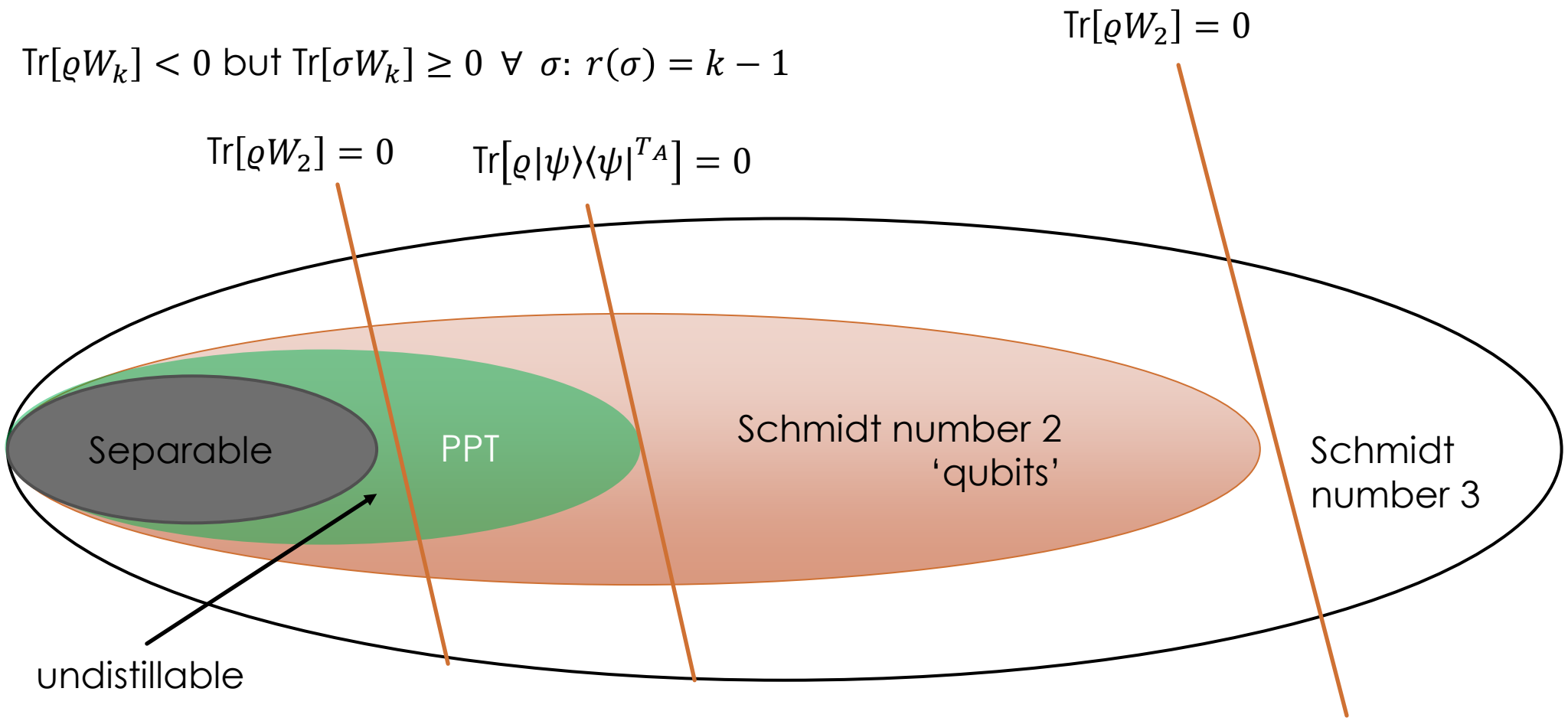
$r(|\psi\rangle)$ …Schmidt rank (dimensionality of entanglement)

$$\varrho = \sum_i p_i|\psi_i\rangle\langle\psi_i| \xrightarrow[|\mathfrak{D}(\varrho)| = \infty]{\mathfrak{D}(\varrho) := \{(p_i, |\psi_i\rangle): \sum_i p_i|\psi_i\rangle\langle\psi_i| = \varrho\}} r(\varrho) := \min_{\mathfrak{D}(\varrho)} \max_{|\psi_i\rangle \in \mathfrak{D}(\varrho)} r(|\psi_i\rangle)$$

$r(\varrho)$ …Schmidt number (dimensionality of entanglement)

SCHMIDT NUMBER WITNESSES

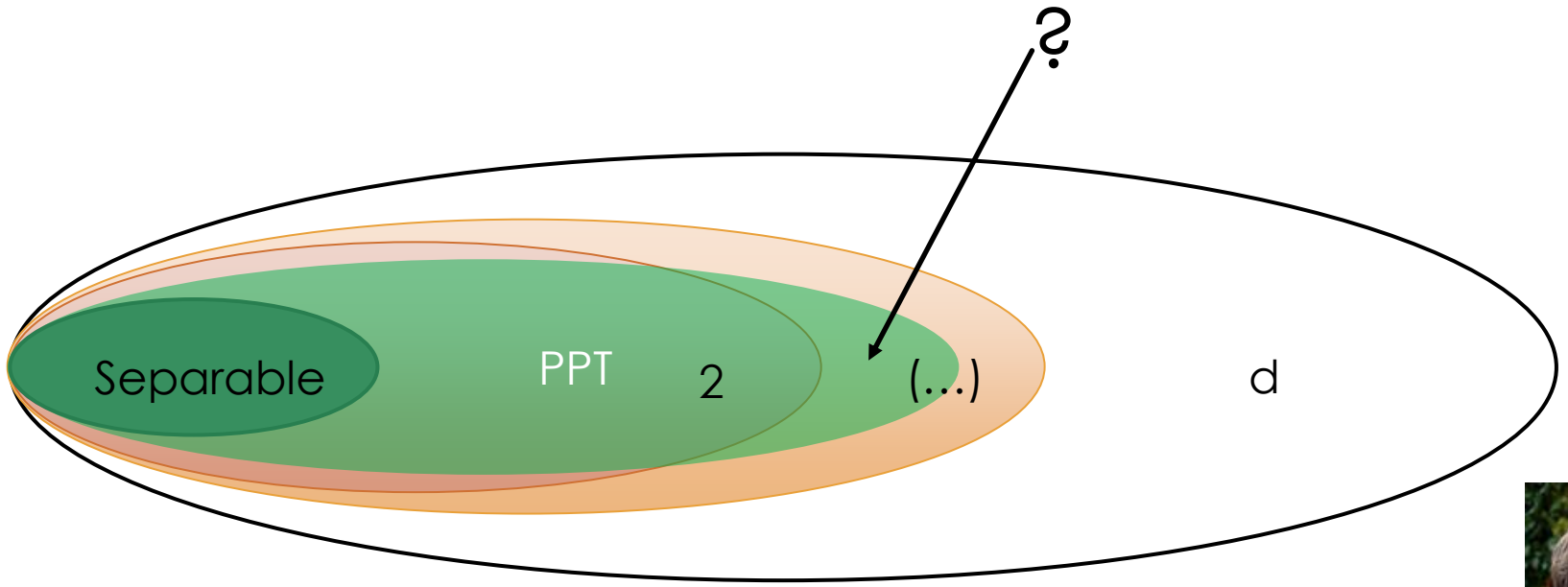
$\mathrm{Tr}[\varrho W_k] < 0$ but $\mathrm{Tr}[\sigma W_k] \geq 0 \;\; \forall \;\; \sigma\colon r(\sigma) = k - 1$



M. Horodecki, P. Horodecki, R. Horodecki, Phys. Rev. Lett. 80, 5239 (1998)

Y. Yang, D.H. Leung, and W-S. Tang. Lin. Alg. Appl. 503:233–247 (2016)

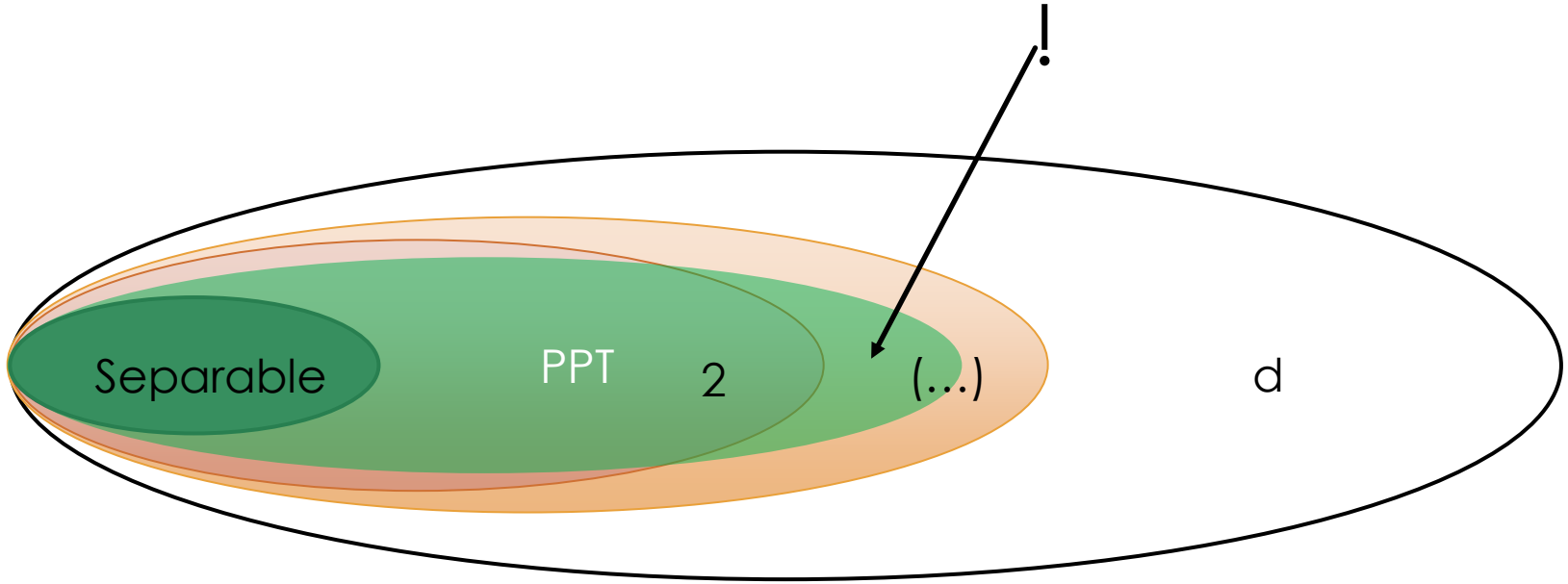
SCHMIDT NUMBER WITNESSES



There $\exists\, \varrho\ s.t.\ \varrho^{T_A} \geq 0$ and $r(\varrho) = \frac{d-1}{4}$

M. Huber, L. Lami, C. Lancien, A. Müller-Hermes, Phys. Rev. Lett. 121, 200503 (2018)

SCHMIDT NUMBER WITNESSES



Flavien Hirsch

There $\exists\, \varrho\ s.t. r(\varrho^{T_A}) \leq 4$ and $r(\varrho) \geq \frac{d-1}{4}$

Implies impossibility of device independent verification of dimension->open question for entanglement

F. Hirsch, M. Huber, arXiv:2003.14189

MULTIPARTITE ENTANGLEMENT

$|\phi_A\rangle = \otimes_{i=1}^{k} |\varphi_{\alpha_i}\rangle \quad A = \{\alpha_1, \alpha_2, (\dots), \alpha_k\}$ k-separability/ $\max_{\alpha_i \in A} |\alpha_i|$ -producibility

Multipartite entanglement $\quad k = 1, \max_{\alpha_i \in A} |\alpha_i| = \mathrm{n}$

Full separability $\quad k = n, \max_{\alpha_i \in A} |\alpha_i| = 1$

- THERE IS NO SCHMIDT DECOMPOSITION, BUT

$[\vec{r}]_i(|\psi\rangle) = rank(Tr_{\overline{\alpha_i}}(|\psi\rangle\langle\psi|))$ Schmidt rank vector

Nontrivial constraints, e.g. $r_A \leq r_{AB} r_{AC}$

J. Cadney, M. Huber, N. Linden, A. Winter, Lin. Alg. Appl., 452, 153-171 (2014)

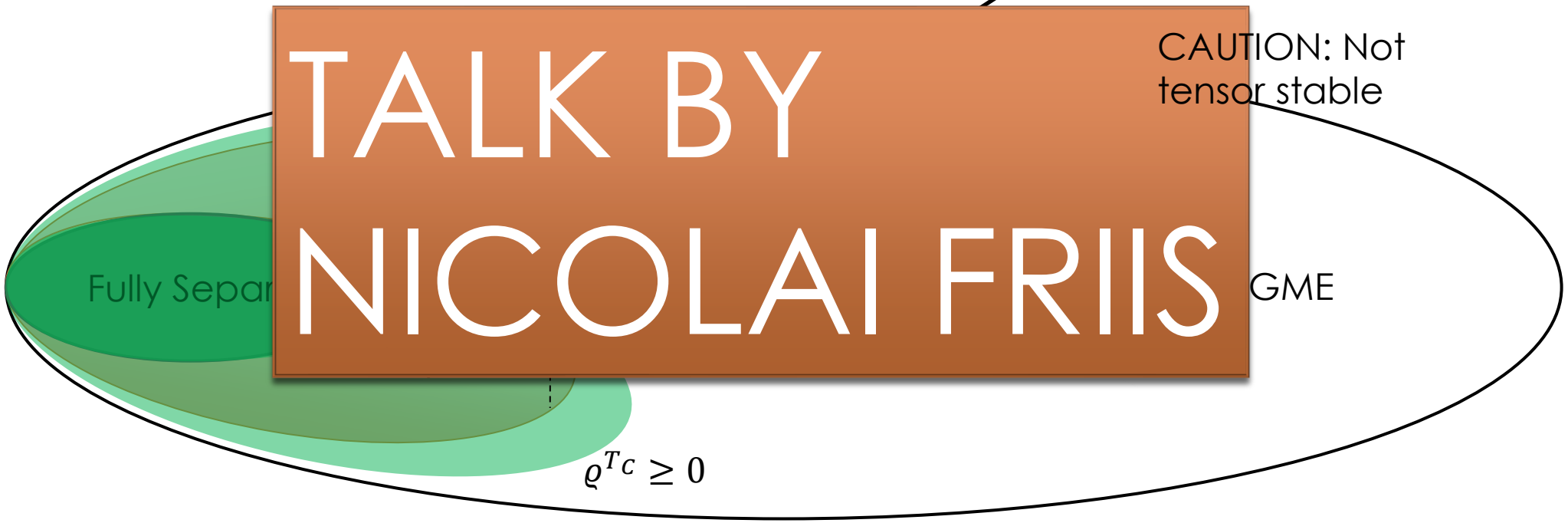
Andreas Winter

Noah Linden

MULTIPARTITE ENTANGLEMENT-MIXED STATES

$$\sigma_k = \sum_{|A|\geq k} \sum_i p_i^A \, |\phi_i^A\rangle\langle\phi_i^A|$$

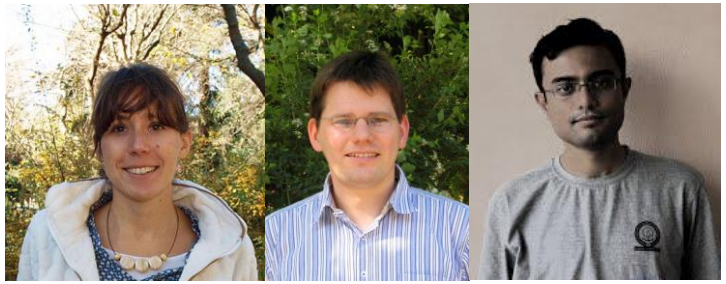
$$\sigma_2 = p^{\{1,23\}} |\varphi^1\rangle\langle\varphi^1| \otimes |\varphi^{23}\rangle\langle\varphi^{23}| + p^{\{12,3\}} |\varphi^{12}\rangle\langle\varphi^{12}| \otimes |\varphi^3\rangle\langle\varphi^3|$$



TALK BY NICOLAI FRIIS

MULTIPARTITE ENTANGLEMENT-MIXED STATES

- ENTANGLEMENT ACROSS EVERY BIPARTITION ON ONE COPY NECESSARY BUT NOT SUFFICIENT FOR GENUINE MULTIPARTITE ENTANGLEMENT
  (SEE ALSO CONJECTURE PRESENTED BY NICOLAI FRIIS)



$W_{A|BC} < 0 \qquad W_{B|AC} < 0 \qquad W_{C|AB} < 0$

Idea: $\quad W_{GME} = W_{A|BC} + W_{B|AC} + W_{C|AB} + M$

Theorem: For every ϱ_{GME} $\exists\, M\ s.t. \langle W_{GME} \rangle < 0$ and W_{GME} is a multipartite entanglement witness

M. Huber, R. Sengupta, Phys. Rev. Lett. 113, 100501 (2014)

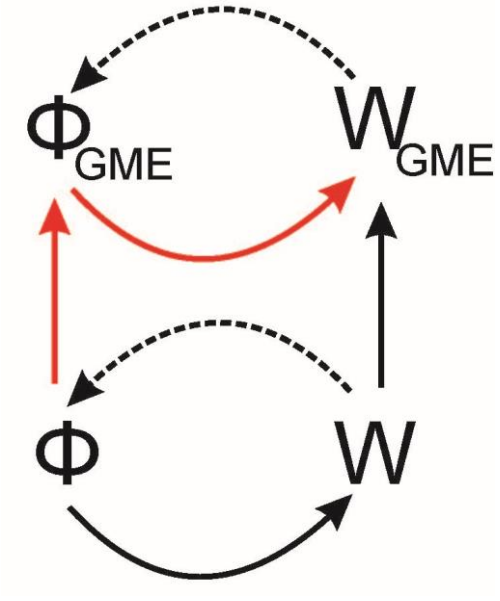
C. Lancien, O. Gühne, R. Sengupta, M. Huber, J. Phys. A: Math. Theor. 48 505302 (2015)

POSITIVE MAPS FOR MULTIPARTITE SYSTEMS

$\Lambda \otimes \mathbb{I}_k[\sigma_{SEP}] \geq 0$

$\Phi_{GME}[\sigma_2] \geq 0$

$\Phi[\sigma_{SEP}] \geq 0$



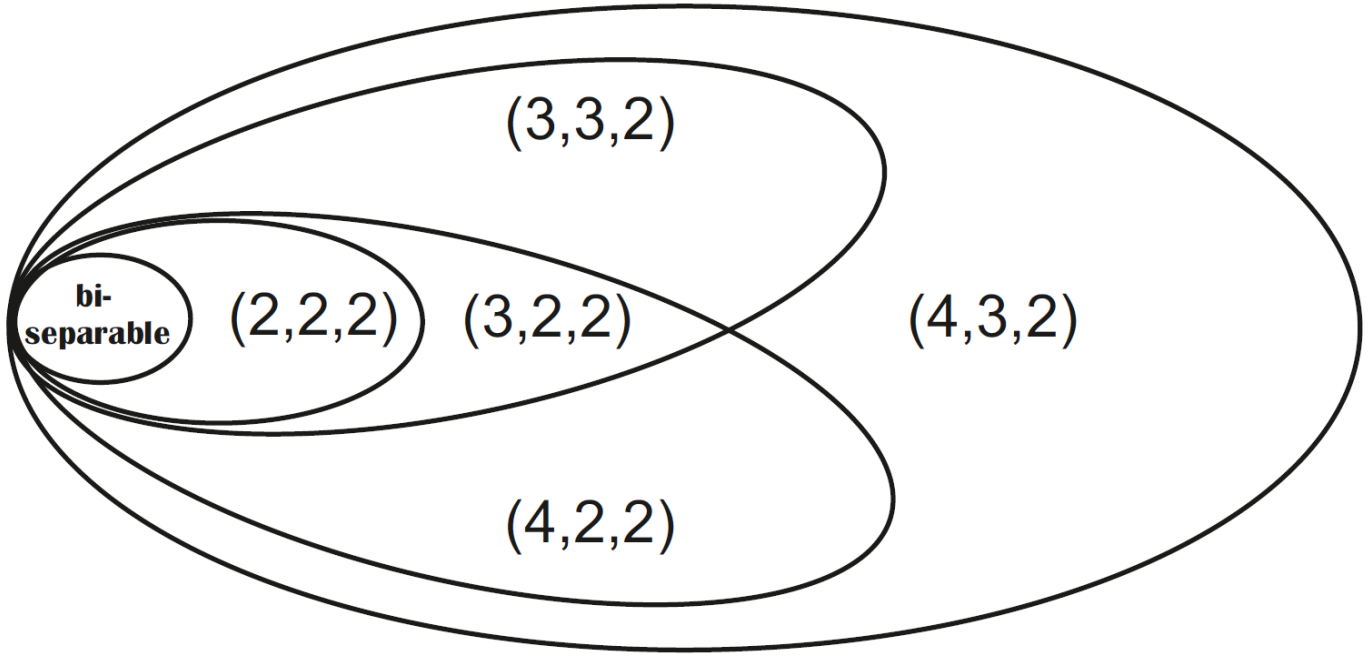
F. Clivaz, M. Huber, L. Lami, G. Murta, J. Math. Phys. 58, 082201 (2017)

MULTIPARTITE ENTANGLEMENT

$$[\vec{r}]_i = rank(Tr_{\overline{\alpha_i}}(|\psi\rangle\langle\psi|))$$

Schmidt rank vector

$$[\vec{r}]_i(\varrho) = \min_{\mathfrak{D}(\varrho)} \max_{|\psi_i\rangle \in \mathfrak{D}(\varrho)} \vec{r}^{\downarrow}(|\psi_i\rangle)$$



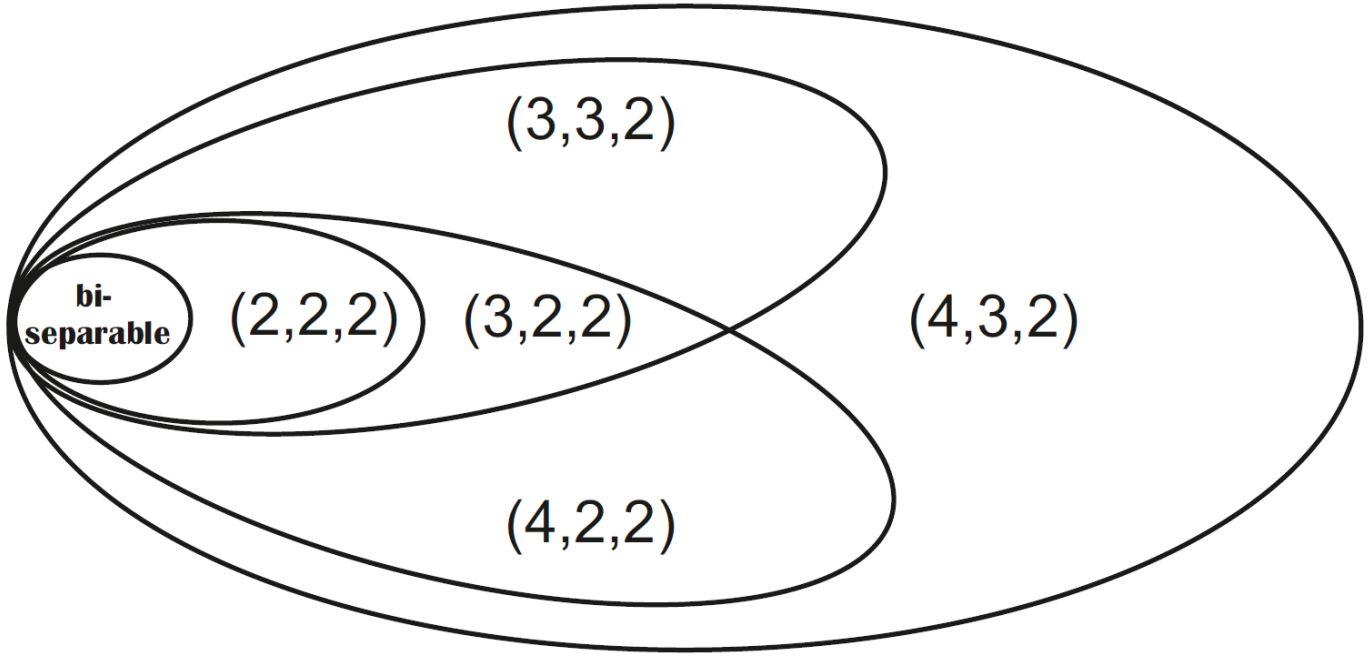
M. Huber, J. de Vicente, Phys. Rev. Lett. 110, 030501 (2013)

MULTIPARTITE ENTANGLEMENT

$$[\vec{r}]_i(\varrho) = \min_{\mathfrak{D}(\varrho)} \max_{|\psi_i\rangle \in \mathfrak{D}(\varrho)} \vec{r}^{\downarrow}(|\psi_i\rangle)$$ Schmidt rank vector

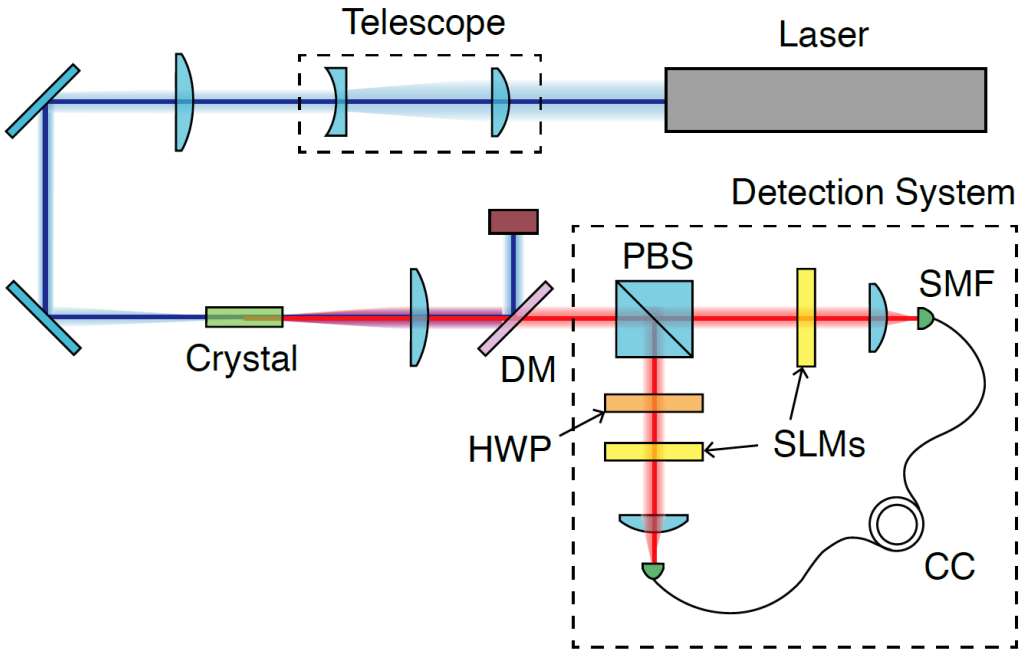
$$\langle W_i \rangle \leq \log_2 [\vec{r}]_i(\varrho)$$

$$|\psi_{332}\rangle = \frac{1}{\sqrt{3}}(|000\rangle + |111\rangle + |221\rangle)$$



M. Huber, J. de Vicente, Phys. Rev. Lett. 110, 030501 (2013)

NOISE ADVANTAGE?



Accidental=coincidence click not caused by intended photon pair

$|\psi_T\rangle = |\phi^+\rangle$ p_A…probability for a coincidence to be accidental

$$\varrho = (1 - p_A)|\phi^+\rangle\langle\phi^+| + p_A \frac{1}{d^2} \mathbb{I}_{d^2} \qquad p_{SEP} = \frac{d}{d+1}$$

Scaling of p_A in d determines existence of physical advantage

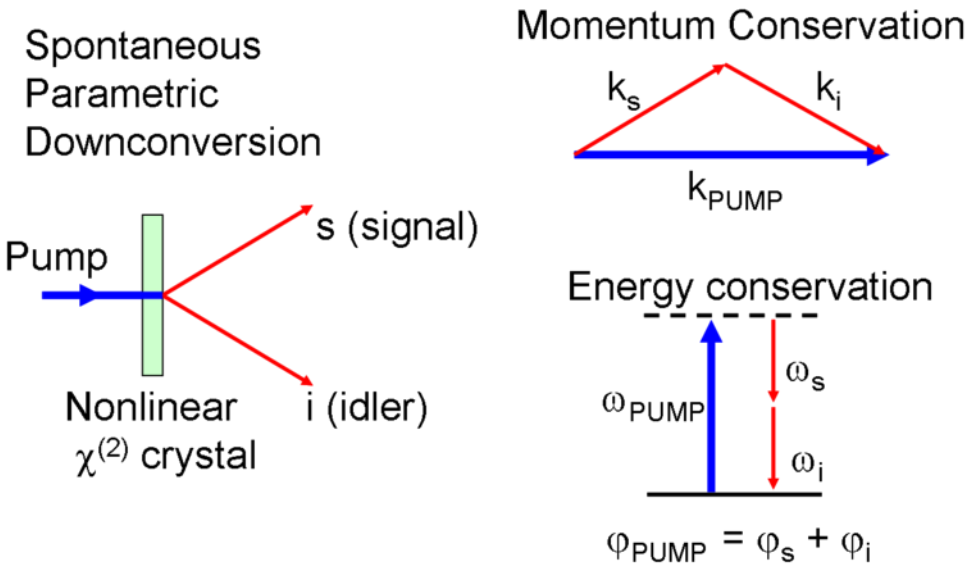
L. Lami, M. Huber, J. Math. Phys. 57, 092201 (2016)

SOME CHALLENGES

- PHYSICAL AVAILABLITY-*WHERE DO WE FIND HIGH-DIMENSIONAL ENTANGLEMENT?*
- MEASUREMENTS AND WITNESSES-*HOW DO WE CONFIRM PRESENCE?*
- NOISE MODELS-*UNDER WHICH PHYSICAL CONDITIONS WILL WE FIND NOISE ADVANTAGES?*
- *APPLICATIONS-CAN WE USE ANY ENTANGLEMENT FOR USEFUL PROTOCOLS?*

PHYSICAL AVAILABLITY-*WHERE DO WE FIND HIGH-DIMENSIONAL ENTANGLEMENT?*

e.g. spontaneous parametric down-conversion (SPDC)



e.g. OAM entanglement $k_{PUMP} = 0 \Rightarrow k_S = -k_i$

$$|\psi_{OAM}\rangle = \sum_{i=-\infty}^{\infty} \lambda_i(|i,-i\rangle)$$

M. Krenn, M. Malik, Th. Scheidl, R. Ursin, A. Zeilinger, Optics in Our Time (pp. 455-482), Springer (2016)

PHYSICAL AVAILABLITY-*WHERE DO WE FIND HIGH-DIMENSIONAL ENTANGLEMENT?*

TALK BY GIUSEPPE VITAGLIANO

M. Fadel, A. Usui, M. Huber, N. Friis, G. Vitagliano

MEASUREMENTS AND WITNESSES-*HOW DO WE CONFIRM PRESENCE?*

HOW MANY LOCAL MEASUREMENT SETTINGS DO WE NEED?

Method	Full state tomography	Optimal fidelity	Optimal witness
#of local settings	$d+1$	$d+1$	2
#of global settings	$(d+1)^2$	$d+1$	2

Actually, we can lower bound entanglement / fidelity from measurements in two bases

MEASUREMENTS AND WITNESSES-*HOW DO WE CONFIRM PRESENCE?*

HOW MANY LOCAL MEASUREMENT SETTINGS DO WE NEED?

Paul Erker

Mario Krenn

Mutually unbiased bases: $|\langle i_k | j_{k'} \rangle| = \delta_{ij}\delta_{kk'} + (1-\delta_{kk'})\frac{1}{d}$

$$U \otimes U^* |\phi^+\rangle = |\phi^+\rangle \longrightarrow |\phi^+\rangle := \frac{1}{\sqrt{d}} \sum_{i=0}^{d-1} |ii\rangle = \frac{1}{\sqrt{d}} \sum_{i=0}^{d-1} |i_k {i^*}_k\rangle$$

Measure: $C_{ij} = \langle ij|\varrho|ij\rangle$ and $\tilde{C}_{ij} = \langle i_k j_k^* | \varrho | i_k j_k^* \rangle$

$f(C_{ij}, \tilde{C}_{ij}) \leq E_{oF}(\varrho)$

P. Erker, M. Krenn, M. Huber, Quantum 1, 22 (2017)

MEASUREMENTS AND WITNESSES-*HOW DO WE CONFIRM PRESENCE?*

HOW MANY LOCAL MEASUREMENT SETTINGS DO WE NEED?

$$|\psi_T\rangle := \sum_{i=0}^{d-1} \lambda_i |ii\rangle$$

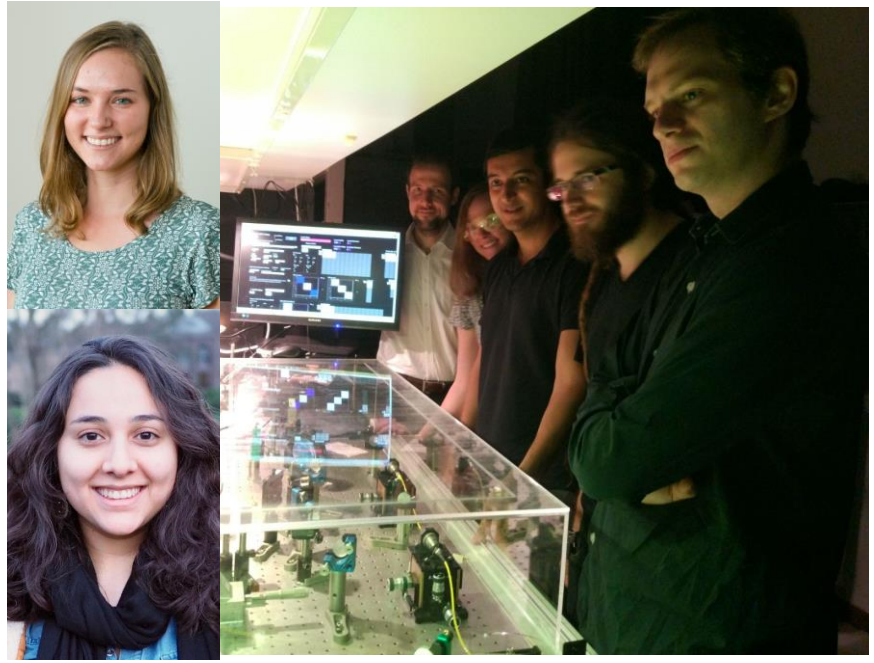
Tilted bases: $|\langle i_k | j_{k'}\rangle| = \delta_{ij}\delta_{kk'} + (1-\delta_{kk'})\lambda_i\lambda_j$

Theorem: *Every pure state is uniquely determined by measurements in Schmidt basis and one tilted basis*

Measure: $C_{ij} = \langle ij|\varrho|ij\rangle$ and $\tilde{C}_{ij} = \langle i_k j_k^* |\varrho| i_k j_k^*\rangle$ $\quad f(C_{ij}, \tilde{C}_{ij}) \leq \mathcal{F}(|\psi_T\rangle\langle\psi_T|, \varrho)$

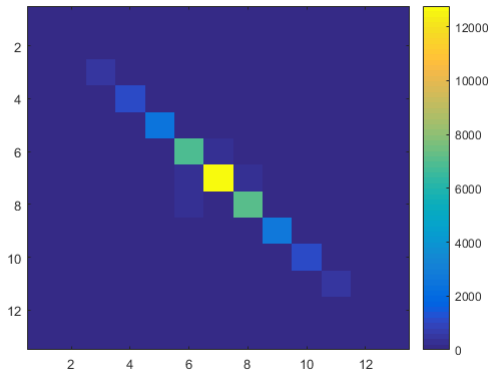
OPEN PROBLEM: Multipartite states?

MEASUREMENTS AND WITNESSES-*HOW DO WE CONFIRM PRESENCE?*

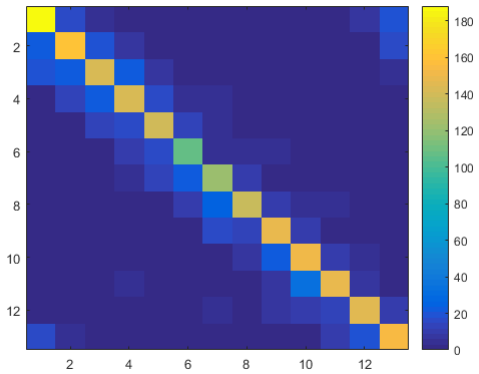


J. Bavaresco, N. Herrera Valencia, C. Klöckl, M. Pivoluska, P. Erker, N. Friis, M. Malik, M. Huber, Nat. Phys. 14, 1032 (2018)

C_{ij}



$\tilde{C}_{ij}$

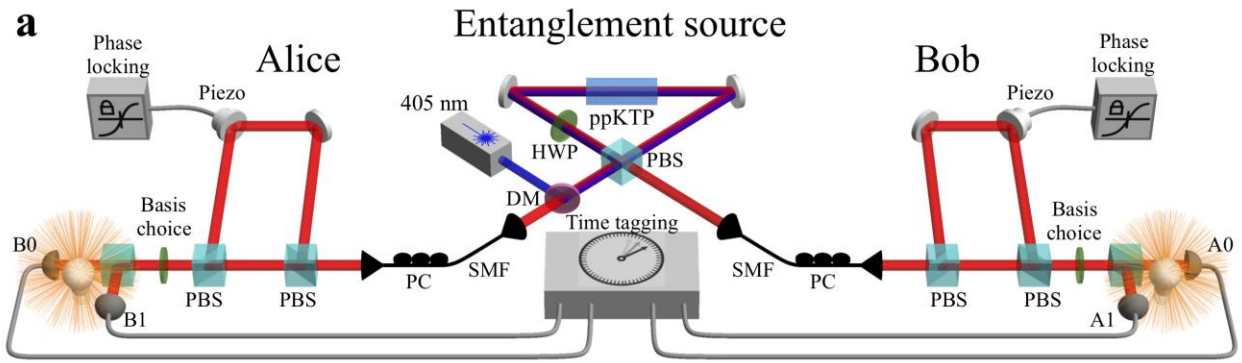


$|\psi_T\rangle$

Experimental results

d	d_{ent}	$\tilde{F}(\rho, \Phi^+)$	$\tilde{F}(\rho, \Phi)$
3	3	91.5±0.4%	92.5±0.4%
5	5	89.9±0.4%	90.0±0.5%
7	6	84.2±0.5%	86.9±0.6%
11	9	74.8±0.4%	76.2 ±0.6%

NOISE MODELS-*UNDER WHICH PHYSICAL CONDITIONS WILL WE FIND NOISE ADVANTAGES?*

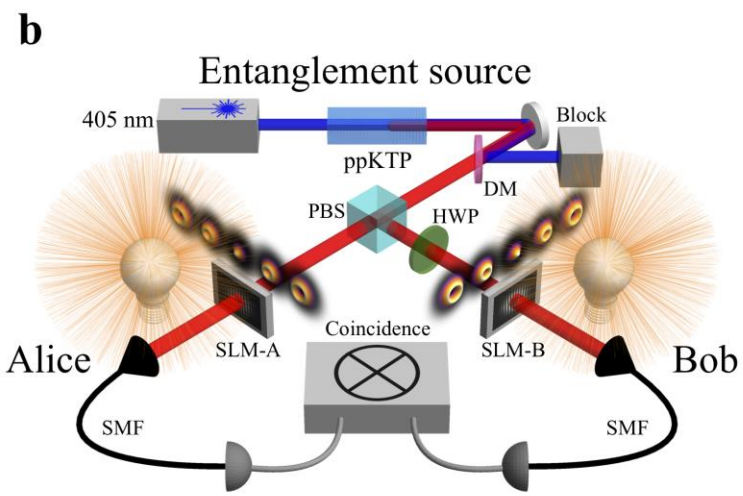


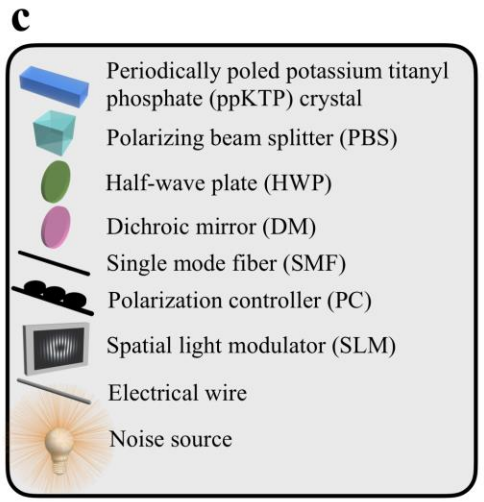
Rupert Ursin

Sebastian Ecker

Lukas Bulla

Mehul Malik



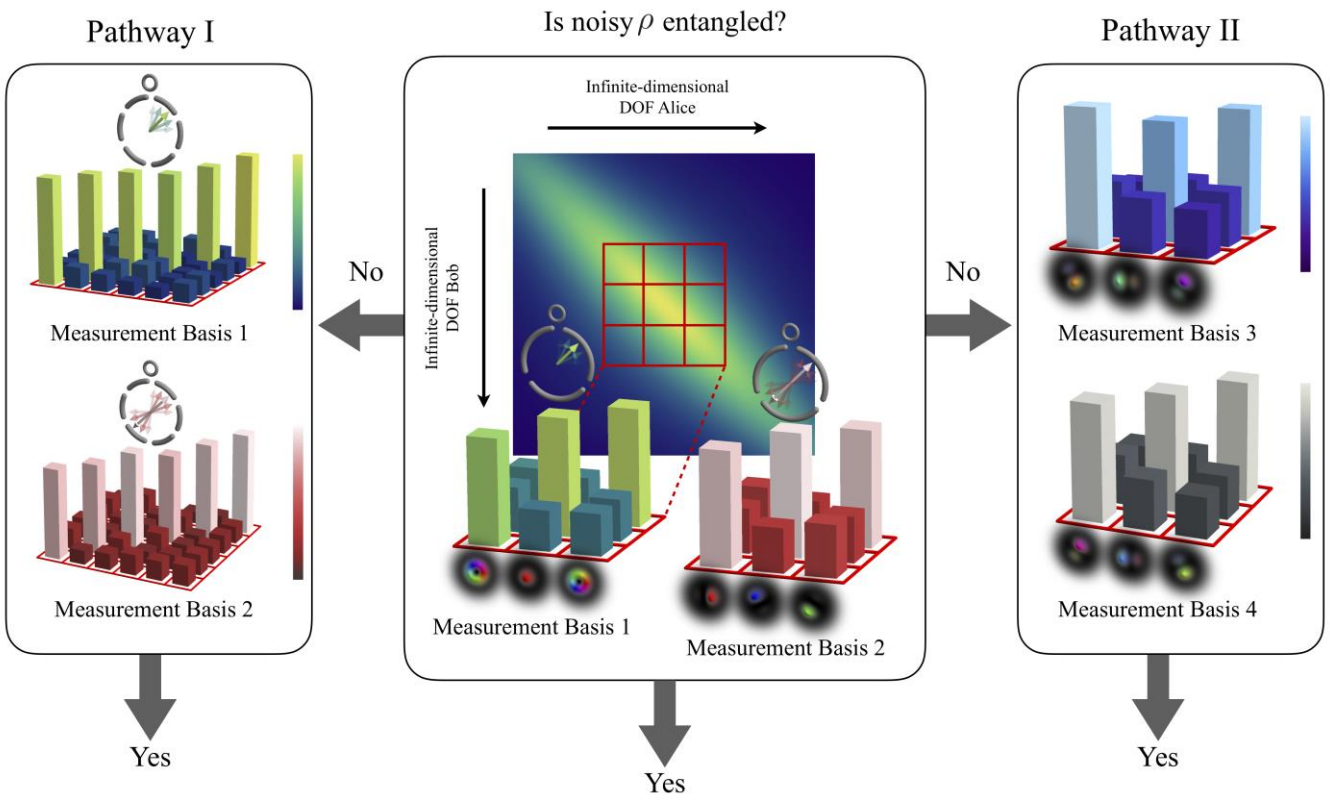


Robert Fickler

Frederic Bouchard

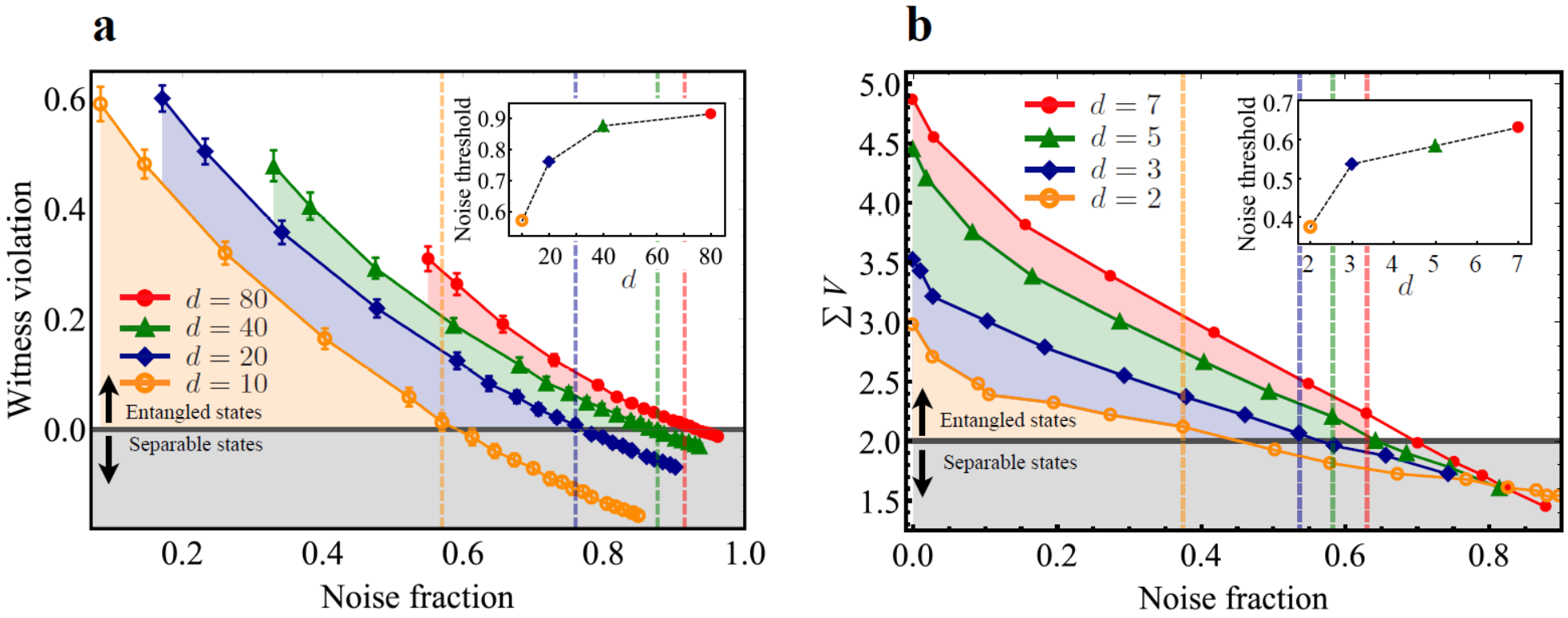
Florian Brandt

NOISE MODELS-*UNDER WHICH PHYSICAL CONDITIONS WILL WE FIND NOISE ADVANTAGES?*



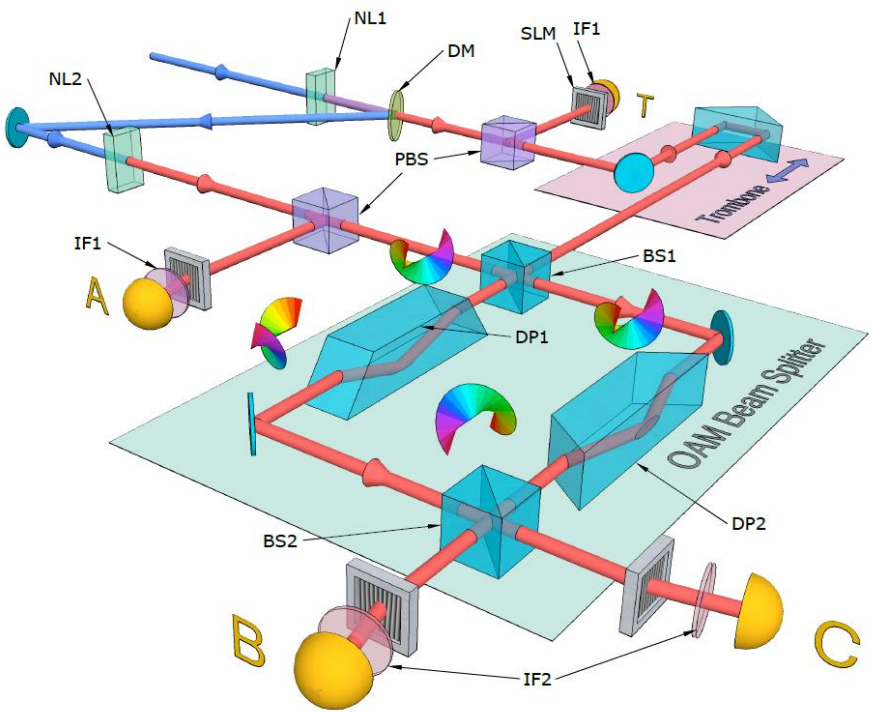
Yelena Guryanova

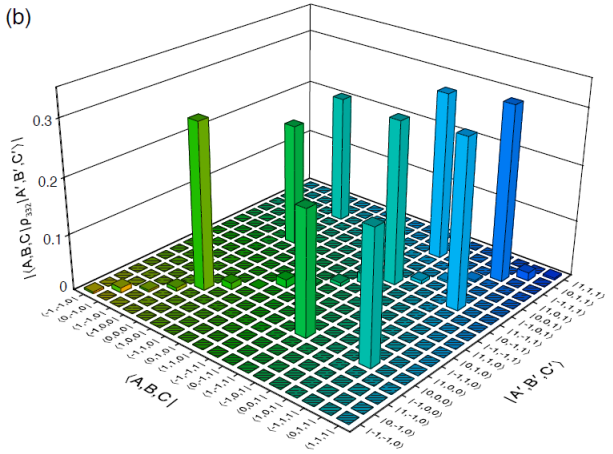
NOISE MODELS-*UNDER WHICH PHYSICAL CONDITIONS WILL WE FIND NOISE ADVANTAGES?*



S. Ecker, F. Bouchard, L. Bulla, F. Brandt, O. Kohout, F. Steinlechner, R. Fickler, M. Malik, Y. Guryanova, R. Ursin, M. Huber, Physical Review X, 9(4), 041042 (2019)

MULTIPARTITE ENTANGLEMENT





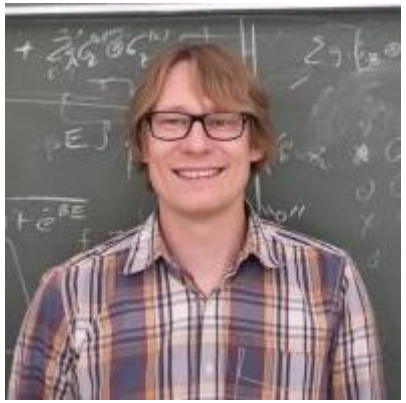
$\longrightarrow \varrho_{exp} \in (3,3,2)$

M. Malik, M. Erhard, M. Huber, M. Krenn, R. Fickler, A. Zeilinger, Nature Photonics 10, 248-252 (2016)

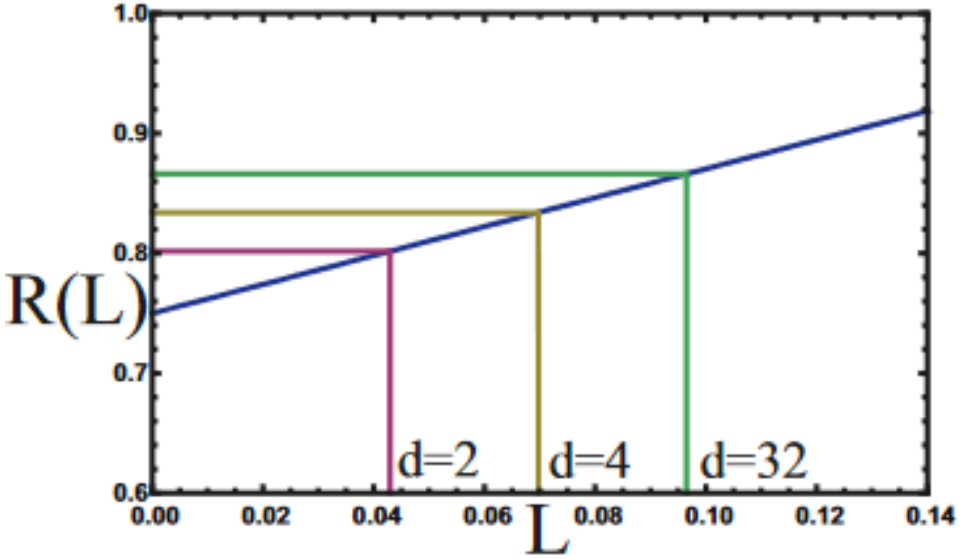
APPLICATIONS-CAN WE USE ANY ENTANGLEMENT FOR USEFUL PROTOCOLS?

- OVERCOMING WEAK RANDOMNESS IN DEVICE INDEPENDENT QKD

M. HUBER, M. PAWLOWSKI, PHYS. REV. A 88, 032309 (2013)

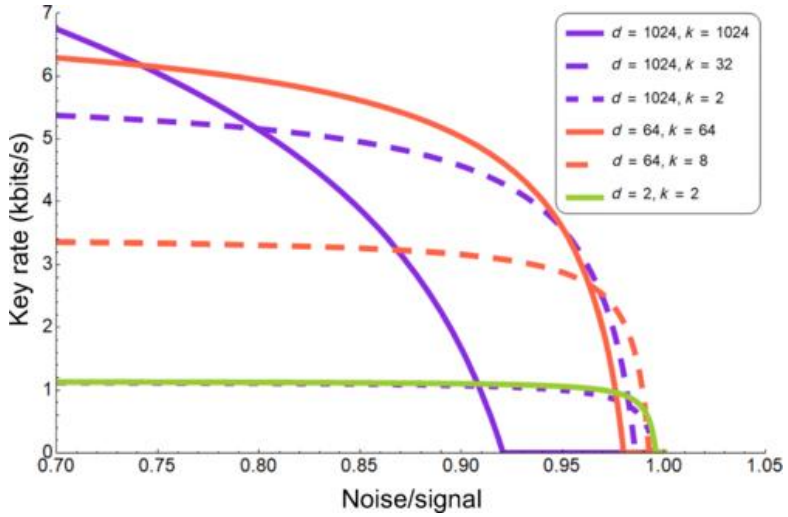


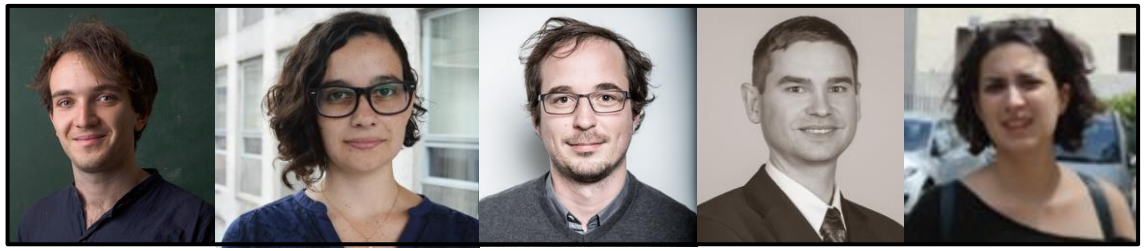
Marcin Pawlowski



APPLICATIONS-CAN WE USE ANY ENTANGLEMENT FOR USEFUL PROTOCOLS?

- QUANTUM KEY DISTRIBUTION OVERCOMING EXTREME NOISE: SIMULTANEOUS SUBSPACE CODING USING HIGH-DIMENSIONAL ENTANGLEMENT





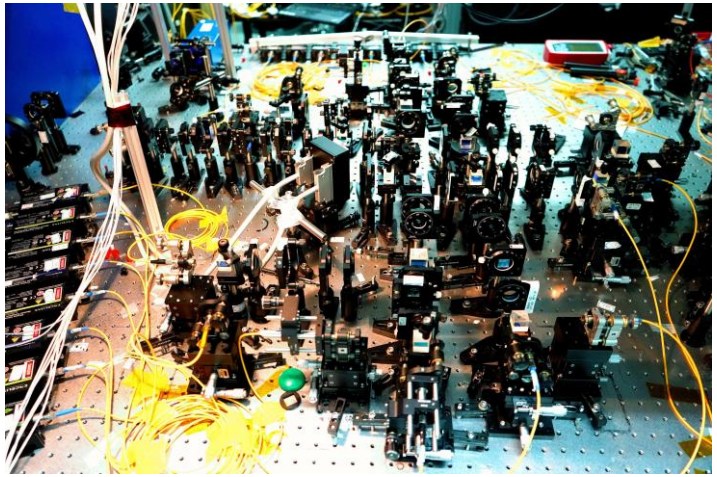
M. Doda, M. Huber, G. Murta, M. Pivoluska, M. Plesch, Ch. Vlachou, Phys. Rev. Applied 15, 034003 (2021)

FIRST IMPLEMENTATION



Xiao-Min Hu Bi-Heng Liu Xiaoqin Gao

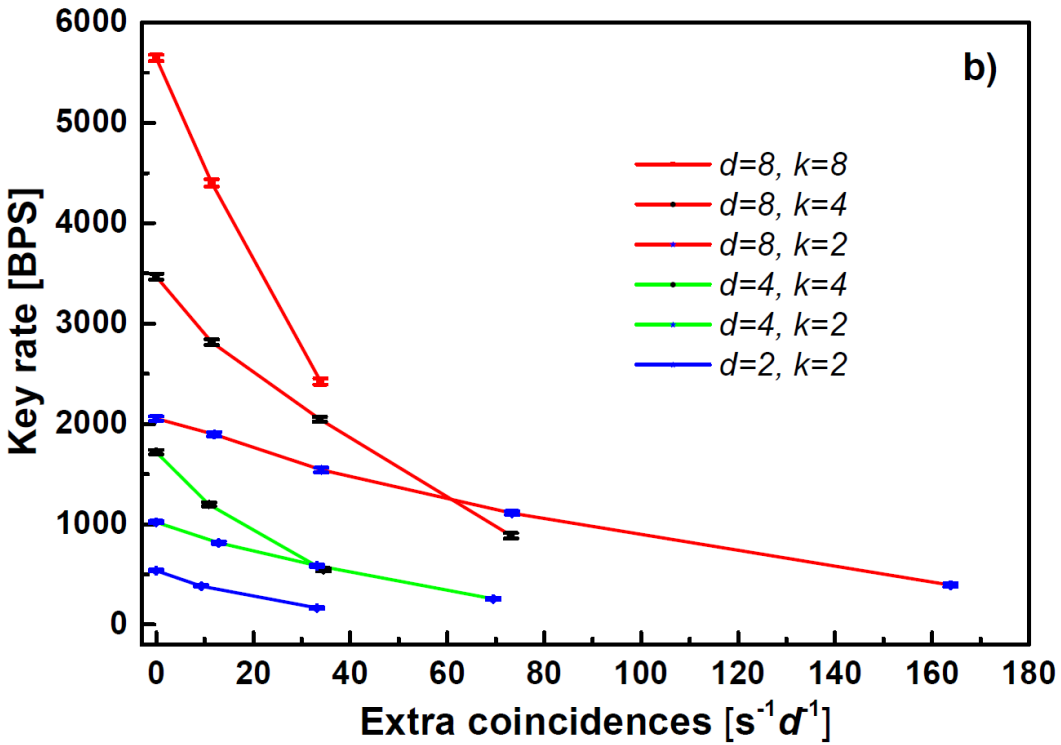

M. Pivoluska Paul Erker



Xiao-Min Hu, Chao Zhang, Yu Guo, Fang-Xiang Wang, Wen-Bo Xing, Cen-Xiao Huang, Bi-Heng Liu, Yun-Feng Huang, Chuan-Feng Li, Guang-Can Guo, Xiaoqin Gao, Matej Pivoluska, Marcus Huber, arXiv:2011.03005

Xiao-Min Hu, Wen-Bo Xing, Bi-Heng Liu, Yun-Feng Huang, Chuan-Feng Li, Guang-Can Guo, Paul Erker, Marcus Huber, Phys. Rev. Lett. 125, 090503 (2020)

PATHWAYS FOR ENTANGLEMENT BASED QUANTUM COMMUNICATION IN THE FACE OF HIGH NOISE



SUMMARY

- RICH STRUCTURE OF ENTANGLEMENT BEYOND QUBITS

N. Friis, G. Vitagliano, M. Malik , M. Huber, Nature Reviews Physics 1, 72–87 (2019)

OPEN PROBLEMS

- Entanglement = ‘non-locality‘?
- Is every entangled state useful for QKD?
- Best use of the full spectrum (Hyperentanglement, etc.)
- Limits to measurements
- Best discretisation
- Efficient verification of multipartite states

THANK YOU FOR YOUR ATTENTION AND MANY THANKS TO FRIENDS AND COLLABORATORS