How many particles do make a fluid?

Searching for hydrodynamic behavior in mesoscopic ultracold gases

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[Floerchinger, Giacalone, Heyen, Tharwat, **PRC 105, 044908 (2022)**]
[Brandstetter, Heintze, Lunt, Subramanian, Holten, Jochim, Heyen, Giacalone, Floerchinger, **in preparation**]

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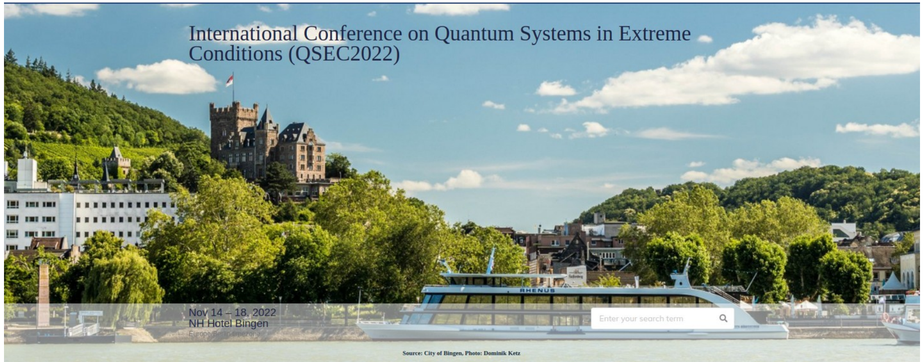


OUTLINE

1. The quest for hydrodynamics in systems that are small.

2. Elliptic flow as a function of particle number.

3. Hydrodynamic predictions.

Conclusion + Prospects.

1.
The quest for hydrodynamics in systems that are small.

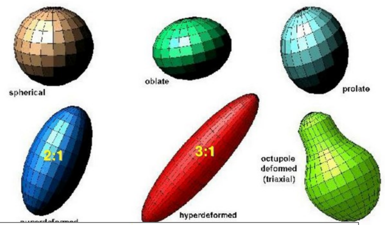
Emergent phenomena are among the most interesting in Nature.

“More is different”, **[P. Anderson, 1972]**

https://en.wikipedia.org/wiki/Emergence

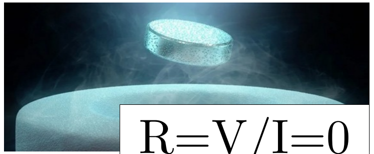
Examples relevant for nuclear / cold atom physics:

Nuclear deformations

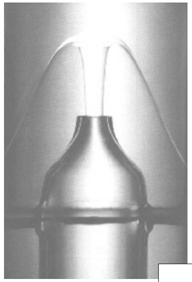


$$R = R_0\left(1 + \sum_n \beta_n Y_n^0\right)$$

Superconductivity



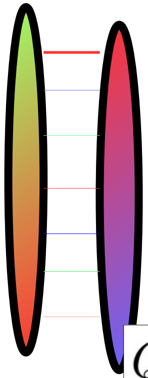
$\mathrm{R=V/I=0}$



$\eta=0$

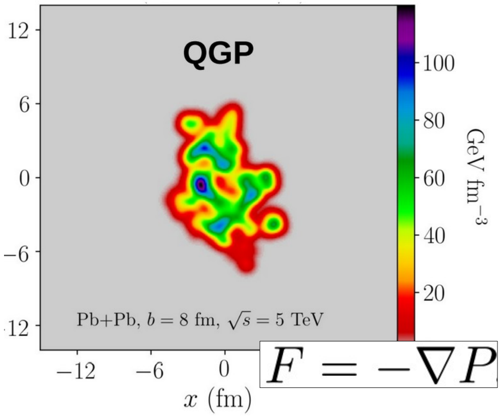
Superfluidity

color glass condensate



$Q_s^2(\mathbf{x})$

quark-gluon plasma



$F = -\nabla P$

Focus of this talk: Hydrodynamics, a prime example of emergent (macroscopic) behavior.

$$F = -\nabla P$$

Emergence in a particle system via collisions (kinetic theory).

The *pressure tensor* is defined as the fluctuation of the velocities of the ensemble from the mean velocity, i.e. as the 2-nd order moment:

$$\boxed{\mathbf{P} = m \int (\mathbf{v} - \mathbf{v}_b)(\mathbf{v} - \mathbf{v}_b) f(\mathbf{v}) d^3 v}$$

Emergence of superfluid motion in BEC (no collisions, but due to interactions in a Fermi gas).

$$\frac{\partial}{\partial t} n + \nabla(v_S n) = 0$$

EOS

$$m \frac{\partial}{\partial t} v_S + \nabla\left(\frac{1}{2} m v_S^2 + \mu(n) + V_{ext}\right) = 0$$

Hydrodynamic equations of superfluids (T=0)
Closed equations for
n and v_S

[from S. Stringari,
Lectures at Collège de France (2004/2005)]

Both situations require a mascroscopic scenario, i.e., very large particle numbers.

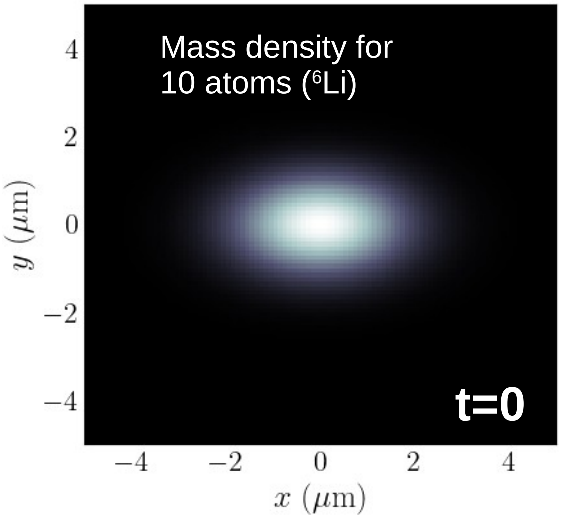
Frontier: mesoscopic systems? What if the particle number is small?

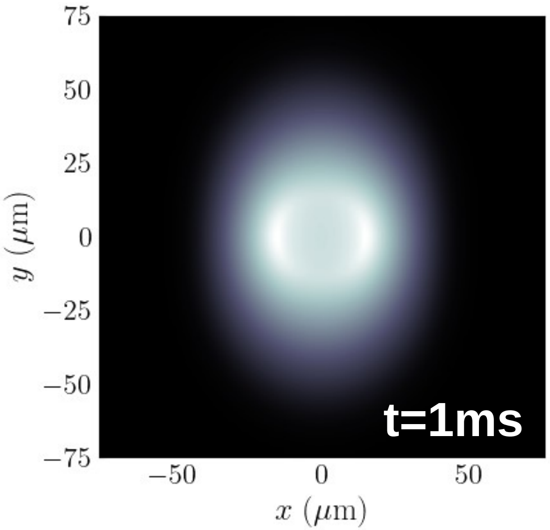
Tool to probe emergent hydrodynamic behavior:
Shape inversion of the gas due to asymmetry in pressure-gradient force (elliptic flow).

[Ollitrault, **PRD 46 (1992) 229-245**

Does not really matter whether system is superfluid or collisional.

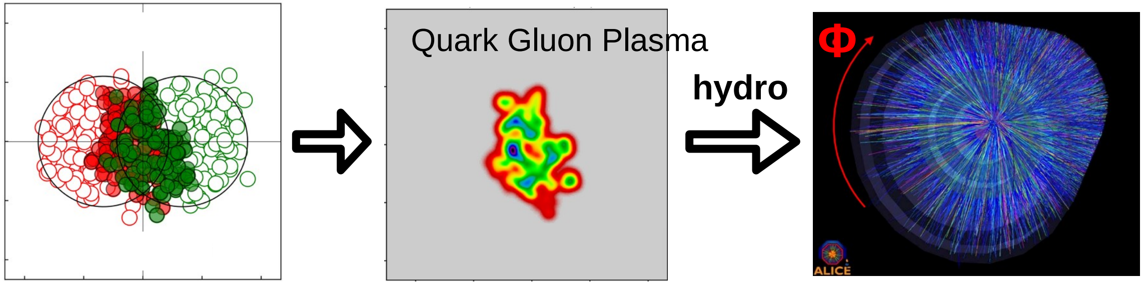
Realistic application: ideal Fermi gas in 2D at zero temperature.





In heavy-ion collisions:
2nd Fourier harmonic of the azimuthal particle distribution.

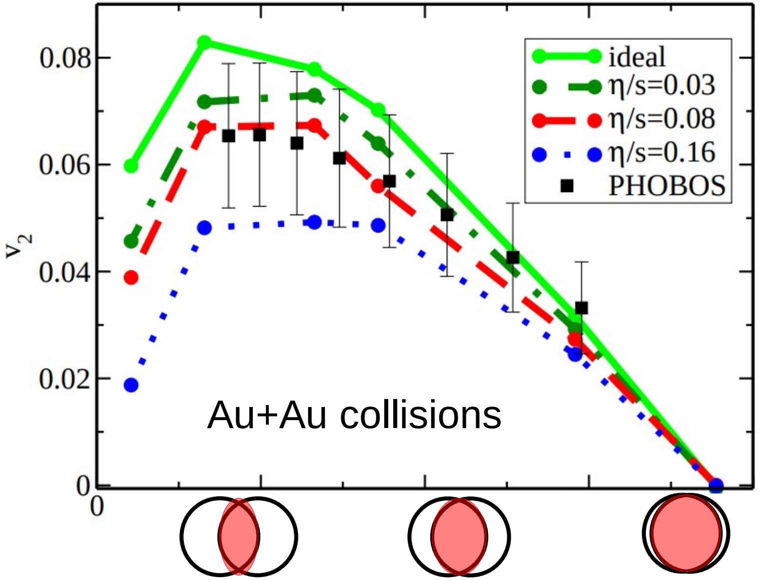
$$V_2 = \frac{1}{N}\int_{\mathbf{p}_t} \frac{dN}{d^2\mathbf{p}_t} e^{-i2\phi_p}$$



Elliptic flow probes viscous corrections.

$$\rho\frac{d\mathbf{v}}{dt} = \underbrace{-\vec{\nabla}P}_{1/R} + \underbrace{\eta\vec{\nabla}^2\mathbf{v} + \vec{\nabla}\left[\vec{\nabla}\cdot\mathbf{v}\left(\zeta+\frac{2}{3}\eta\right)\right]}_{1/R^2}$$

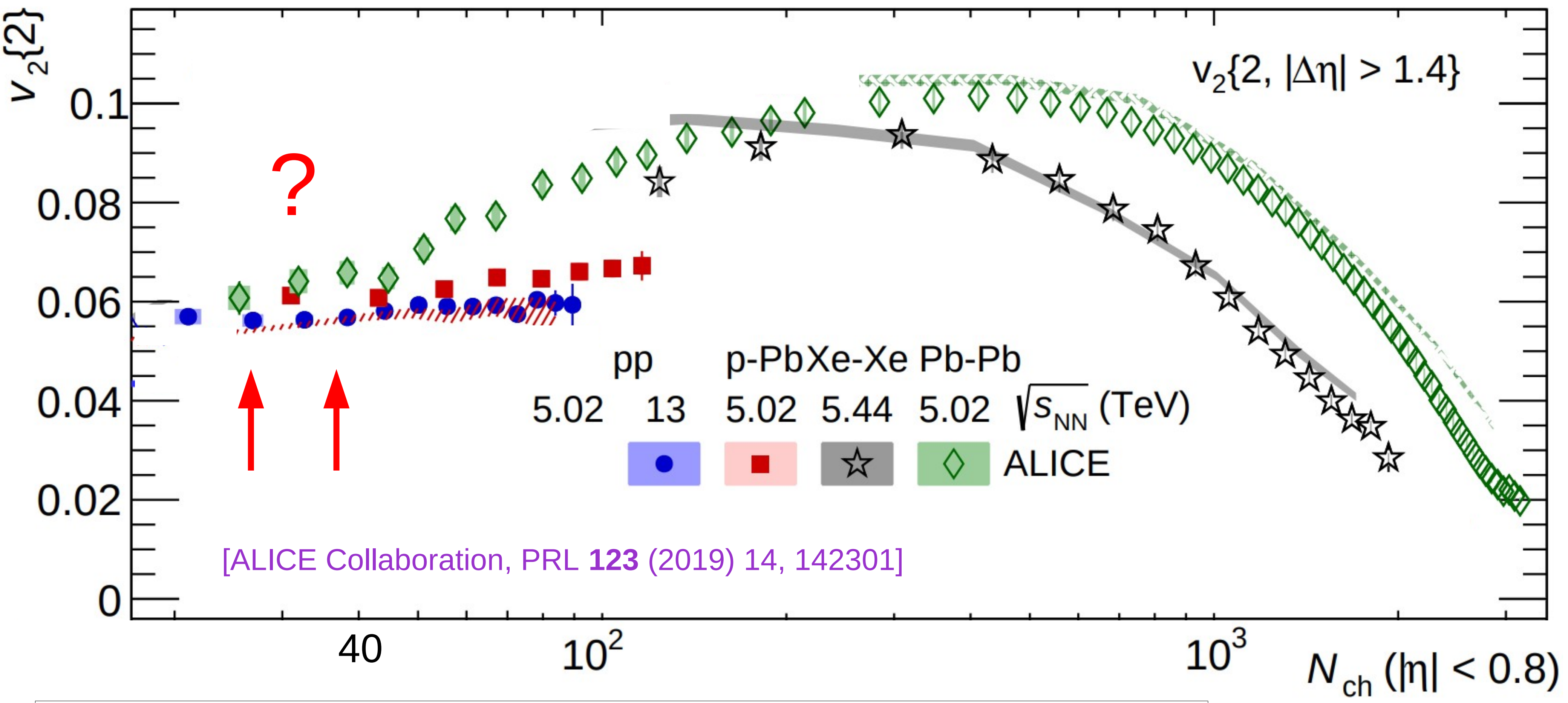
[Romatschke & Romatschke, **PRL 99, 172301 (2007)**]



Clear evidence that the QGP behaves like a strongly-coupled quasi-perfect fluid.

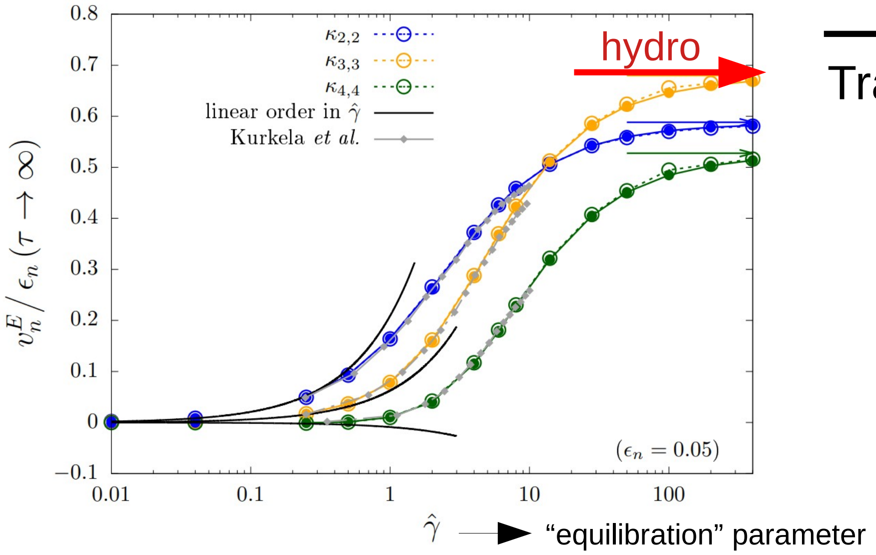
Why mesoscopic systems?

Signal of elliptic flow persists at low particle numbers.



30-40 strongly-interacting particles… can that make a fluid?

Understanding “small systems” is a very active research area.

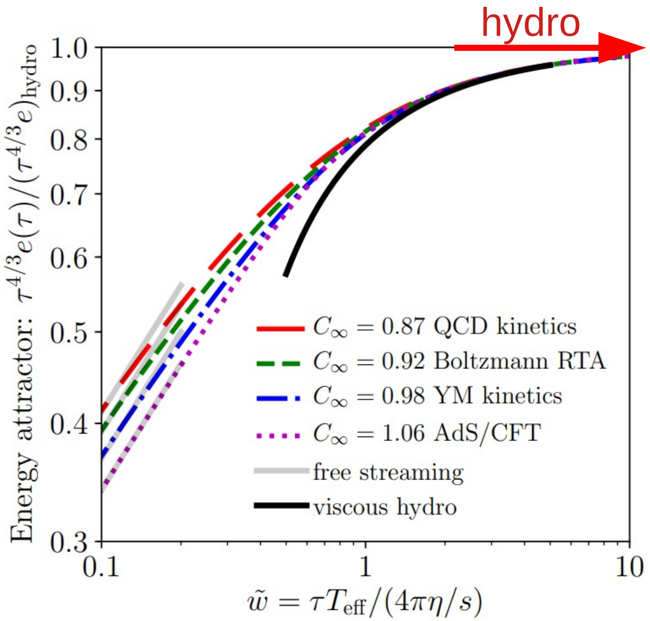


Transition to fluid dynamics.

[Kurkela, Wiedemann, Wu, **EPJC 79 (2019) 11, 965**]
[Ambrus, Schlichting, Werthmann, **PRD 105 (2022) 1, 014031**]

Out-of-equilibrium hydrodynamics.
Emergence of the hydrodynamic attractor.

[Romatschke & Romatschke, **arXiv:1712.05815**]
[Giacalone, Mazeliauskas, Schlichting, **PRL 123, 262301 (2019)**]
[Berges et al., **RMP 93 (2021) 3, 035003**]



Can we attack these questions with cold atom experiments?

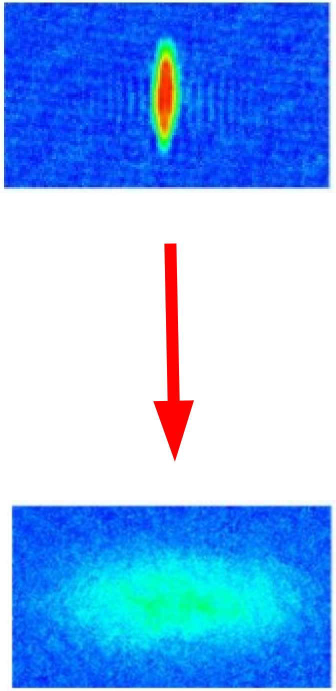
2.

Elliptic flow as a function of particle number.

[Floerchinger, Giacalone, Heyen, Tharwat, **PRC 105, 044908 (2022)**]

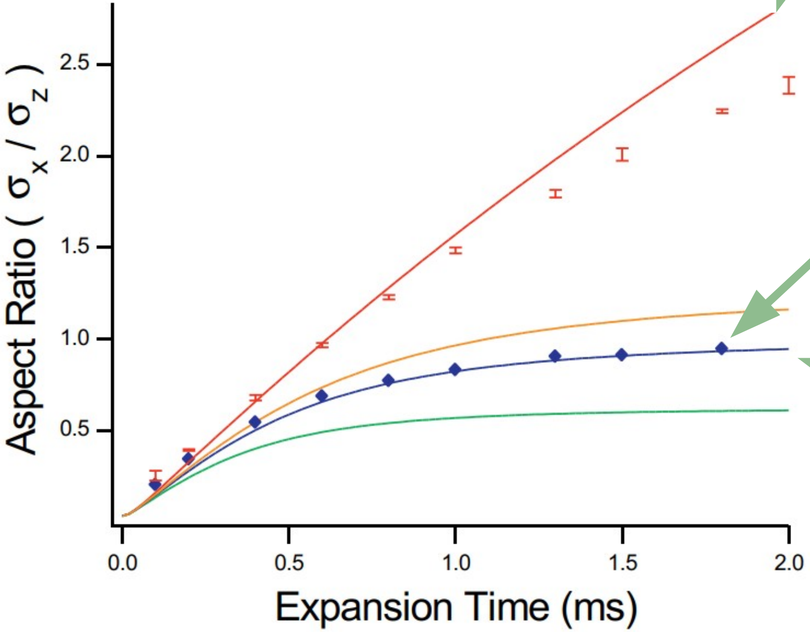
Interaction and system geometry can be tuned.

Elliptic flow used to reveal superfluid behavior of an ultracold Fermi gas.



100 μs

2000 μs



ideal hydrodynamics

strongly-interacting gas

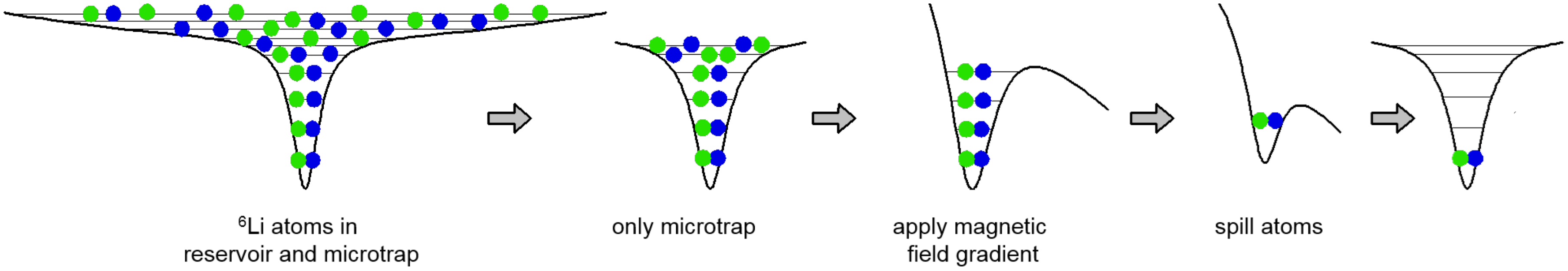
non-interacting gas

ballistic expansion

[Menotti, Pedri, Stringari, PRL **89**, 250402 (2002)]
[O’Hara et al., Science **298** (2002) 2179-2182]

Controlled transition from few-body to many-body physics.

[Serwane et al., Science **332** (2011) 6027]



[methods: next talk by Sandra Brandstetter]

Our proposal:
Study elliptic flow to assess emergent hydrodynamic behavior as a function of particle number (in two dimensions).

Measuring elliptic flow in mescoscopic samples.

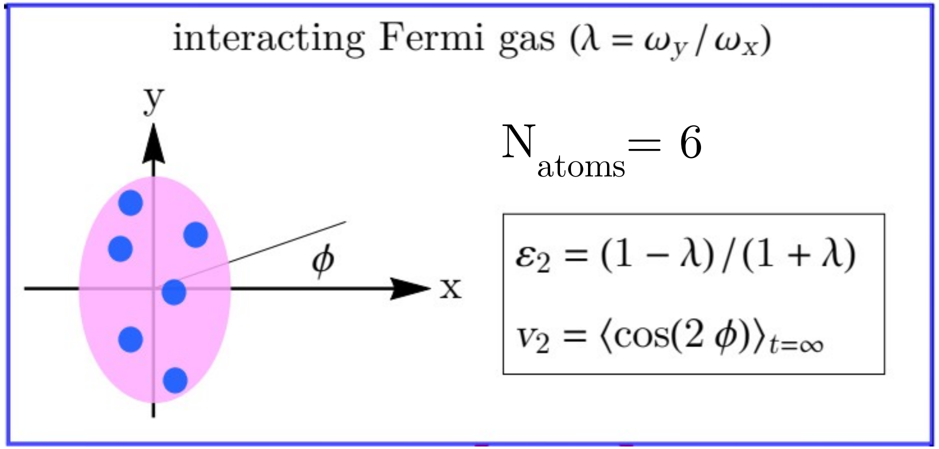
1 – Statistical description, i.e., repeat the experiment many times like in heavy-ion collisions.

2 – Unlike in heavy-ion collisions, orientation of the ellipse and ellipticity, ε_2, can be chosen.

3 – Let the system expand and measure anisotropy $<\cos 2\Phi_p>$ with respect to the fixed axis.

single-particle measurement!

4 – Repeat the experiments for different number of atoms in the cloud.



Assessing the “background”.

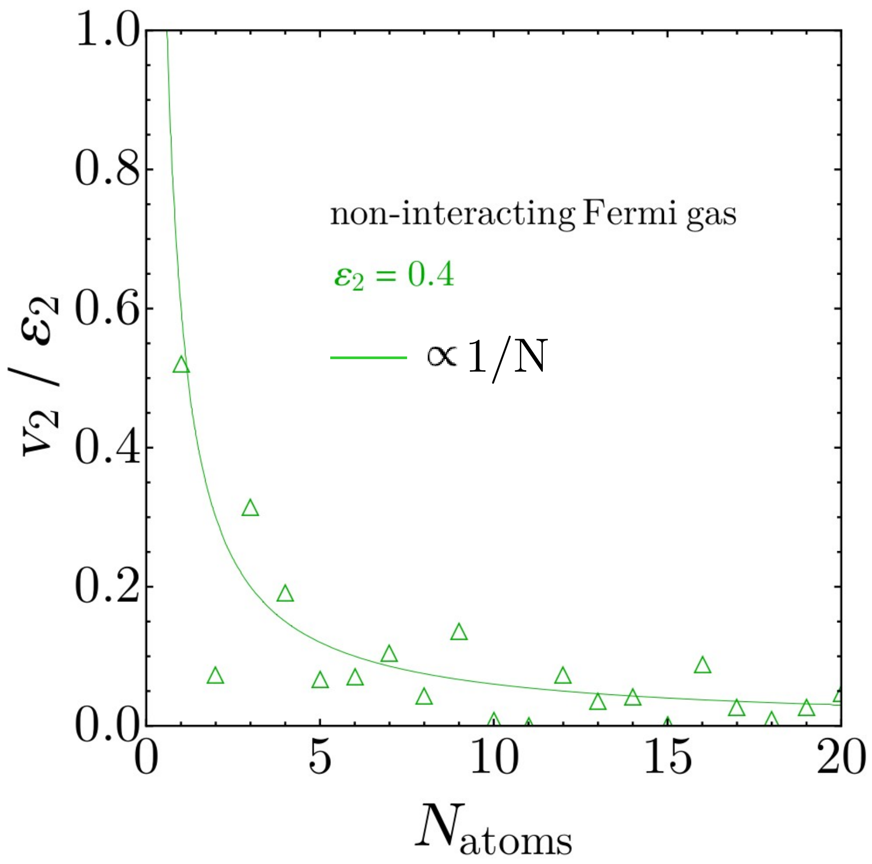
Imposing an elliptical potential has a strong impact on the initial momentum distribution.

$$\Delta p_x \Delta x \geq \frac{h}{4\pi}$$

Calculate v2 from the quantum harmonic oscillator (initial momentum anisotropy).

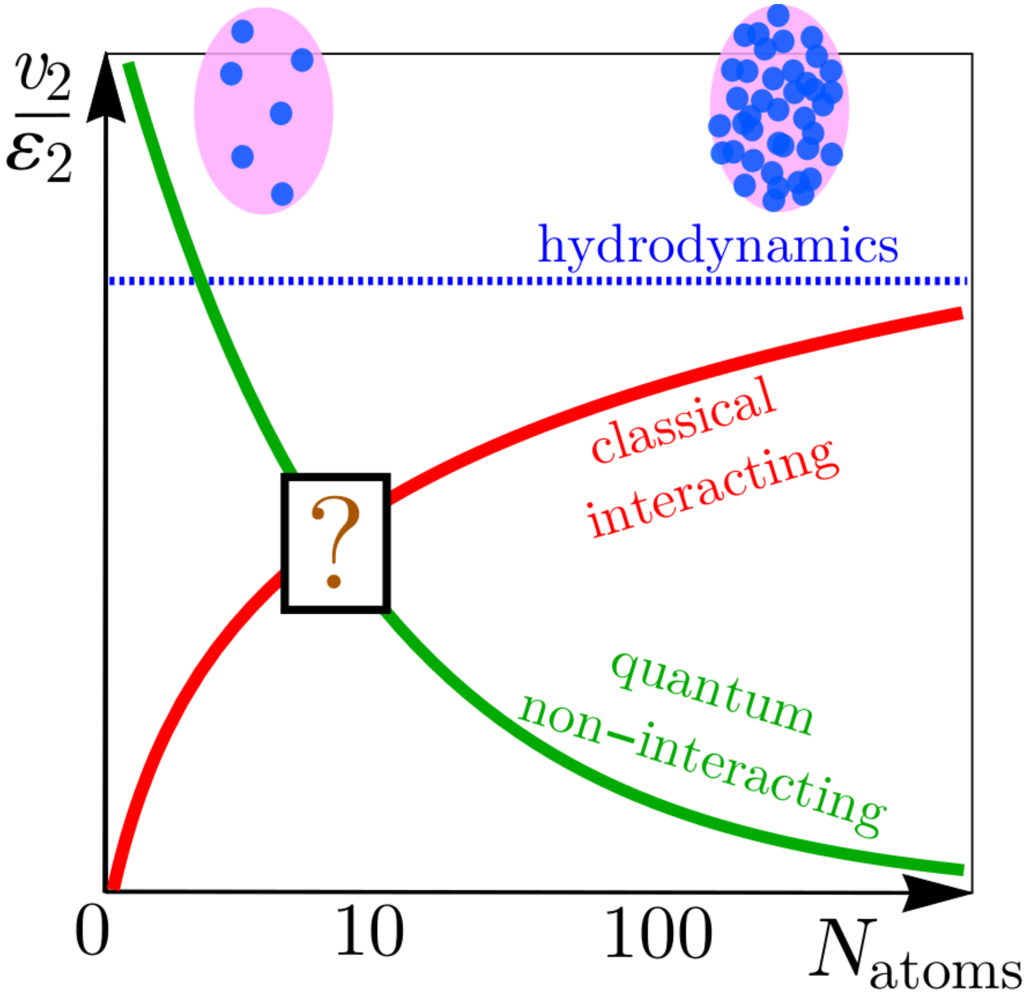
$$v_2 = \langle \cos(2\phi_p) \rangle_\Psi$$

trapped non-interacting fermions



It disappears quickly, like 1/N.

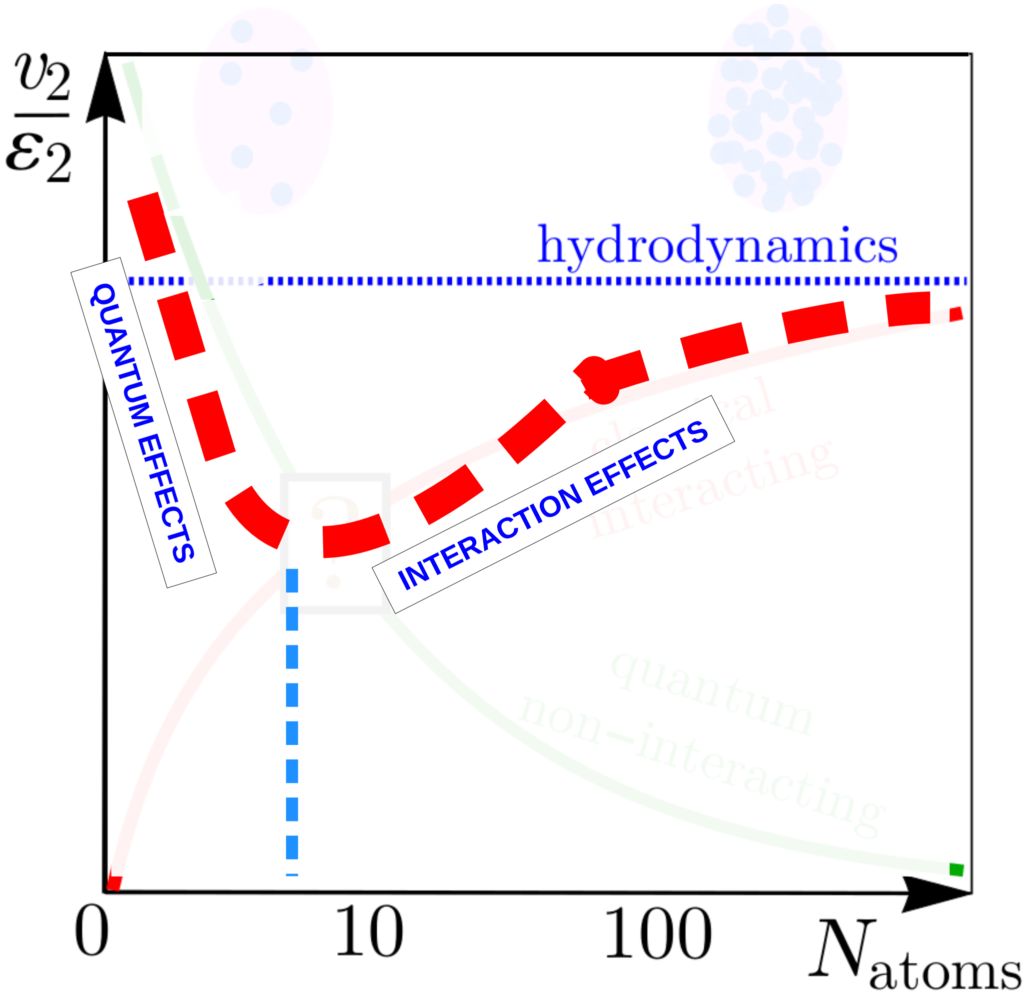
Qualitative expectations.



Combining the curves…

Could there be a minimum?

Transition from quantum effects to interaction effects?



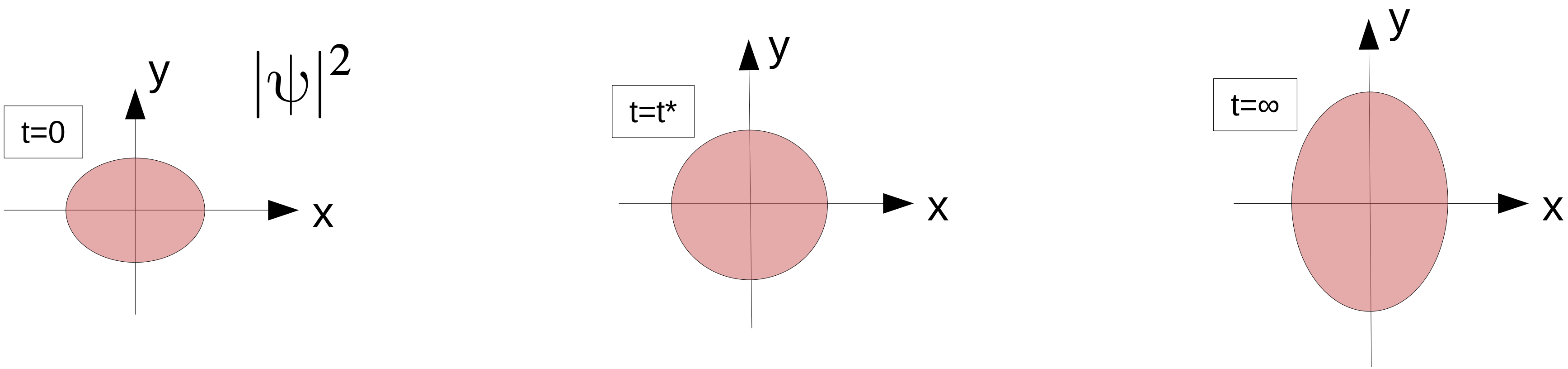
[experiment: next talk by Sandra Brandstetter]

3.
Hydrodynamic predictions.

How to test an emergent fluid description?

Momentum space anisotropy can not be directly predicted by hydrodynamics.

Idea: look at the evolution of the "average cloud" → $|\psi|^2(x,y)$



Our approach:

– Take the same geometry as experiment at t=0.

– Assume it represents a fluid with a total mass of 10 ^{6}Li atoms.

– Evolve in time according to ideal (superfluid?) hydrodynamics.

We only need the pressure.

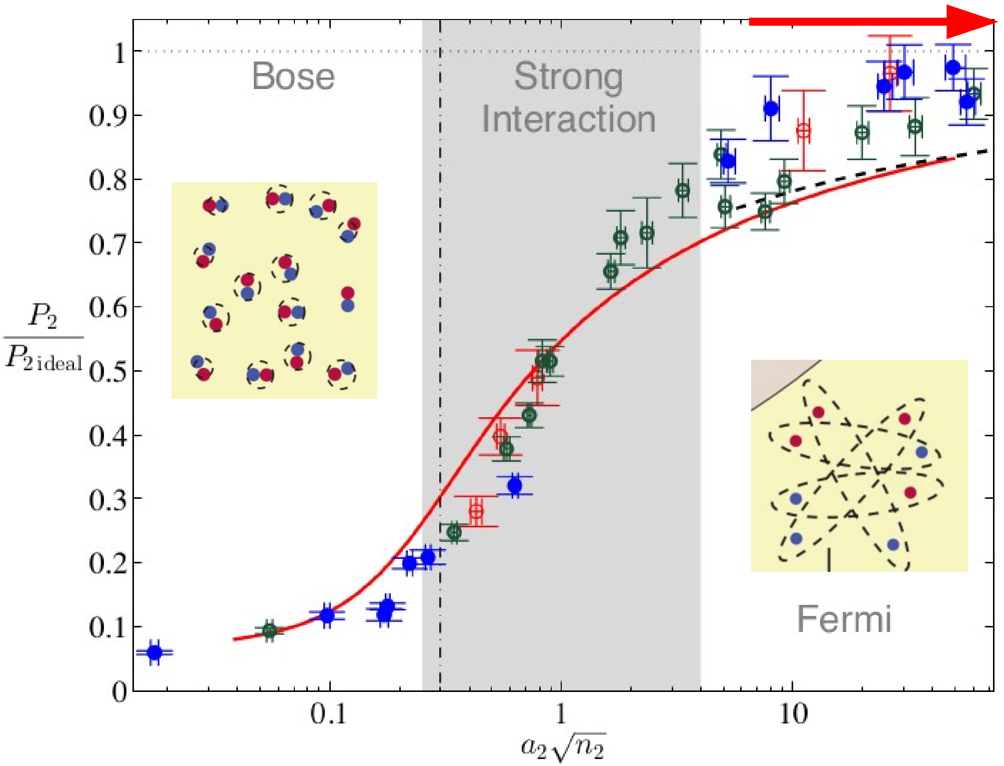
$$F = -\nabla P$$

[Levinsen, Parish, Annual Review of Cold Atoms and Molecules, arXiv:1408.2737]

EoS for ideal Fermi gas:

$$P_{\text{ideal}} = \frac{\pi\hbar^2}{2M} n^2$$

Mass of ^{6}Li

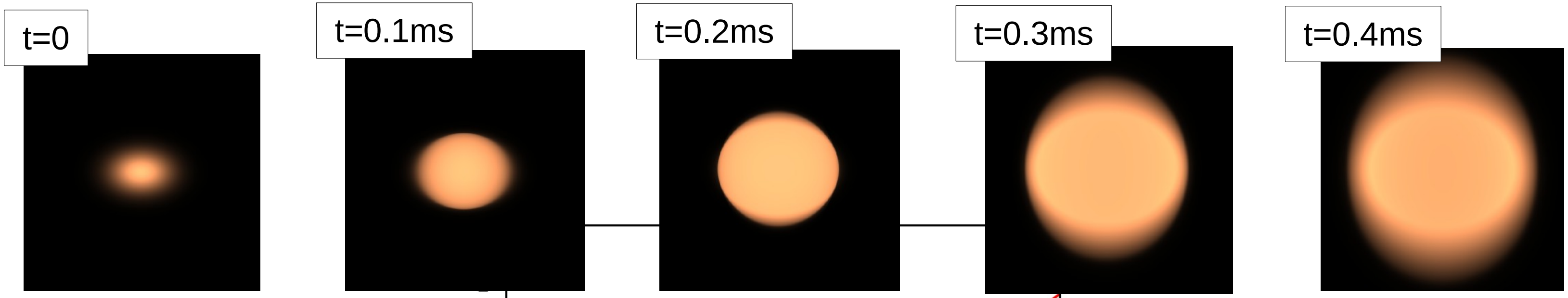


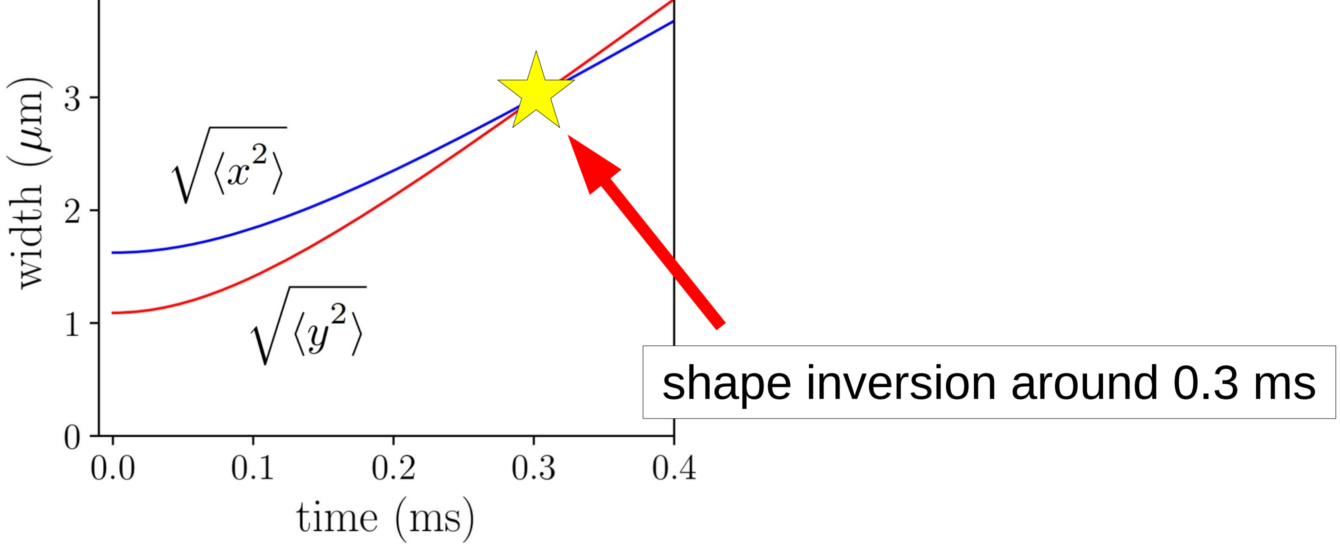
Realistic pressure at trap center for experimental setup:

$a_{2D} \approx 2.4\ \mu\text{m}$ ⇨ $P \approx 0.7 P_{\text{ideal}}$

HYDRODYNAMIC PREDICTIONS

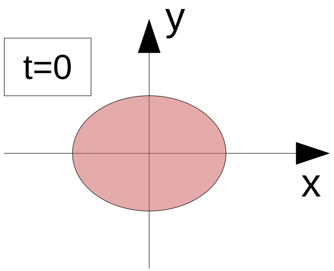
Compressible hydro solver developed at Stony Brook:
https://pyro2.readthedocs.io/en/latest/index.html#





$P = P_{ideal}$

10 atoms



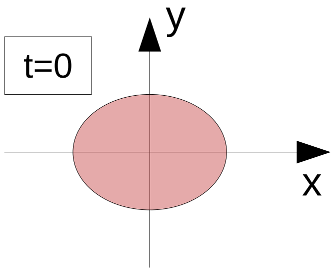
[experiment: next talk by Sandra Brandstetter]

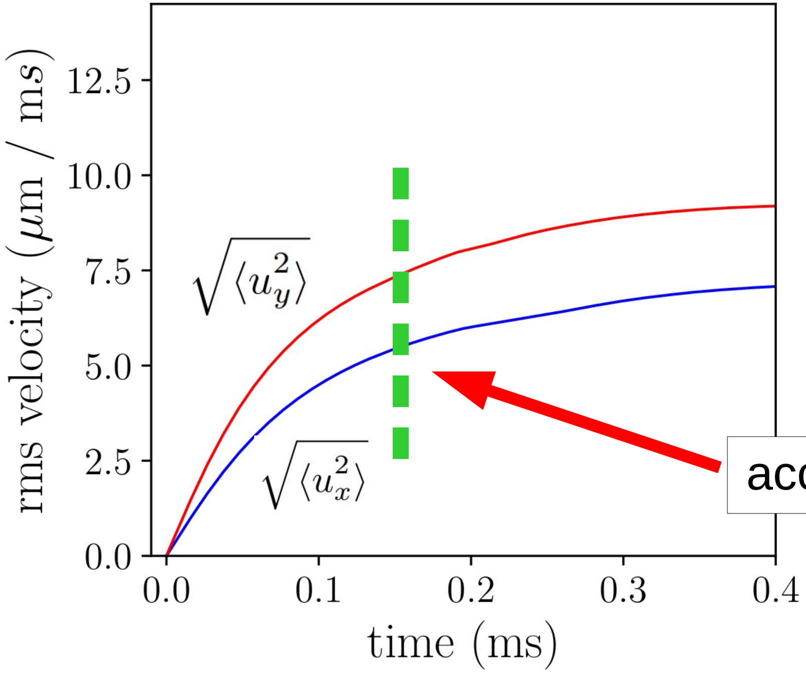
HYDRODYNAMIC PREDICTIONS

Over what time scale are gradients effective? $F = -\nabla P$

$P = P_{ideal}$

10 atoms





acceleration ends around 0.15 ms

Should be detected as well in momentum space distributions

[experiment: next talk by Sandra Brandstetter]

CONCLUSION

- Emergent fluid behavior observed across scales (superfluids: $T=0$, QGP: $T\sim10^{12}$ K)
- Cold atoms to assess fluid behavior with tunable particle number and interactions.
- <u>Method from high-energy nuclear collisions to measure elliptic flow with few particles.</u>
- “Background” effects leading to elliptic flow vanish quickly with the particle number.
- **Ideal hydrodynamic predictions for $|\psi|^2$ in the experimental setup.**

PROSPECTS

- Study more observables (e.g. triangular flow, mean momentum)
- Going beyond $|\psi|^2$? Event-by-event analysis?
- Further signals of superfluidity (rotational properties?)
- Microscopic approach?

THANK YOU!