Universal induced interaction between heavy polarons in superfluid

— Effective field theory approach to polaron physics —

KF, M. Hongo, & T. Enss, Phys. Rev. Lett. (in press)
[arXiv:2206.01048] (2022)

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14 November 2022

Plan of this talk

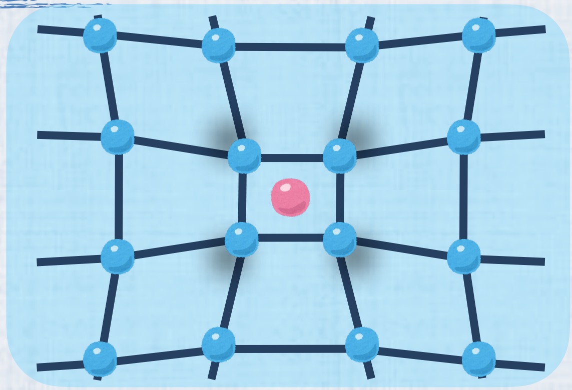
1. **Introduction of the polaron**
   - Polaron in ultracold atoms
   - Induced interaction between polarons

2. **Induced interaction between polarons**
   - Theoretical formulation of polaron physics
   - EFT approach : focusing on linear dispersion phonons

3. **Magnitude of the potential in the BCS-BEC crossover**

4. **Summary**

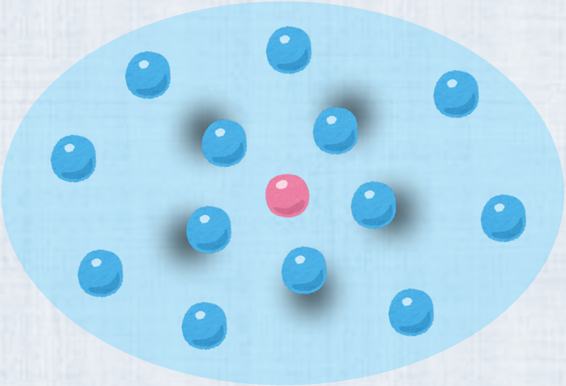
What is the polaron?



Polaron (Landau's original definition)

: an electron interacting with phonons in a crystal

lattice wave inducing **polarization**



Polaron in ultracold atoms

: an impurity interacting with quantum gas particles

▸ Ultracold atoms provide a simple and ideal research platform.

✓ **High experimental controllability**

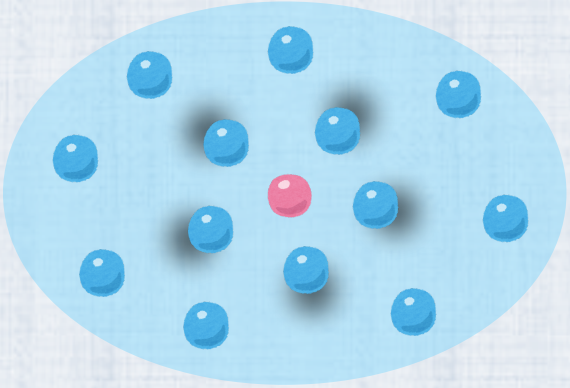
- quantum statistics & internal degrees of freedom
- impurity-medium and medium-medium interaction

From one-body to two-body

Focusing on impurities immersed in a superfluid

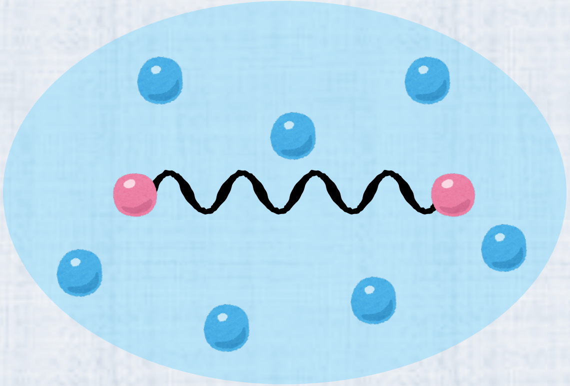
One impurity problem

: effective mass, mobility, dressing cloud, etc.



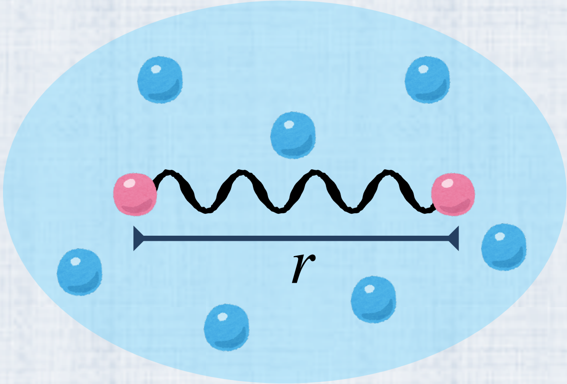
Two impurity problem

: induced interaction, bipolaron state, etc.



Induced Interaction between impurities

Focusing on impurities immersed in a superfluid



Two impurity problem

: induced interaction, bipolaron state, etc.

mediated by exchanging bosonic quanta

Superfluid phonons

✓ **The Yukawa potential** at weak impurity-medium interaction in BEC

$$V(r) \sim -\frac{e^{-\sqrt{2}r/\xi}}{r}$$ (ξ : healing length)

See e.g. Pethick & Smith's text book "Bose-Einstein condensation in Dilute gases"

- Short-range potential mediated by a **gapped** mode
- There is a **gapless** mode (superfluid phonon) governing long-range physics

Is there a long-range induced interaction mediated by gapless modes?

Plan of this talk

1. Introduction of the polaron

Is there a long-range induced interaction mediated by gapless modes?

→ Yes!! Long-range van der Waals force

2. **Induced interaction between polarons**
   - Theoretical formulation of polaron physics
   - EFT approach : focusing on linear dispersion phonons

3. Magnitude of the potential in the BCS-BEC crossover

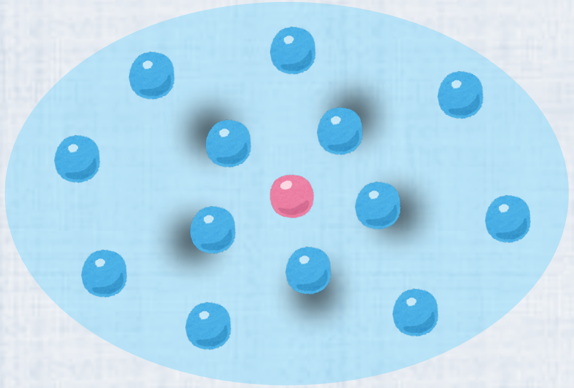
4. Summary

Theoretical formulation of polaron physics

✓ Microscopic model :

Impurities interacting with a medium

$$\mathcal{L}_{\mathrm{micro}}(x) = \mathcal{L}_{\mathrm{imp}}(x) + \mathcal{L}_{\mathrm{medium}}(x) + \mathcal{L}_{\mathrm{int}}(x)$$



▸ Impurity-medium interaction in the contact *s*-wave channel

$$\mathcal{L}_{\mathrm{int}}(x) = -g_{IM} \underbrace{\Phi^\dagger(x)\Phi(x)}_{\text{Impurity density}} \underbrace{\psi^\dagger(x)\psi(x)}_{\text{Medium density}}$$

✓ Our problem is to find $S_{\mathrm{polaron}}[\Phi, \Phi^\dagger]$ by integrating out the medium

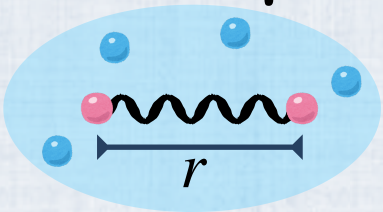
$$\exp\left[iS_{\mathrm{polaron}}[\Phi, \Phi^\dagger]\right] = \int \mathcal{D}(\psi, \psi^\dagger)\ \exp\left[i \int dt d^3x\, \mathcal{L}_{\mathrm{micro}}(x)\right]$$

▸ Formally simple, but difficult to perform the integration

Effective field theory method: Superfluid EFT

Focus only on the long-distance behavior of the induced potential

$$r \gg \xi \quad (\xi \text{ : healing length})$$



➡ Focus only on the linear dispersion phonon

From **the Galilean invariance** of the medium, the medium Lagrangian is **generally** given in

✓ Galilean superfluid EFT for the medium

$$\mathcal{L}_{\text{medium}}(x) = \mathcal{P}(\theta(x)) \quad \text{with} \quad \theta(x) = \mu - \partial_t \phi(x) - \frac{(\nabla\phi(x))^2}{2m}$$

Galilean invariant combination

$\mathcal{P}(\mu)$: Pressure as a function of μ $\qquad$ $\phi(x)$: phonon field showing a linear dispersion

M. Greiter, F. Wilczek, & E. Witten, Mod. Phys. Lett. B **3**, 903 (1989);
D. T. Son & M. Wingate, Ann. Phys. **321**, 197 (2006).

✓ Interaction term

$$\mathcal{L}_{\text{int}}(x) = -g_{IM}\Phi^\dagger(x)\Phi(x)n(\theta(x))$$

$$\text{with} \quad n(\mu) = \mathcal{P}'(\mu)$$

cf.

$$\mathcal{L}_{\text{int}}(x) = -g_{IM}\Phi^\dagger(x)\Phi(x)\psi^\dagger(x)\psi(x)$$

Medium density

Effective theory for impurities in a superfluid

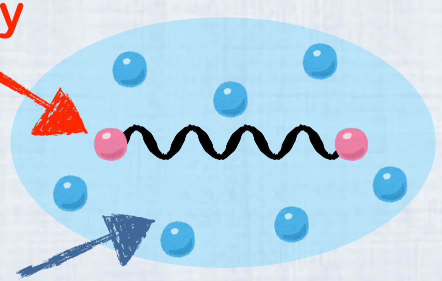
✓ Our effective theory valid at long distances $r \gg \xi$

$$\mathcal{L}_{\text{eff}}(x) = \mathcal{L}_{\text{imp}}(x) + \mathcal{P}(\theta(x)) - g_{IM}\Phi^\dagger(x)\Phi(x)n(\theta(x))$$

▸ **Galilean invariant combination** $\theta(x) = \mu - \partial_t\phi(x) - \dfrac{(\nabla\phi(x))^2}{2m}$

▸ Our assumptions are only two:

- **Galilean invariant medium**
- **Contact s-wave impurity-medium coupling**

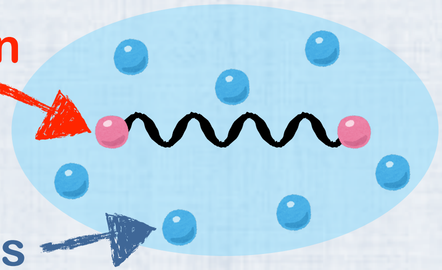


➡ Universal !! : **Independent of the details of the medium**

Our remaining task is **to calculate induced interactions from our effective theory**

cf. nuclear forces are computed from chiral effective field theory

See e.g., R. Machleidt & D. R. Entem,
"Chiral effective field theory and nuclear forces," Phys. Rept. **503**, 1 (2011).



Induced interaction mediated by phonons

Expanding $\mathcal{P}(\theta)$ & $n(\theta)$ and keeping the leading terms with rescaling $\varphi = \sqrt{\chi}\phi$

$$\mathcal{L}(x) = \mathcal{L}_{\rm imp}(x) - g_{IM} n \Phi^\dagger \Phi + \underline{\frac{1}{2}(\partial_t \varphi)^2 - \frac{1}{2}c_s^2 (\nabla \varphi)^2} + \underline{g_{IM}\left[\sqrt{\chi}\partial_t \varphi + \frac{(\nabla \varphi)^2}{2m}\right]\Phi^\dagger \Phi} + \cdots$$

$\chi = n'(\mu)$: compressibility

$c_s = \sqrt{n/(m\chi)}$: speed of sound

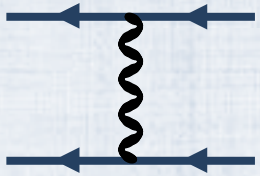
Kinetic term for phonons showing the linear dispersion

✓ Interaction terms between impurities and phonons

$g_{IM}\sqrt{\chi}\partial_t \varphi \Phi^\dagger \Phi$: one-body coupling $\quad$ $g_{IM}\dfrac{(\nabla \varphi)^2}{2m}\Phi^\dagger \Phi$: two-body coupling

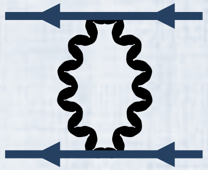
- **The coefficients are constrained by the Galilean invariance**

- One-body coupling ➡ one-phonon exchange $\tilde{V}(k) \sim$



$$V(r) = \int \frac{d^3k}{(2\pi)^3} \underline{\tilde{V}(k)} e^{i\boldsymbol{k}\cdot\boldsymbol{x}}$$

- Two-body coupling ➡ two-phonon exchange $\tilde{V}(k) \sim$



One-phonon exchange potential

Expanding $\mathcal{P}(\theta)$ & $n(\theta)$ and keeping the leading terms with rescaling $\varphi = \sqrt{\chi}\phi$

$$\mathcal{L}(x) = \mathcal{L}_{\mathrm{imp}}(x) - g_{IM} n \Phi^\dagger \Phi + \frac{1}{2}(\partial_t \varphi)^2 - \frac{1}{2} c_s^2 (\boldsymbol{\nabla}\varphi)^2 + g_{IM}\left[\sqrt{\chi}\partial_t \varphi + \frac{(\boldsymbol{\nabla}\varphi)^2}{2m}\right]\Phi^\dagger\Phi + \cdots$$

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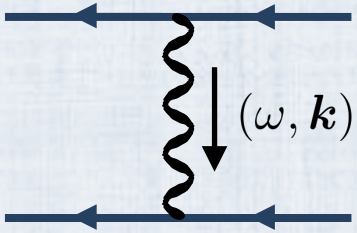
✓ **Static potential = exchanging purely spatial modes with** $(\omega = 0,\ \boldsymbol{k})$

$\tilde{V}(k) \sim$  $\propto \omega^2$

$g_{IM}\sqrt{\chi}\partial_t \varphi \Phi^\dagger \Phi$: one-body coupling

One-phonon exchange potential

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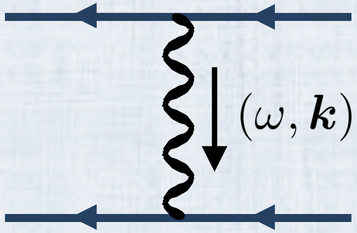
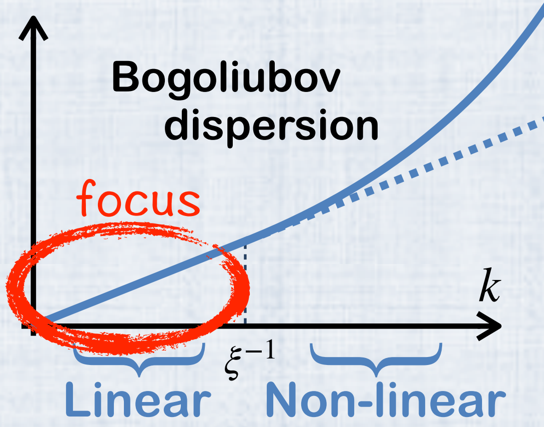
✓ Static potential = exchanging purely spatial modes with $(\omega = 0, \boldsymbol{k})$

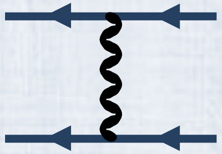
$\tilde{V}(k) \sim$  $\propto \omega^2$ ➡ **0** at $\omega = 0$



▸ **Consistent with the well-known Yukawa potential result**

: The Yukawa potential effectively vanishes at $r \gg \xi$

cf. In the Bogoliubov approach, one-mode exchange has only a contribution from the non-linear dispersion part.

$\tilde{V}(k) \sim$  $\sim -g_{IM}^2 \dfrac{n}{V}\dfrac{1}{\epsilon_{\boldsymbol{k}} + 2\mu}$ $\quad E_{\boldsymbol{k}} = \sqrt{\epsilon_{\boldsymbol{k}}(\epsilon_{\boldsymbol{k}} + 2\mu)}$

Non-linear part

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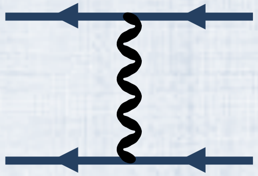
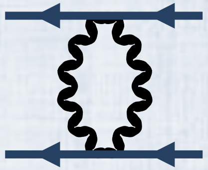
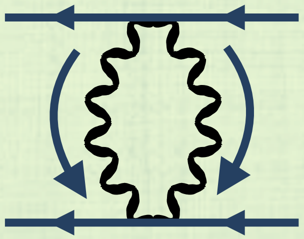
- One-body coupling ➡ one-phonon exchange $\tilde{V}(k) \sim$  = 0

- Two-body coupling ➡ two-phonon exchange $\tilde{V}(k) \sim$  = finite

Van der Waals force from two-phonon exchange

✓ **Two-phonon exchange potential** from $g_{IM}\frac{(\nabla\varphi)^2}{2m}\Phi^\dagger\Phi$



At zero temperature

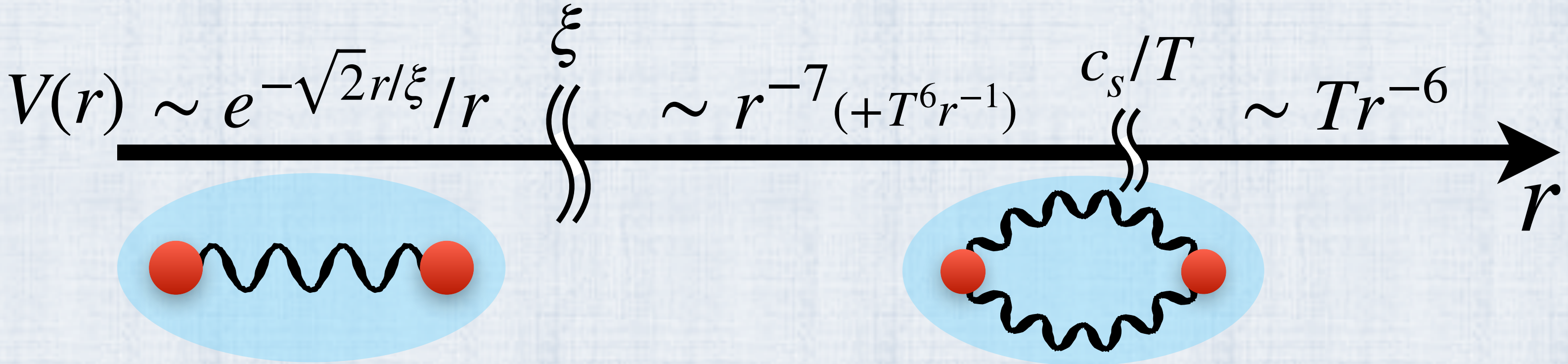
$$V_{T=0}(r) = -g_{IM}^2\frac{43}{128\pi^3 m^2 c_s^3}\frac{1}{r^7}$$

relativistic van der Waals

At finite temperatures (c_s/T : temperature length scale)

$$V(r) = \begin{cases} V_{T=0}(r) - g_{IM}^2\frac{\pi^3 T^6}{135 m^2 c_s^9}\frac{1}{r} & (r \ll c_s/T) \\ -g_{IM}^2\frac{3T}{16\pi^2 m^2 c_s^4}\frac{1}{r^6} & (r \gg c_s/T) \end{cases}$$

non-relativistic van der Waals



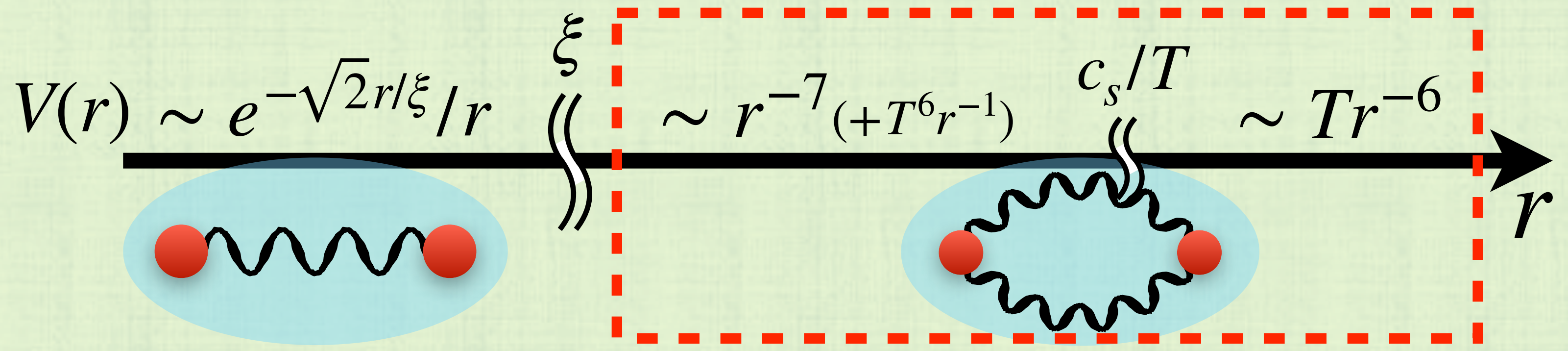
Plan of this talk

1. Introduction of the polaron

2. Induced interaction between polarons

Is there a long-range induced interaction mediated by gapless modes?

→ Yes!! Long-range van der Waals force



3. Magnitude of the potential in the BCS-BEC crossover

4. Summary

Our potential vs. Yukawa potential

Comparing our result with the Yukawa potential

✔ **The obtained potential** $V_{T=0}(r) = -g_{IM}^2 \frac{43}{128\pi^3 m^2 c_s^3} \frac{1}{r^7}$

✔ **The Yukawa potential** $V_{\text{Yukawa}}(r) = -g_{IM}^2 \frac{mn}{2\pi} \frac{e^{-\sqrt{2}r/\xi}}{r}$

$$\frac{V_{T=0}}{V_{\text{Yukawa}}} = \frac{43}{16\sqrt{2}\pi^2} \frac{1}{n\xi^3} \frac{e^{\sqrt{2}x}}{x^6} \quad \text{with} \quad x = r/\xi \qquad \xi = \frac{1}{\sqrt{2}mc_s}$$

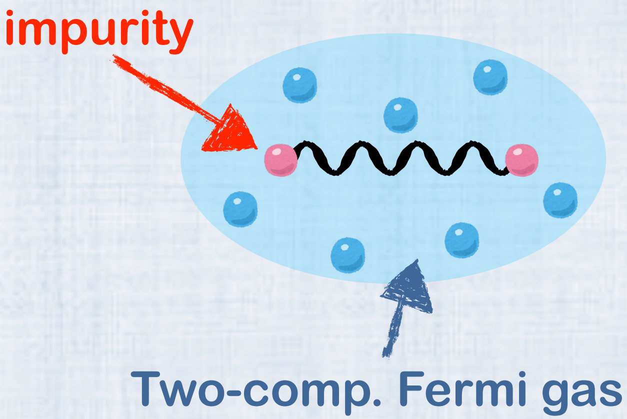
so-called gas parameter

▸ Larger for strongly interacting BEC

- Both are proportional to g_{IM}^2
- Our power-law potential stronger than the Yukawa potential at large distance
- Our potential is of higher order in terms of the gas parameter

Induced potential in BCS-BEC crossover

Our results are valid in the entire BCS-BEC crossover



▸ Our results are based only on two assumptions

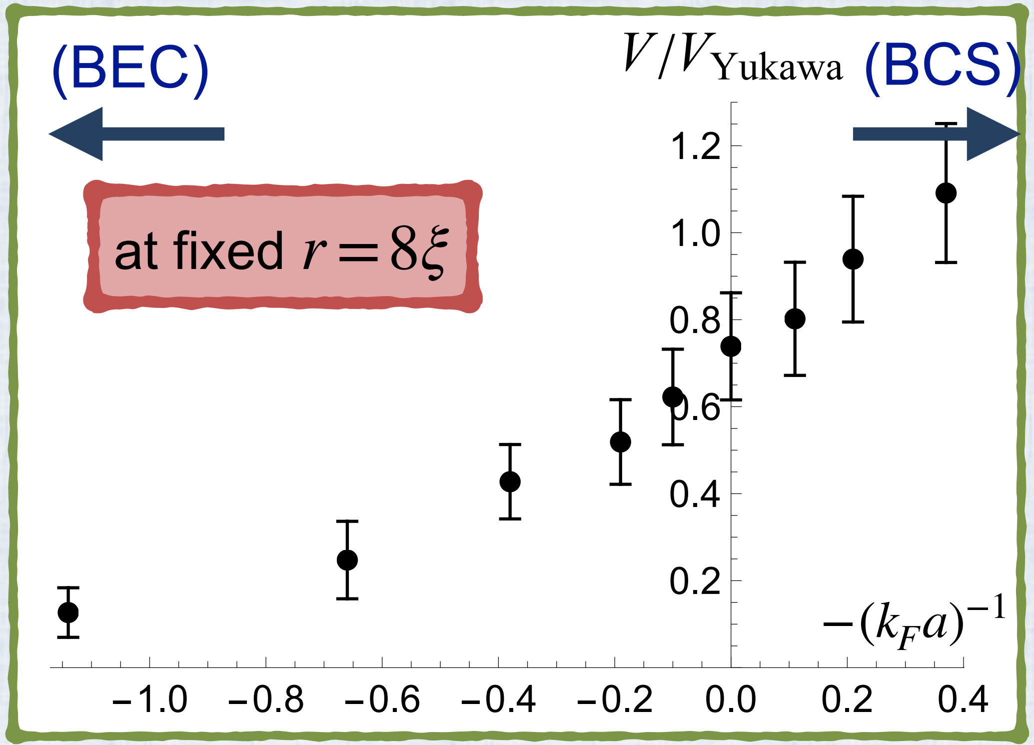
- **Galilean invariant medium**
- **Contact s-wave impurity-medium coupling**

✓ Plotting the ratio as a function of the scattering length

$$\frac{V_{T=0}}{V_{\text{Yukawa}}} = \frac{43}{16\sqrt{2}\pi^2}\frac{1}{n\xi^3}\frac{e^{\sqrt{2}x}}{x^6} \quad \text{with} \quad x = r/\xi$$

with the use of the experimental data

S. Hoinka, et al., Nature Physics **13**, 943 (2017)



▸ The van der Waals potential is small in the BEC side, but becomes relatively larger when $-(k_F a)^{-1}$ increases

▸ **At unitarity, the van der Waals potential is dominant in $r \gtrsim 8\xi$.**

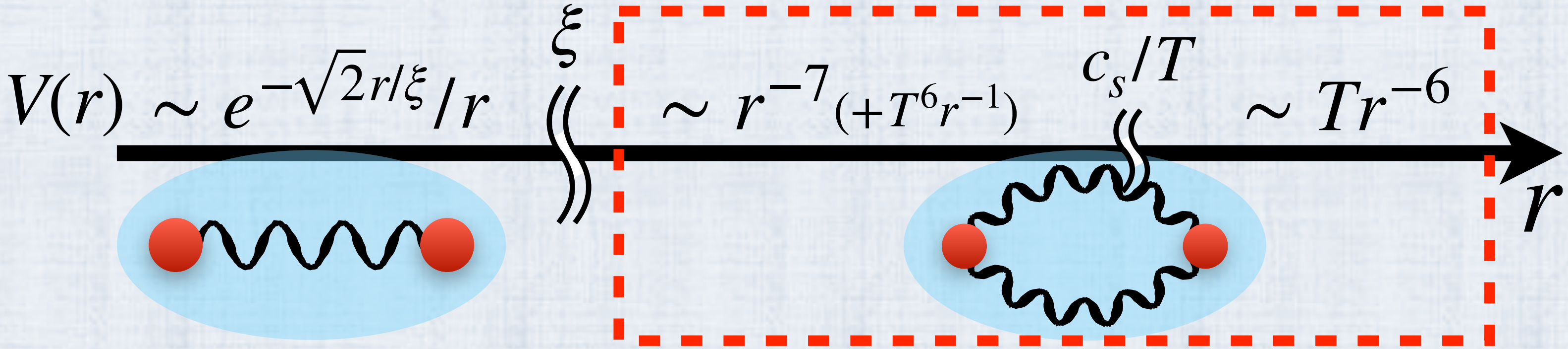
At unitarity, our EFT is robust because of small ξ

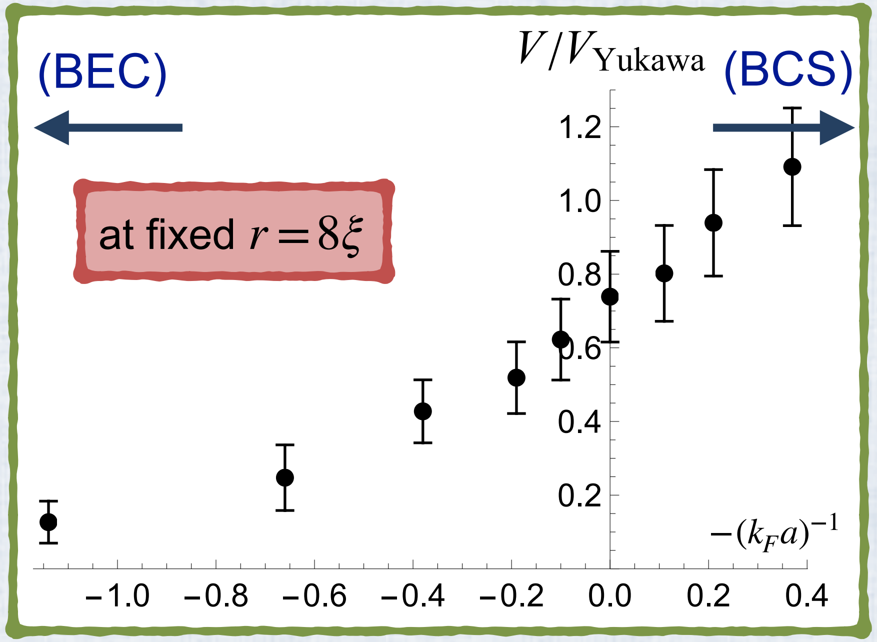
Summary

✓ Induced interaction between impurities in a superfluid

KF, M. Hongo, & T. Enss, Phys. Rev. Lett. (in press)
[arXiv:2206.01048] (2022)

Our results

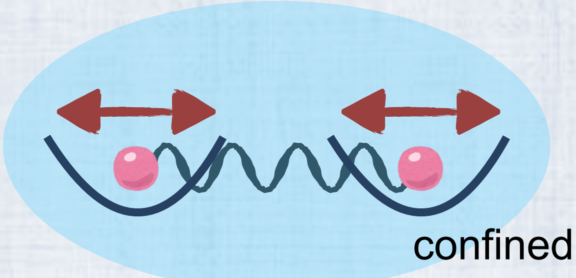




- based on only two assumptions:
  - **Galilean invariant medium**
  - **Contact s-wave impurity-medium coupling**
- The van der Waals potential becomes relatively larger when $-(k_F a)^{-1}$ increases

Experimentally measurable ?

- Ramsey interferometry
- Frequency shift of the out-of-phase mode



confined to separate micro-traps

Thank you!!