Extending the fluid dynamic description of heavy-ion collisions to times before the collision

by Andreas Kirchner

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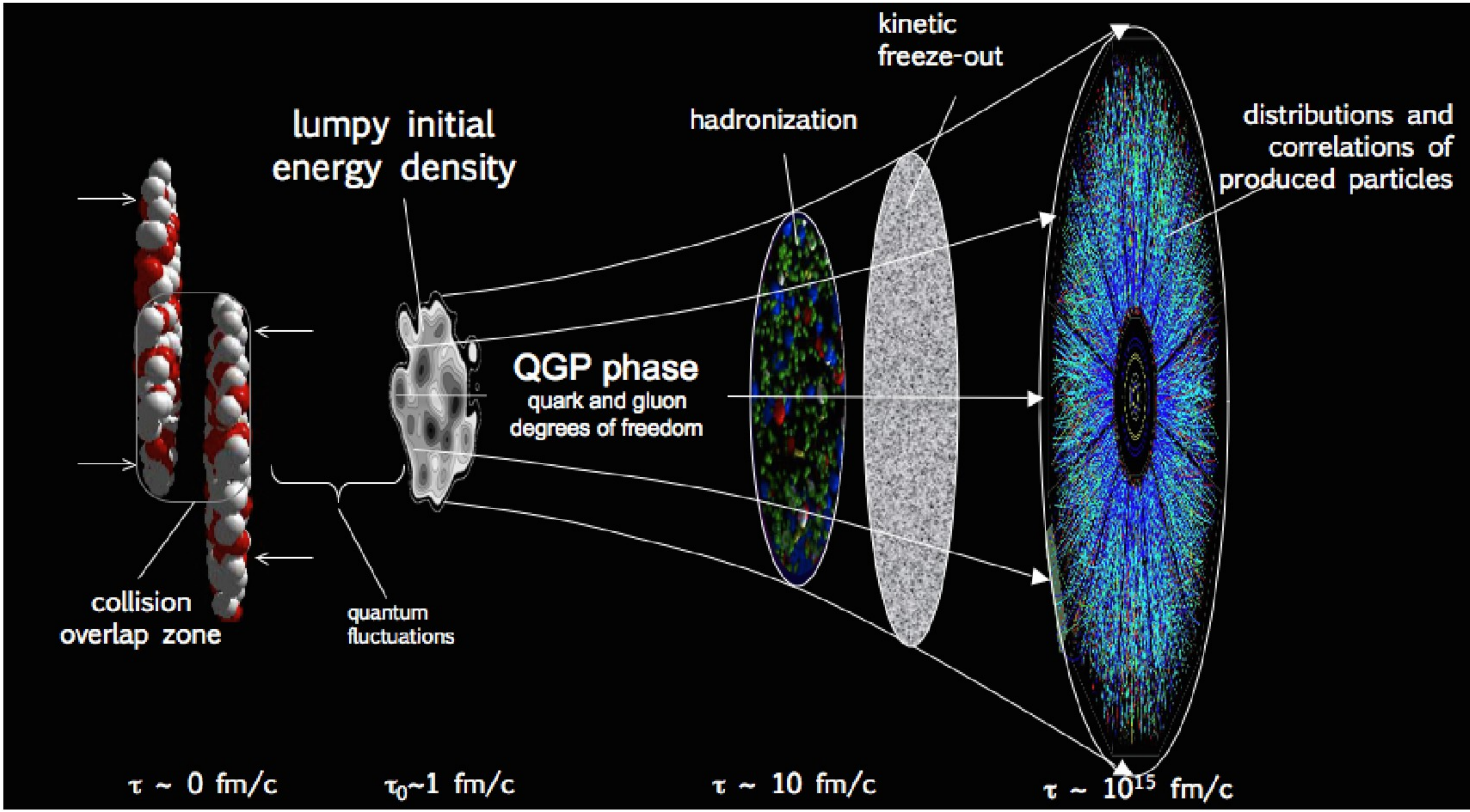
F. Capellino, A. Erschfeld, S. Floerchinger, E. Grossi, A. Kirchner

UNIVERSITÄT HEIDELBERG
ZUKUNFT SEIT 1386

IsoQuant SFB1225

Standard model of heavy-ion collisions



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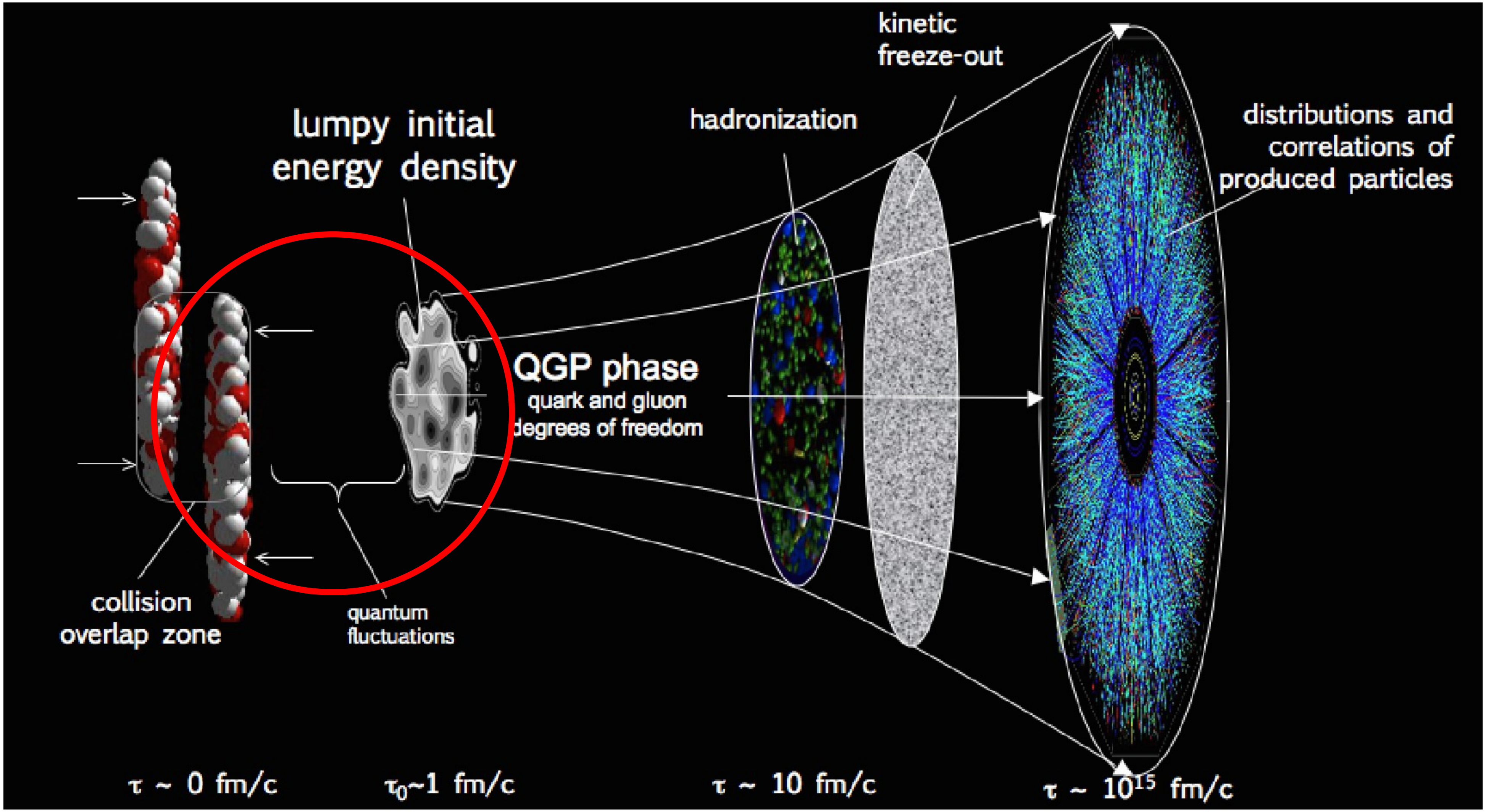


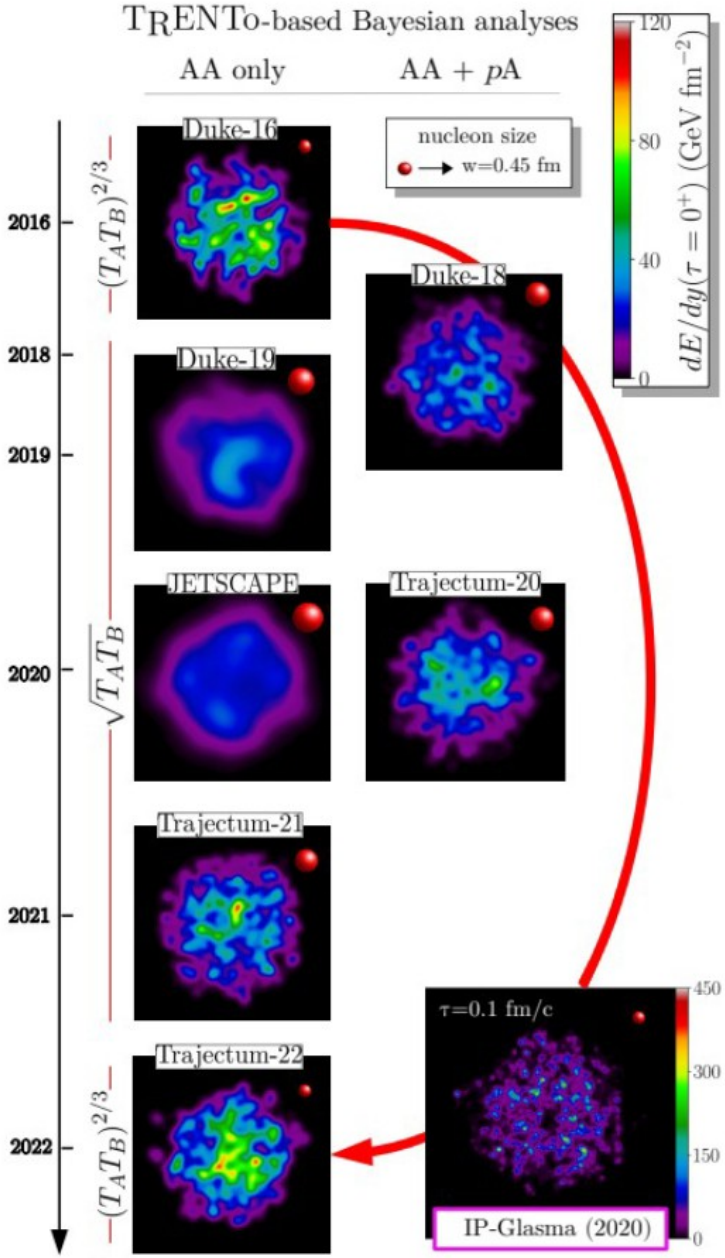
Current model of initial conditions

- Different "models on market" for initializing hydro:
  - TrenTo
  - IP-Glasma
  - Color Glass Condensate
- TrenTo checks geometrically for collisions of nucleons
  → Reduced thickness functions $T_{A/B}$

$$\frac{dE}{dy} \propto \left(\frac{T_A^p + T_b^p}{2} \right)^{q/p}$$

➡ What are the optimal values for p and q ?

Little history of initial conditions



- Values of p and q determined by bayesian analysis, fit to data

  → Values changed back and forth

  → Interpretation of geometric picture changes between entropy and energy density

  → Can this step in modelling be evaded?

➡ Can hydro describe full collision?

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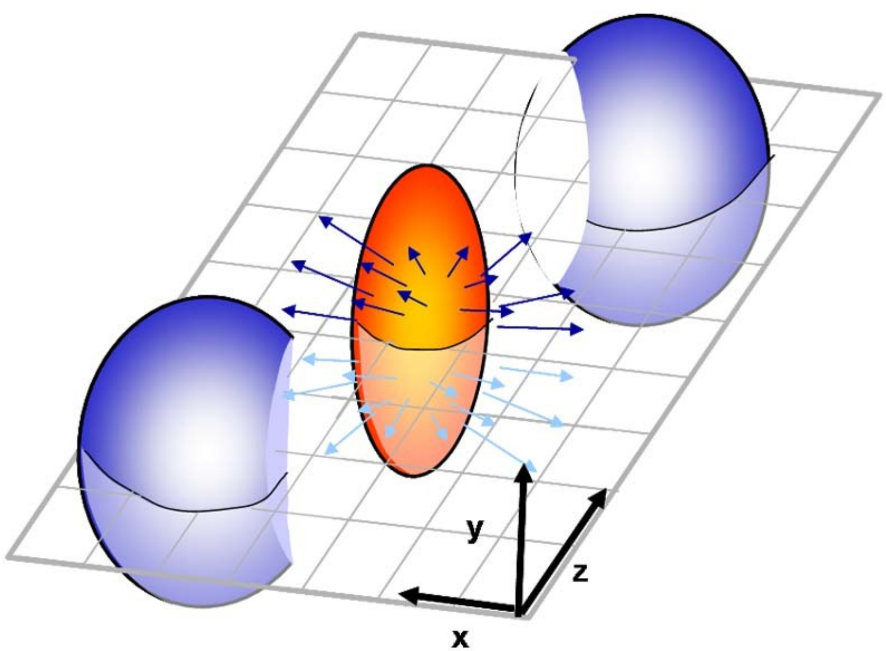
Working principle

- Cartesian coordinates (t, x, y, z)
- For now: only consider longitudinal expansion
- Fluid fields reduce to

$$\phi = (T, u, \pi^{zz}, \pi_{\mathrm{Bulk}}, \nu, \mu)$$



Hydrodynamic setting

- EoM derived from energy-momentum and baryon number current conservation

$$\nabla_\mu T^{\mu\nu} = 0 \qquad \nabla_\mu n^\mu = 0$$

$$u^\mu \partial_\mu \epsilon + (\epsilon + p + \pi_{\mathrm{Bulk}}) \nabla_\mu u^\mu + \pi^{\mu\nu} \nabla_\mu u_\nu = 0$$

$$(\epsilon + p + \pi_{\mathrm{Bulk}}) u^\mu \nabla_\mu u^\nu + \Delta^{\mu\nu} \partial_\mu (p + \pi_{\mathrm{Bulk}}) + \Delta^\nu_\alpha \nabla_\mu \pi^{\mu\alpha} = 0$$

$$u^\mu \partial_\mu n + n \nabla_\mu u^\mu + \nabla_\mu \nu^\mu = 0$$

- Energy-mometetum tensor is based on tensor decomposition with respect to time-like eigenvector (i.e. Fluid velocity)

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + (p + \pi_{\mathrm{Bulk}}) \Delta^{\mu\nu} + \pi^{\mu\nu}$$

$$n^\mu = n u^\mu + \nu^\mu$$

Hydrodynamic setting

- Supplemental equations for $\pi^{\mu\nu}$, π_{Bulk} and ν^{μ} : Second order Israel-Stewart and diffusion current equations

$$\tau_H \Delta^{\alpha}_{\beta} u^{\mu} \nabla_{\mu} \nu^{\beta} + \nu^{\alpha} + \kappa \left(\frac{nT}{\epsilon + p} \right)^2 \Delta^{\alpha\beta} \partial_{\beta} \left(\frac{\mu_B}{T} \right) = 0$$

$$P^{\mu\nu\rho}_{\sigma} \left[\tau_S (u^{\lambda} \nabla_{\lambda} \pi^{\sigma}_{\rho} - 2\pi^{\sigma\lambda} \omega_{\rho\lambda}) + 2\eta \nabla_{\rho} u^{\sigma} \right] + \pi^{\mu\nu} = 0$$

$$\boxed{\tau_{\mathrm{Bulk}} u^{\mu} \partial_{\mu} \pi_{\mathrm{Bulk}}} + \boxed{\pi_{\mathrm{Bulk}}} + \boxed{\zeta \nabla_{\mu} u^{\mu}} = 0$$

Israel-Stewart | **Ideal** | Navier-Stokes

- Isreal-Stewart: Introduce relaxation time τ
  $\rightarrow$ Equations remain valid outside equilibrium

Nuclear droplet model

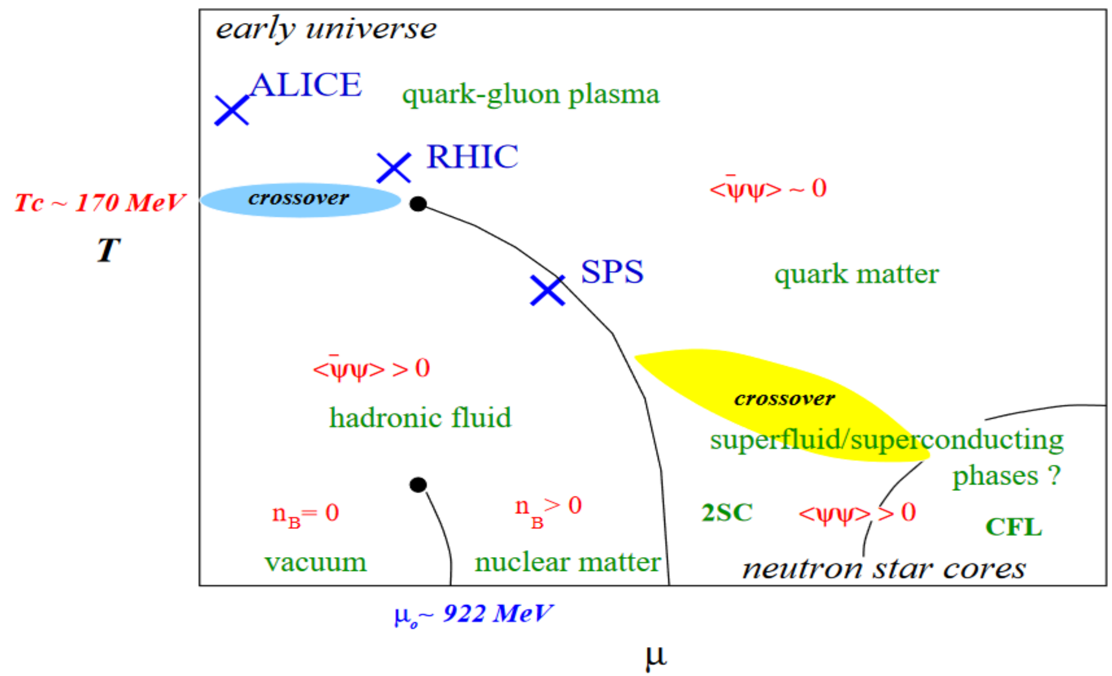
- Idea of describing nucleus as liquid not new!

  → Bethe-Weizsäcker-formula (1935)

- Use fluid variables to describe nucleus

  → Fluid system described by energy-momentum tensor

- Single nucleus sits at vacuum-nuclear matter phase transition $\rightarrow T^{\mu\nu}$ describes nucleus and vacuum!



Adding energy & momentum

- Incomming nuclei described by

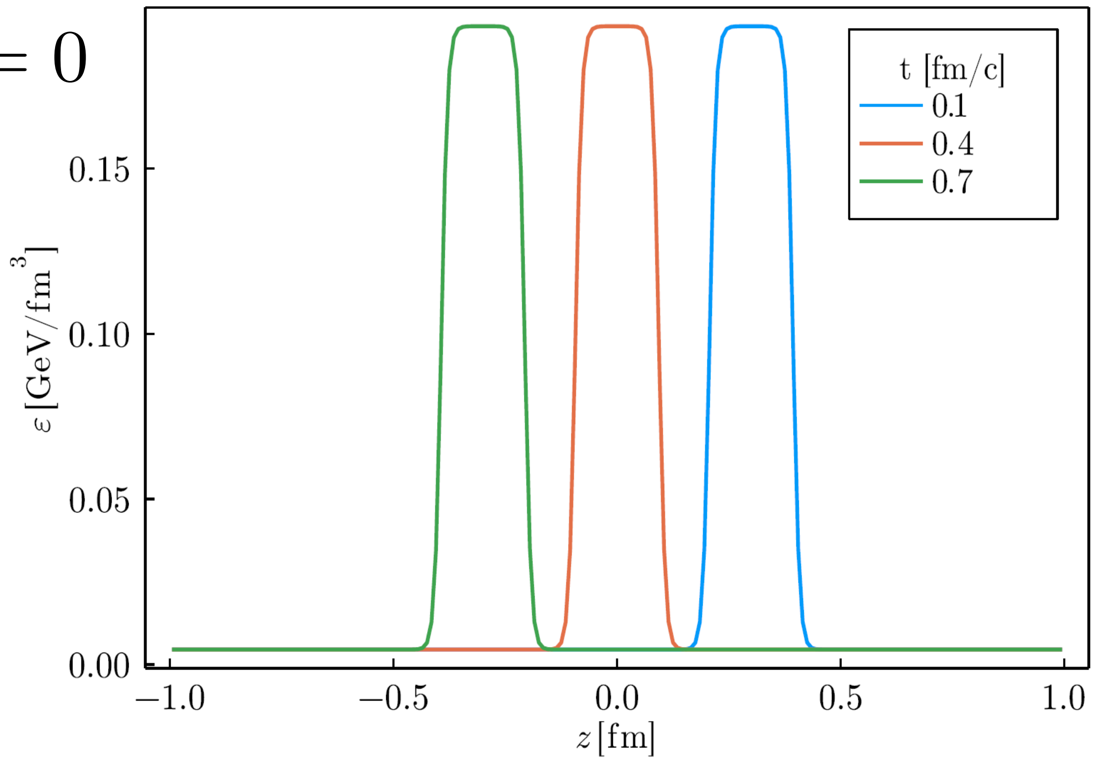
$$T^{\mu\nu}_{\rightarrow/\leftarrow} = \epsilon_{\rightarrow/\leftarrow} u^{\mu}_{\rightarrow/\leftarrow} u^{\nu}_{\rightarrow/\leftarrow}$$

- Nuclei sit at nuclear phase transition

➡ Initial energy density $\epsilon = \mu_{crit} n$

➡ EoM simplify to $u^{\mu}\partial_{\mu} n = 0$

➡ Free streaming nuclei



Adding energy & momentum

- Energy-momentum tensor and number density current of collision system

$$T^{\mu\nu}_{\text{coll}} = T^{\mu\nu}_{\rightarrow} + T^{\mu\nu}_{\leftarrow} \qquad\qquad n^{\mu}_{\text{coll}} = n^{\mu}_{\rightarrow} + n^{\mu}_{\leftarrow}$$

- Obtain fluid variables via Landau matching

$$T^{\mu}_{\nu} u^{\nu} = -\epsilon u^{\mu}, \quad u_{\mu} u^{\mu} = -1$$

- Number density and diffusion current given by

$$\begin{pmatrix} \gamma_{\rightarrow} n_{\rightarrow} + \gamma_{\leftarrow} n_{\leftarrow} \\ 0 \\ 0 \\ \gamma_{\rightarrow}\beta_{\rightarrow} n_{\rightarrow} + \gamma_{\leftarrow}\beta_{\leftarrow} n_{\leftarrow} \end{pmatrix} \stackrel{!}{=} \begin{pmatrix} \gamma n + \beta\nu \\ 0 \\ 0 \\ \gamma\beta n + \nu \end{pmatrix}$$

Adding energy & momentum

- Viscous corrections given by

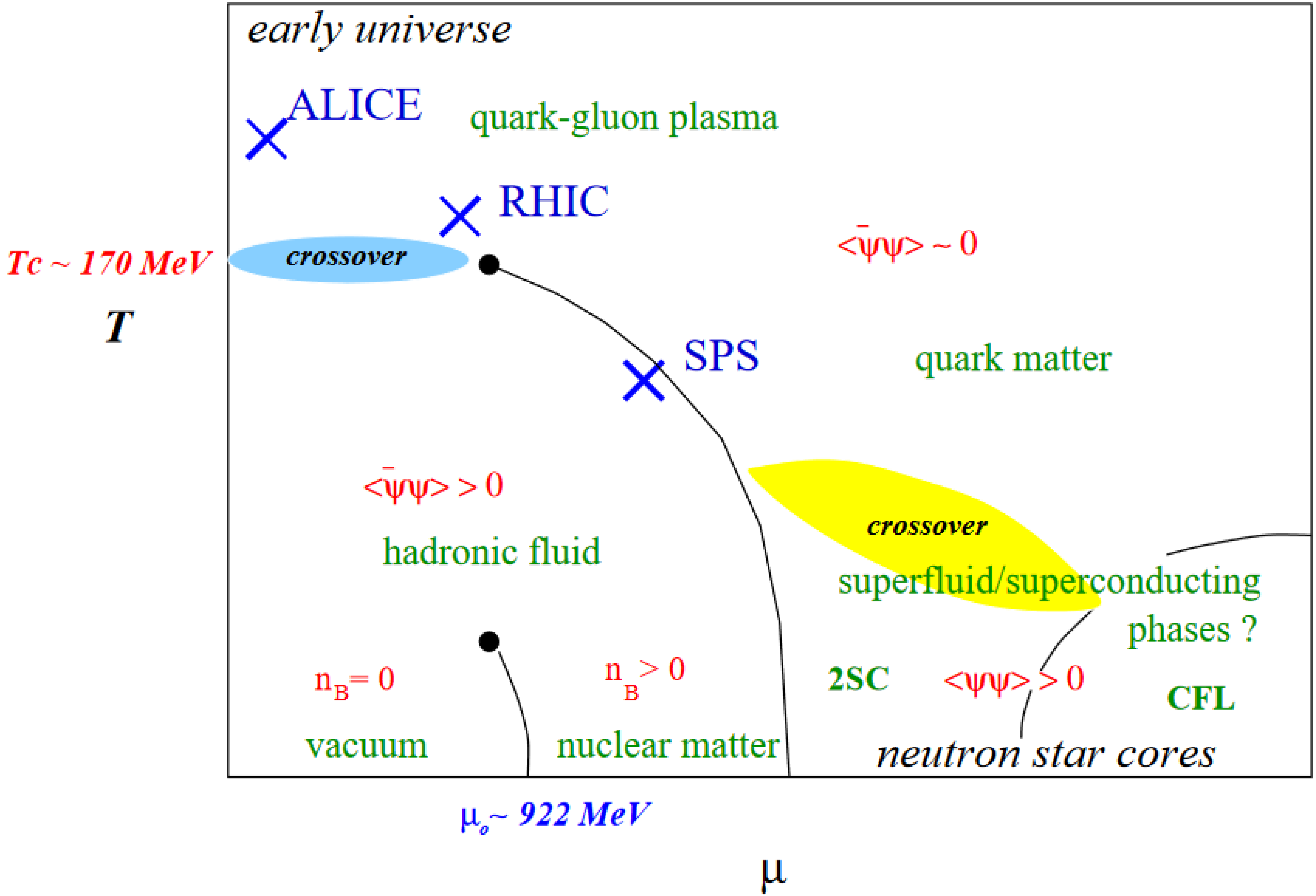
$$\pi_{\mathrm{Bulk}} = \frac{1}{3}\Delta^{\mu\nu}T_{\mu\nu} - p(T,\mu) \quad \pi^{\mu\nu} = T^{\mu\nu} - \epsilon u^{\mu}u^{\nu} - (p + \pi_{\mathrm{Bulk}})\Delta^{\mu\nu}$$

- Invert EoS to obtain full set of fluid variables

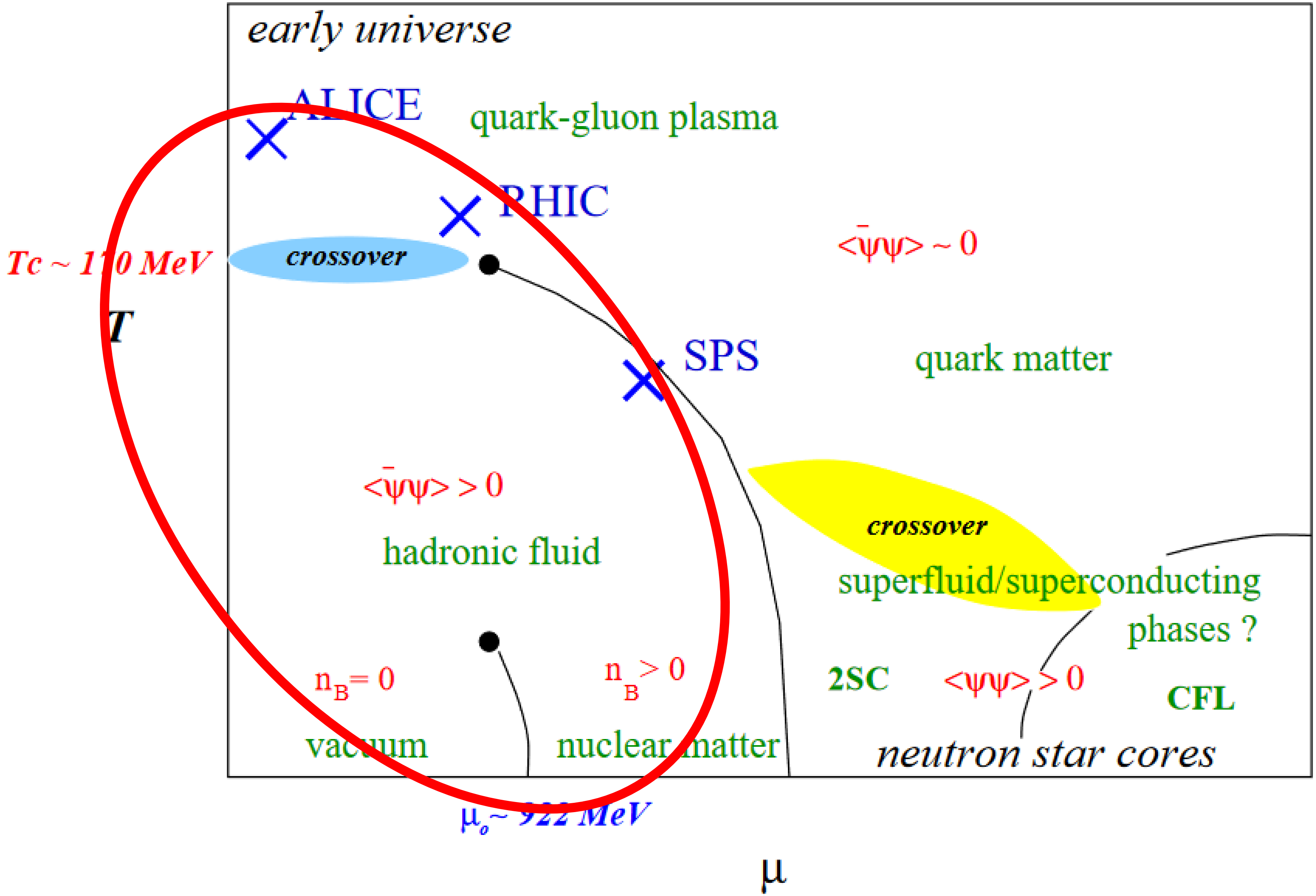
$$n(T,\mu) = n_{\mathrm{coll}}$$
$$\epsilon(T,\mu) = \epsilon_{\mathrm{coll}}$$

➞ Which EoS to use to cover large range in T and μ?

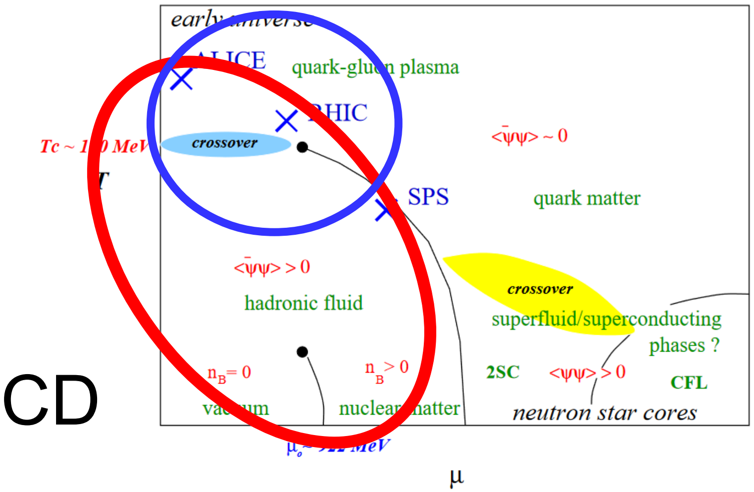
Equation of state



Equation of state

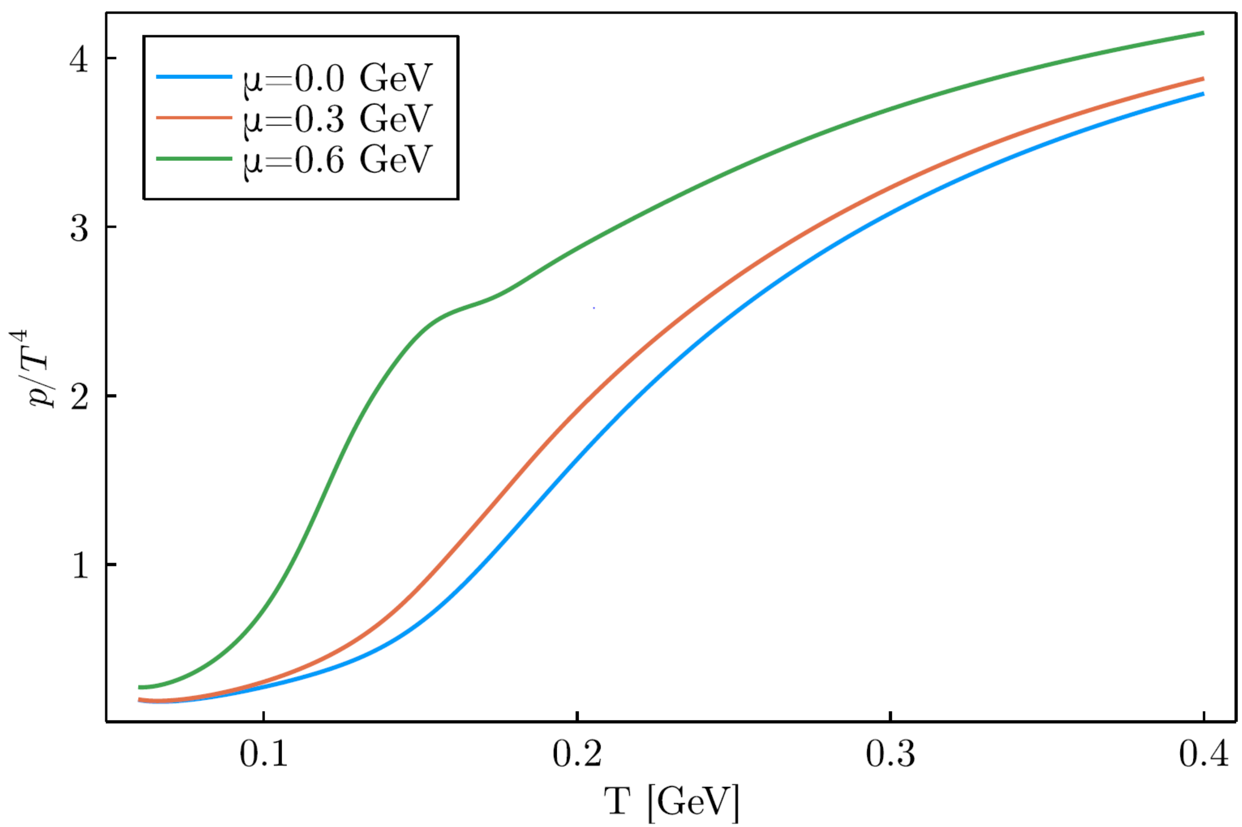


EoS – High T

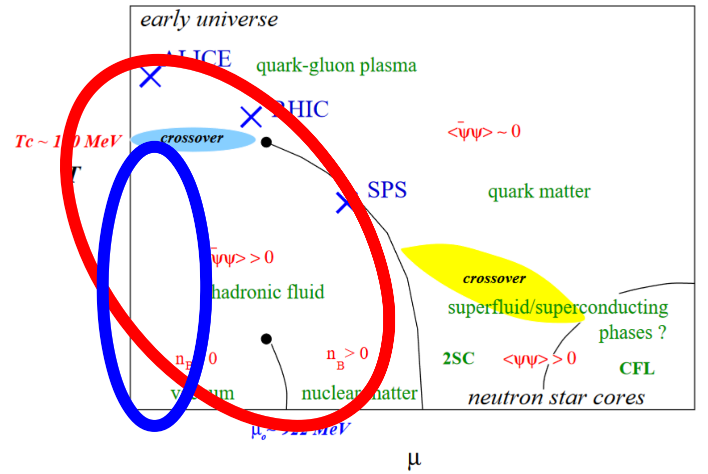


- At high T and low density → Lattice QCD

$$p_{\mathrm{LQCD}} = p(T) + T^4 \left(\frac{1}{2!}\chi_{2B}(T) \left(\frac{\mu}{T}\right)^2 + \frac{1}{4!}\chi_{4B}(T) \left(\frac{\mu}{T}\right)^4 \frac{1}{6!}\chi_{6B}(T) \left(\frac{\mu}{T}\right)^6 \right)$$



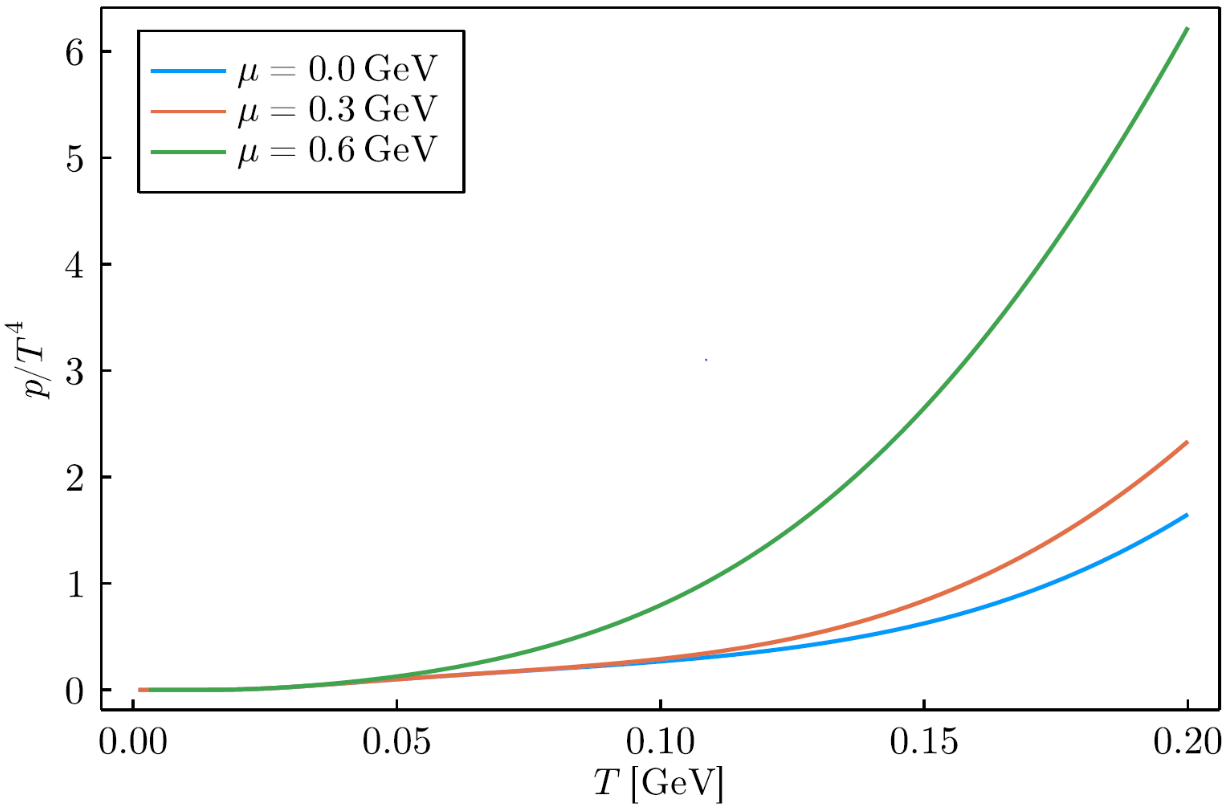
EoS – Low T, low density



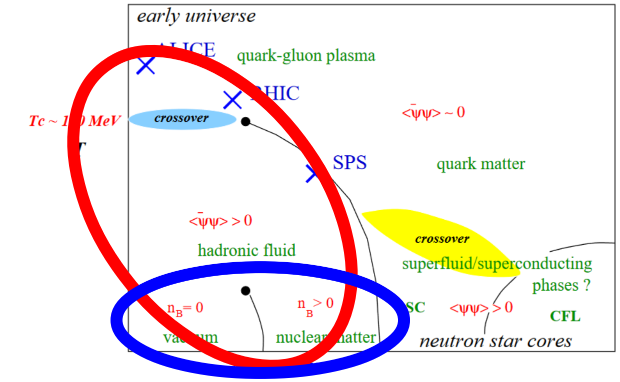
- Low T and low density → Hadron Resonance Gas

$$p_{\mathrm{HRG}}(T,\mu) = \sum_i d_i p_{\mathrm{FG}}(T, B_i\mu; m_i) + \sum_i d_i p_{\mathrm{BG}}(T; m_i)$$

- Pressure given by sum of partial pressure of constituents



EoS – Low T, high density



- Low T and high density → Walecka model

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m_N + g_\sigma \sigma - g_\omega \gamma^\mu \omega_\mu)\psi + \frac{1}{2}(\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2) - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu$$

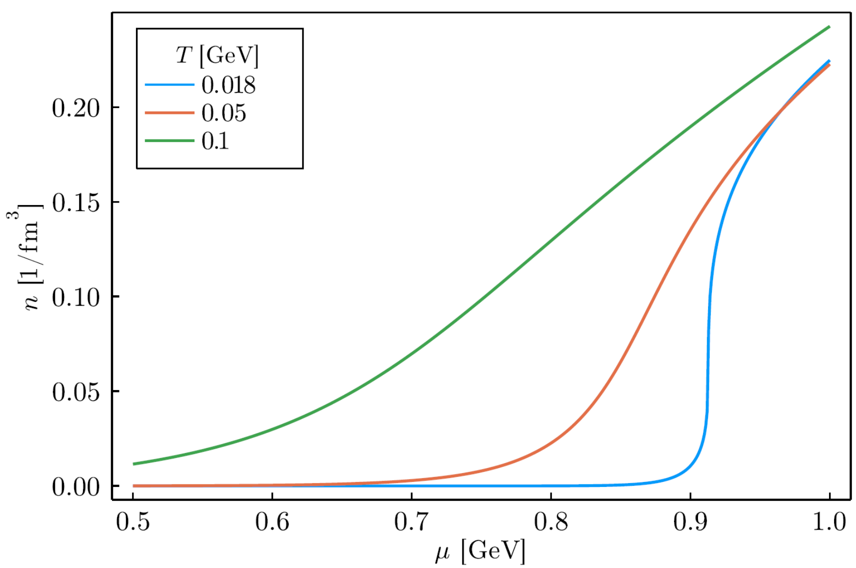
- Effective model of protons and neutrons with omega and scalar meson exchange
- Pressure in mean-field approximation: [1202.1671]

$$p_{\mathrm{WM}}(T,\mu) = 4p_{\mathrm{FG}}(T,\mu^*; m_N^*) + 4p_{\mathrm{FG}}(T,-\mu^*; m_N^*) - \tfrac{1}{2} m_\sigma^2 \bar{\sigma}^2 + \tfrac{1}{2} m_\omega^2 \bar{\omega}_0^2$$

- Mean-fields determined by gap equations

$$\bar{\omega}_0 = \frac{g_\omega}{m_\omega^2} \frac{\partial P_{FD}}{\partial \mu^*}$$

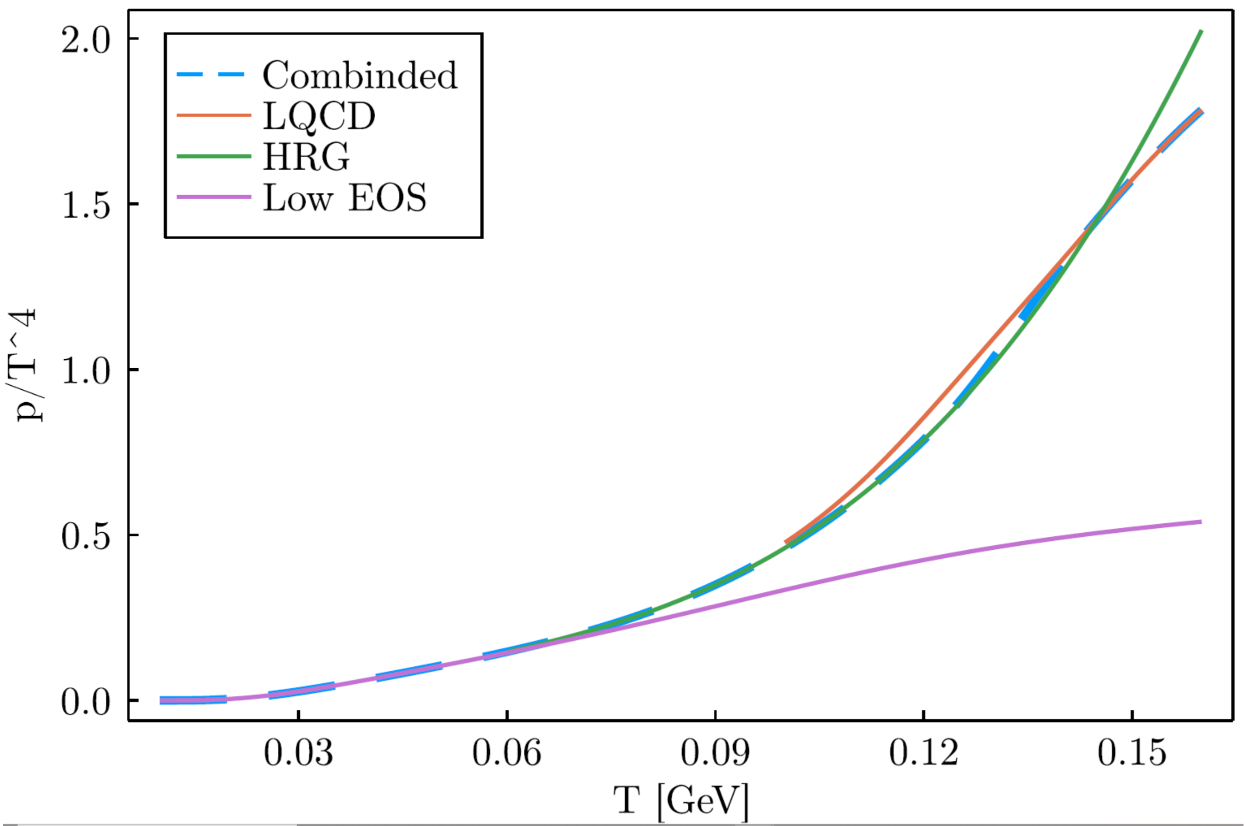
$$\bar{\sigma} = -\frac{g_\sigma}{m_\sigma^2} \frac{\partial P_{FD}}{\partial m_N^*}$$



Combining the EoS

- Low T: HRG w/o proton and neutron + Walecka
- High T: Transition from HRG to LQCD

$$p(T,\mu) = \Theta(T,\mu) p_{\mathrm{LQCD}}(T,\mu) + (1 - \Theta(T,\mu)) p_{\mathrm{HRG}}(T,\mu)$$



Finite Temperature Landau matching

- EoS at low T tricky to handle (degenerate Fermi gas)

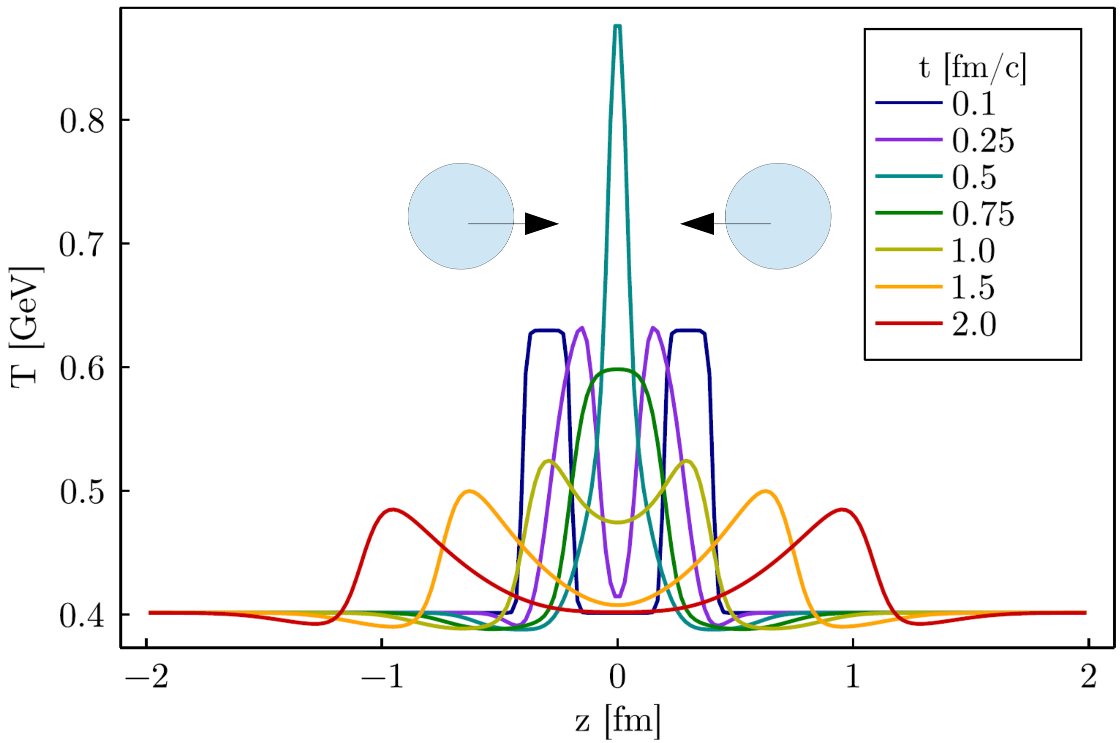
$$p_{\mathrm{HRG}} \propto K_2(\frac{m}{T}) \qquad p_{\mathrm{Walecka}} \propto \mathrm{Li}_{5/2}(-e^{\frac{\mu-m}{T}})$$

- Solution: Sommerfeld & asymptotic expansion
- For now: Do Landau matching at finite temperature

➡ Invert $n(T_{BG},\mu) = n_{\mathrm{drop}}$

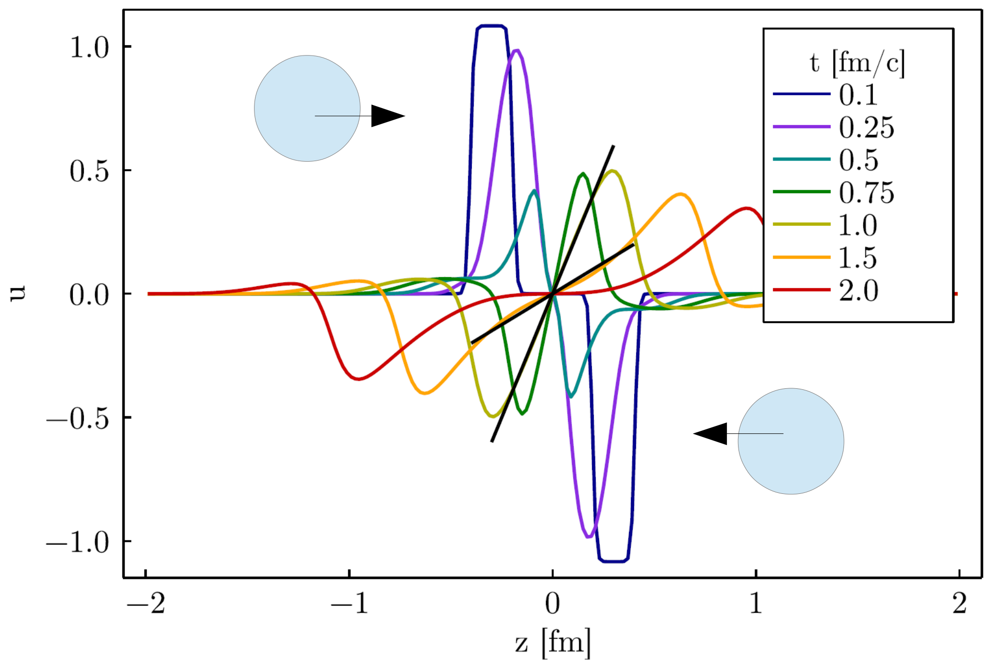
➡ Do Landau matching with $T^{\mu\nu} = \epsilon(T_{BG},\mu)u^\mu u^\nu$

First results

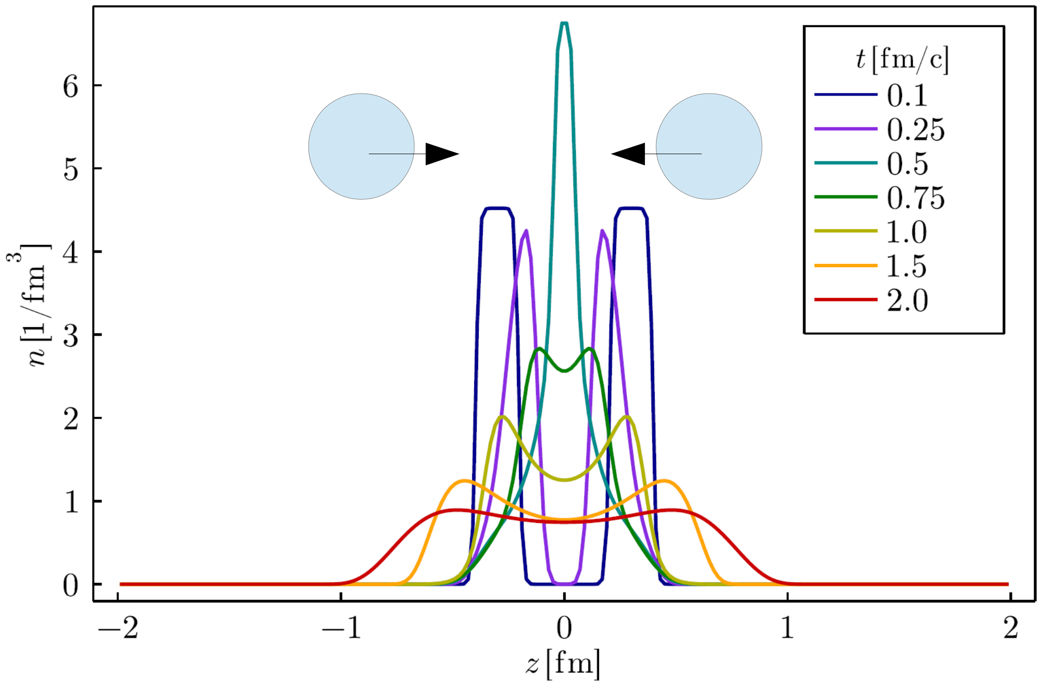


Significant heating up
z-invariant at late times

Emergent Bjorken flow $(v \approx \frac{z}{t})$

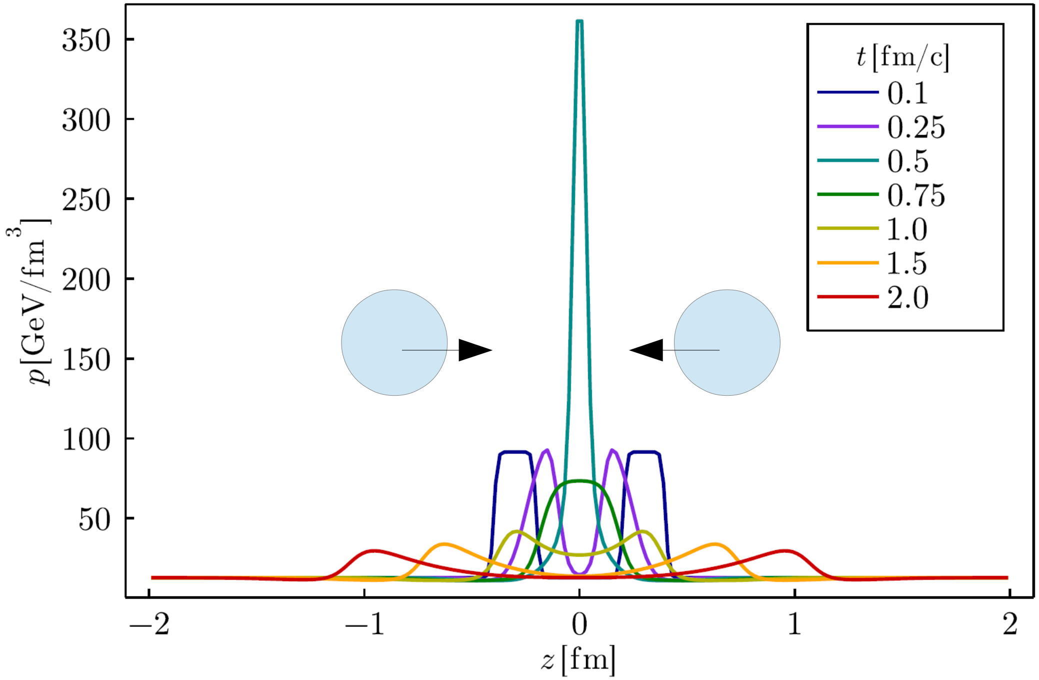


First results



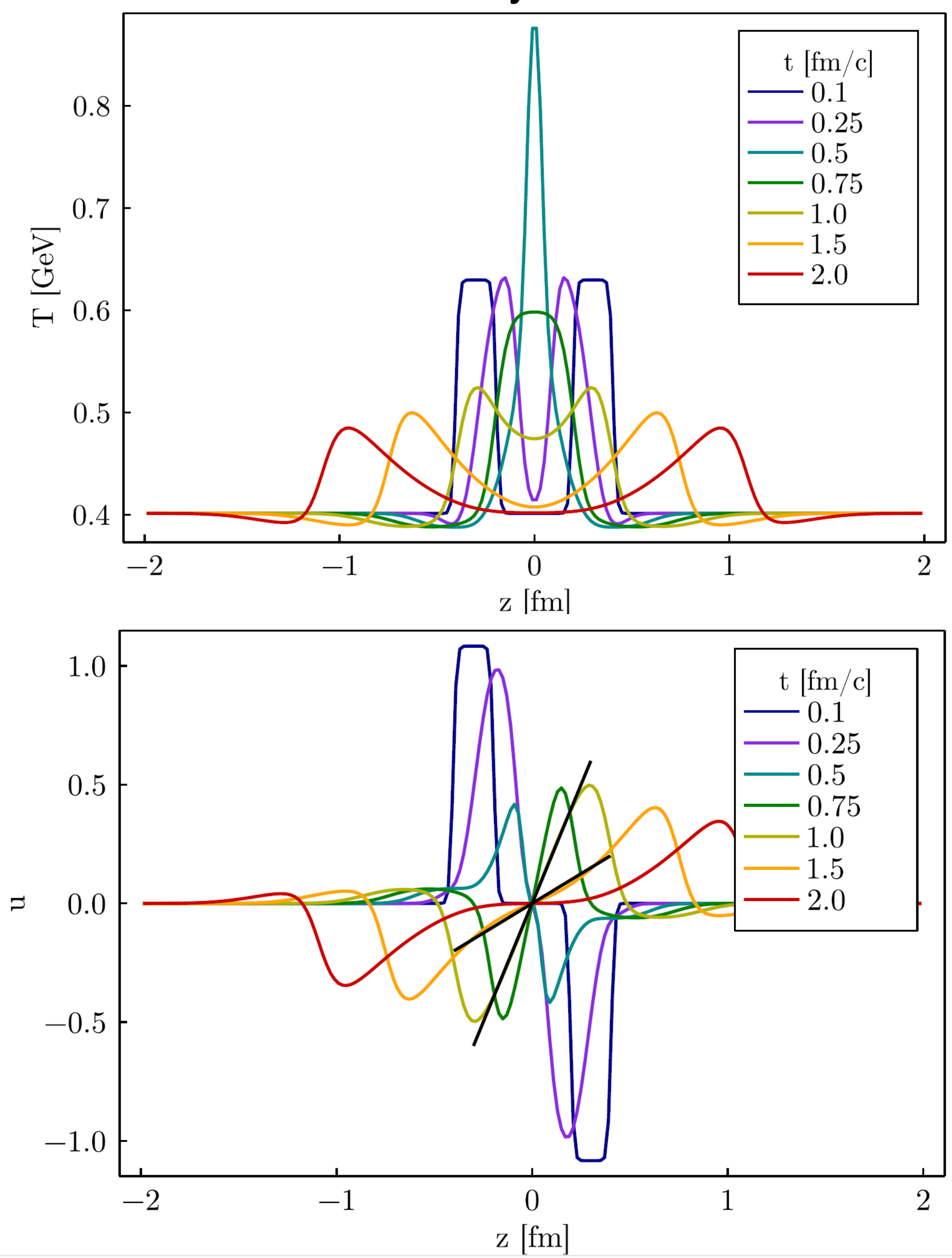
Matter clumps up in collision zone

Huge initial pressure gradient
→ longitudinal expansion

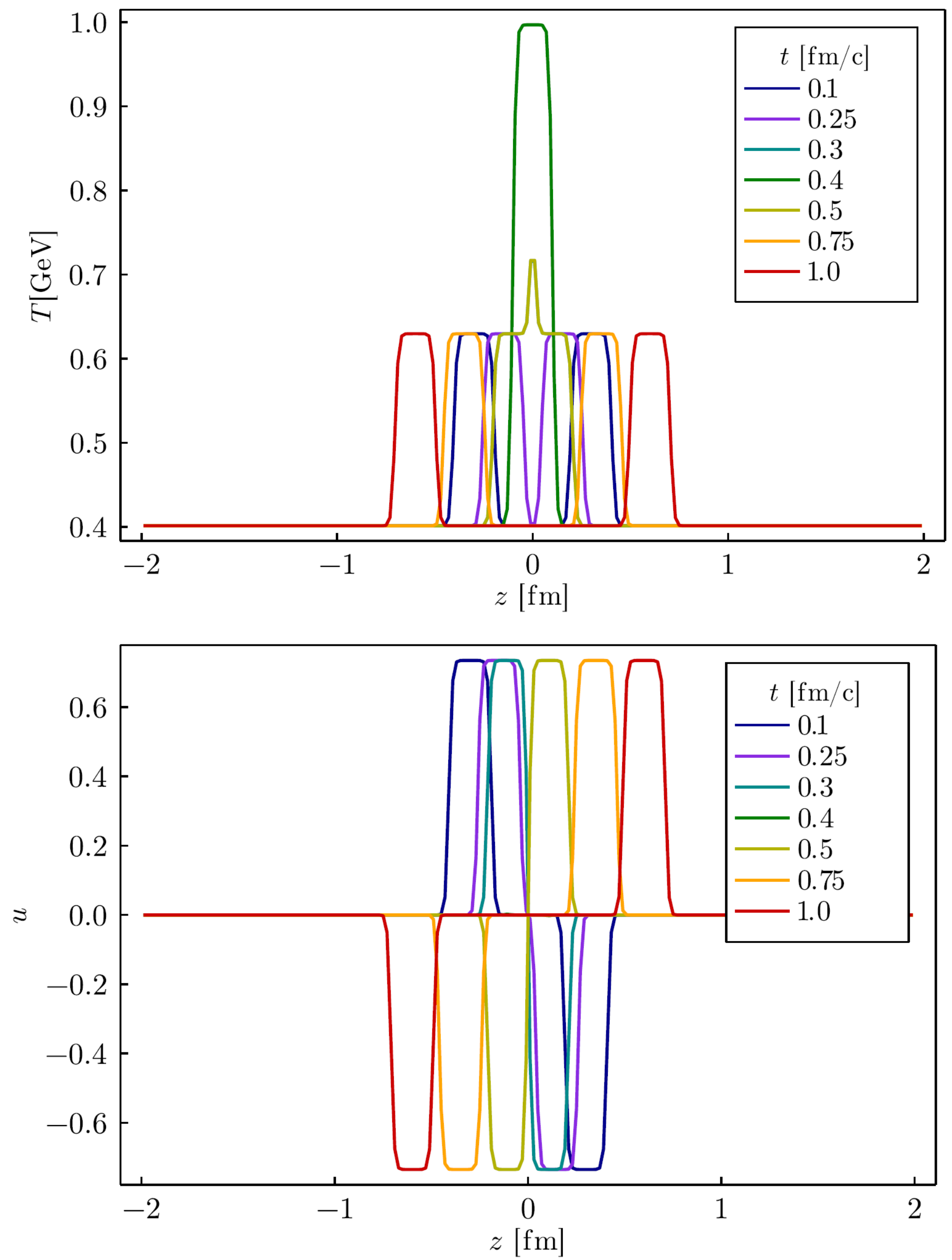


First results

Hydro

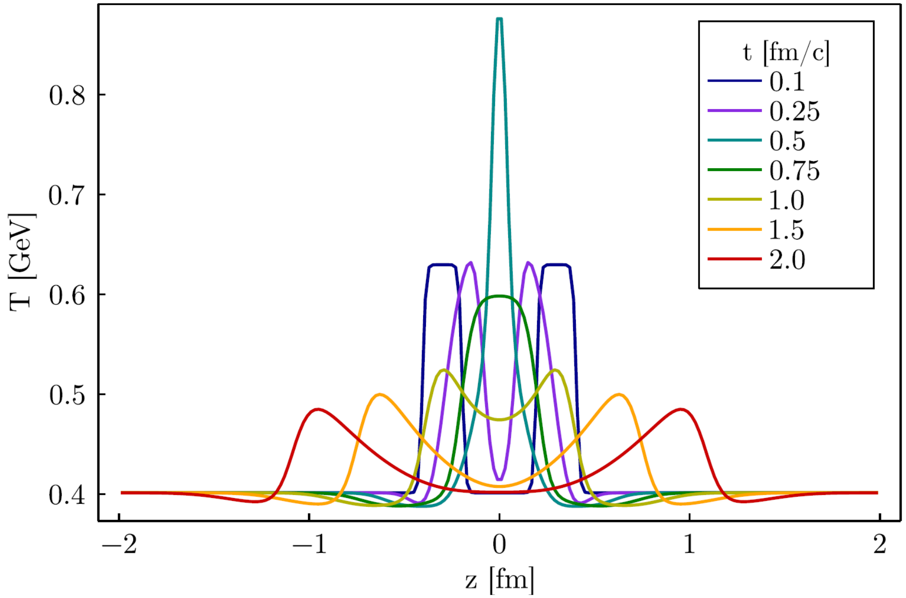


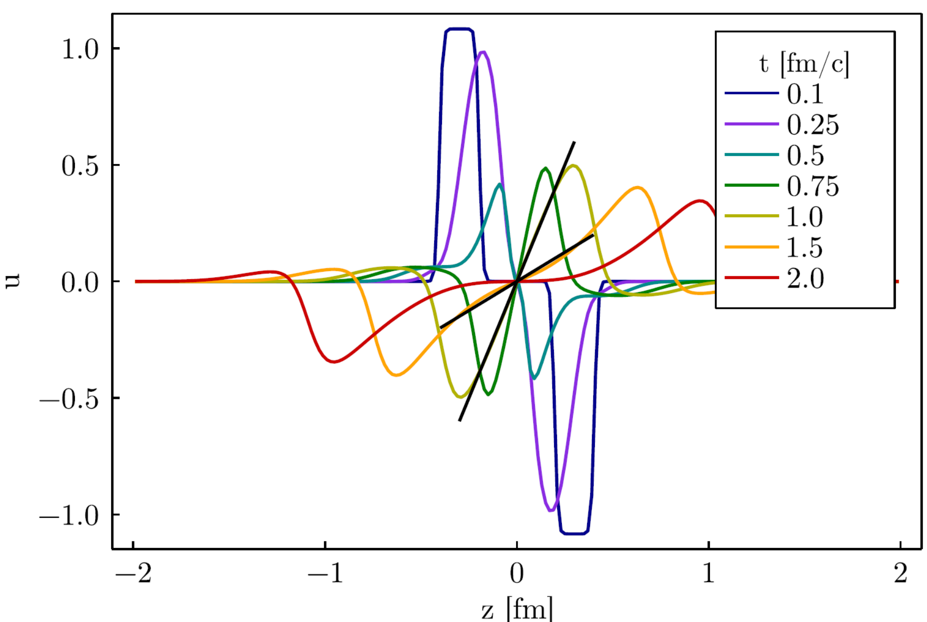
non-interacting



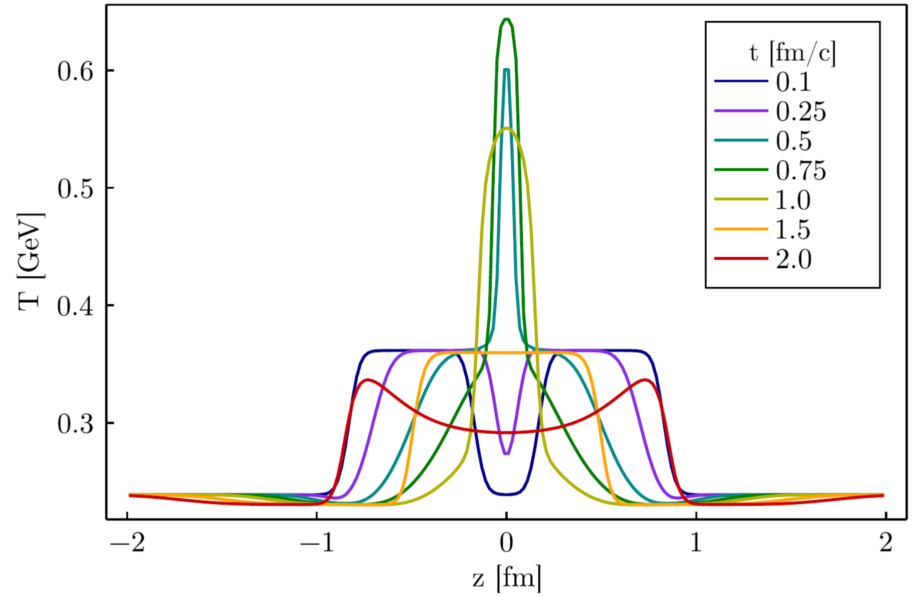
First results

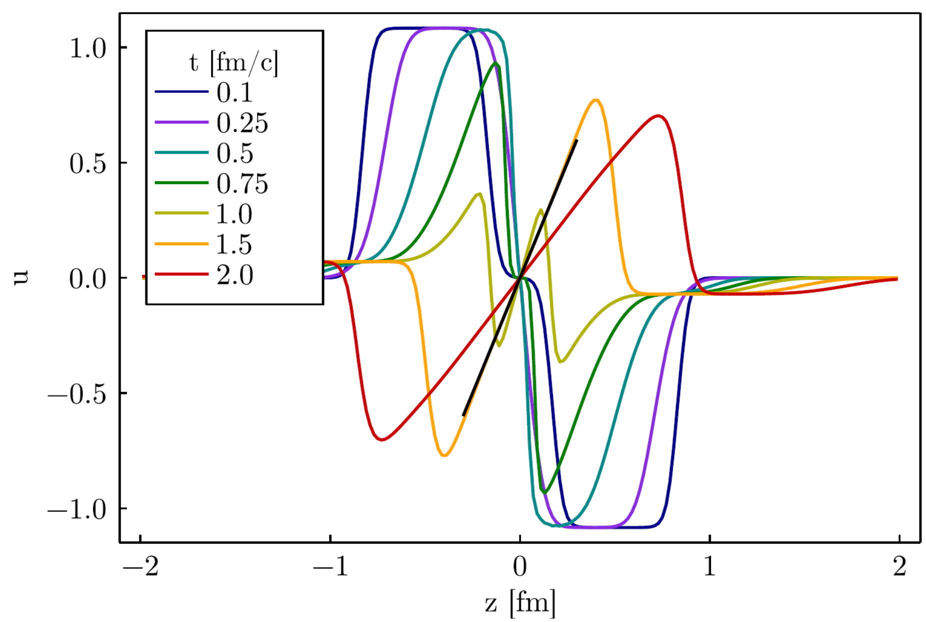
$v = 0.9999 \Rightarrow \gamma \approx 71 \Rightarrow \sqrt{s} \approx 133 \text{ GeV}$





$v = 0.999 \Rightarrow \gamma \approx 22 \Rightarrow \sqrt{s} \approx 42 \text{ GeV}$





Summary & Outlook

- Applied hydro to time before collision

  $\rightarrow$ Heating up

  $\rightarrow$ Emergent Bjorken flow

- Outlook:
  - Improve initial conditions $T^{\mu\nu} = T^{\mu\nu}_{in} - T^{\mu\nu}_{out}$
  - Initialize viscous fields

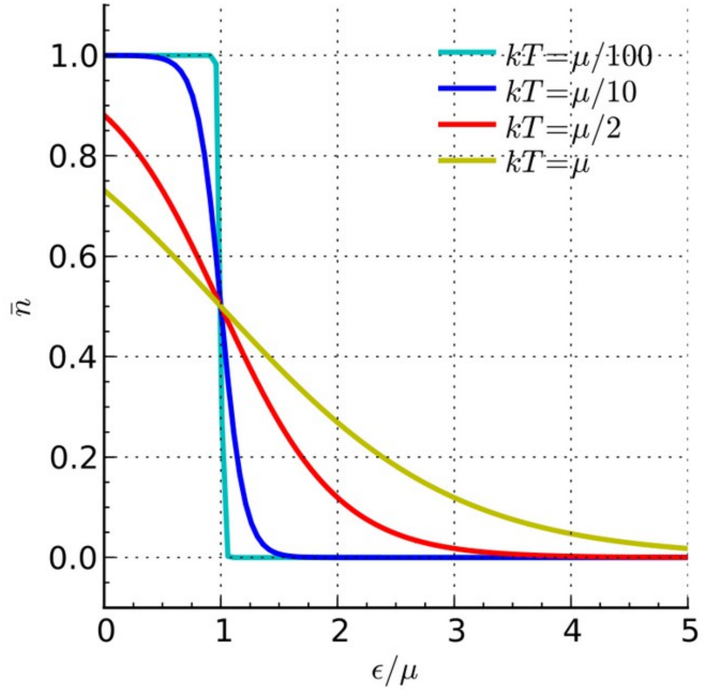
    $\rightarrow$ Examine causality
  - Apply results to traditional hydro

Sommerfeld Expansion

- At $T = 0$: Fermions occupy Fermi-sphere

  $\rightarrow$ Fermi distribution becomes step function
- Including first correction in temperature yields

$$\int_{-\infty}^{\infty} \frac{H(\epsilon)}{e^{\beta(\epsilon - mu)} + 1} \, \mathrm{d}\epsilon = \int_{-\infty}^{\mu} H(\epsilon) \, \mathrm{d}\epsilon + \frac{\pi^2}{6} \frac{1}{\beta^2} H'(\epsilon) + \mathcal{O}\left(\beta^{-4}\right)$$



Validity of hydrodynamics

- Hydrodynamics assumes local thermal equilibrium
- Deviations from equilibrium

  $\rightarrow$ Dissipative corrections
- Navier-Stokes-type equation $\pi_{\text{Bulk}} = -\zeta \nabla_\mu u^\mu$

  $\rightarrow$ Bulk pressure reacts instantly

  $\rightarrow$ Out-of-equilibrium effects get resolved quickly

  $\rightarrow$ Causality can be violated
- Add relaxation time $\tau_{\text{Bulk}} u_\mu \partial^\mu \pi_{\text{Bulk}} + \pi_{\text{Bulk}} = -\zeta \nabla_\mu u^\mu$

  $\rightarrow$ System can stay out of equilibrium for longer

Landau matching

- Decompose $T^{\mu\nu}$ tensor with respect to vector
  $\rightarrow$ Logical choice: fluid velocity u^μ
- Decompose in orthogonal & parallel to u^μ
- Have trace and traceless part

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + (p + \pi_{\mathrm{Bulk}})\Delta^{\mu\nu} + \pi^{\mu\nu}$$

Numeric scheme

- Finite difference method based on Jour. Of Comp. Phys. 230 (2011) 232-244
- Time Derivative given by

$$\phi_j^{n+1} = \phi_j^n - \Delta t \left\{ H\left(\frac{\phi_{j+1}^n - \phi_{j-1}^n}{2\Delta x}\right) - \frac{\alpha}{2}\left(\frac{\phi_{j+1}^n - 2\phi_j^n + \phi_{j-1}^n}{\Delta x}\right) \right\}$$

First order time derivative

Numeric viscosity

- Julia implementation with Tsit5() solver