

# Cosmological Tensions Lecture 2

## Measuring the Hubble constant – the Hubble tension

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## How to measure $H_0$ ?

Always a good idea in cosmology:

measure distances to measure the expansion rate

Luminosity distance:

$$d_L(z) = (1+z) \frac{1}{H_0 \sqrt{\Omega_K}} \sinh \left[ H_0 \sqrt{\Omega_K} \int_0^z \frac{dz'}{H(z')} \right]$$

Angular diameter distance:

$$d_A(z) = \frac{1}{1+z} \frac{1}{H_0 \sqrt{\Omega_K}} \sinh \left[ H_0 \sqrt{\Omega_K} \int_0^z \frac{dz'}{H(z')} \right]$$

## Standard candles and standard rulers

In practice “infer distances” = “measure fluxes or angles”

Fluxes:

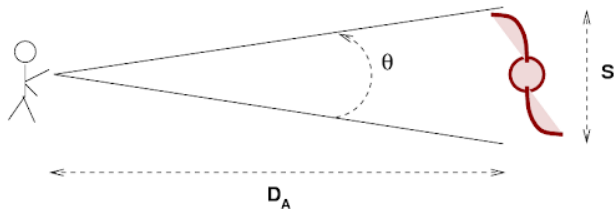
$$d_L = \sqrt{\frac{L}{4\pi F}}$$

$L$ =intrinsic luminosity

Angles:

$$d_A = \frac{s}{\theta}$$

$s$ =intrinsic physical size



# Measuring $H_0$ via the local distance ladder

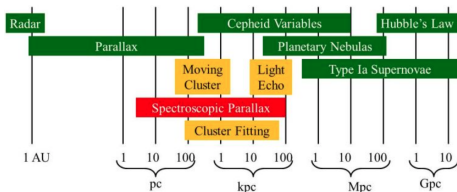
Only strictly empirical (cosmology model-independent) way to measure  $H_0$

Idea: measure  $d$ - $z$  relation, extract  $H_0$  from intercept

Difficulty 1: need to extend distance ladder into the Hubble flow so measured  $z$  is predominantly cosmological (no  $v_{\text{pec}}$ ...but not too far else parameters such as  $\Omega_m$  start to matter )

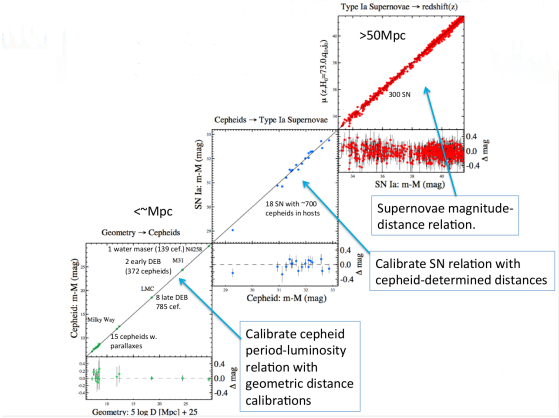
Difficulty 2: each distance indicator has limited range of applicability

Solution: combine different distance indicators in different rungs, as long as two consecutive indicators have a (even limited) range of overlap



# Calibrating the local distance ladder with Cepheids

Best known 3-rung distance ladder: Cepheid-calibrated SNIa



Credits: adapted from Adam Riess and Silvia Galli

## Applying the ladder

Units of  $H_0$  always implicitly km/s/Mpc from now

SH0ES analysis: 75 MW Cepheids with *Gaia* EDR3 parallaxes (plus other geometric distances), >90 Cepheids, 42 calibrator SNela in 37

SNela+Cepheid hosts, 277 SNela in  $0.0233 < z < 0.15$

⇒ 1.4% measurement of  $H_0$ !

$$H_0 = 73.04 \pm 1.04$$

*(Cepheid-calibrated SNela, R22)*

Riess *et al.*, *ApJ Lett.* 934 (2022) L7

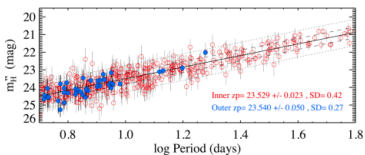
Notes:

- need intermediate rung as SNela are rare events, not enough of them in the local Universe for direct parallax calibration
- Cepheids are standard candles through period-luminosity relation

# Dissecting the local distance ladder

Calibrator (second rung)

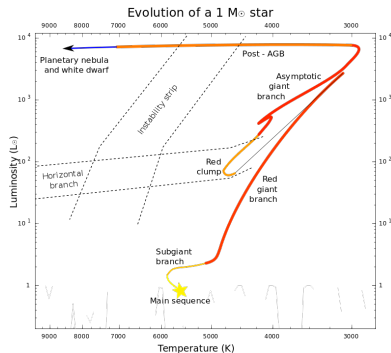
Cepheids



Riess *et al.*, *ApJ Lett.* 934 (2022) L7

Different reanalyses fall between 72.8 and 74.3

Tip of the Red Giant Branch (TRGB)



CCHP analysis:  $H_0 = 69.8 \pm 0.8 \pm 1.7$  *Freedman et al.*, *ApJ* 882 (2019) 34

Later reanalyses fall between 69.6 and 76.9

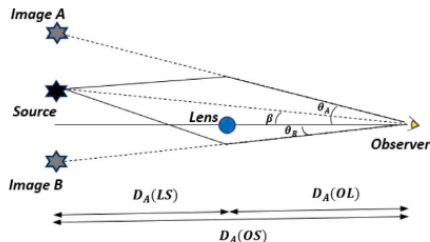
# Dissecting the local distance ladder

- $2^{\text{nd}}$  rung – Cepheids vs TRGB: currently most credible contenders, but no complete consensus on TRGB [See review by Freedman, ApJ 919 \(2021\) 16](#)
- $2^{\text{nd}}$  rung – Mira variables (Miras; highly-evolved low-mass AGB stars) as SNeIa calibrators:  $H_0 = 73.3 \pm 4.0$  [Huang et al., ApJ 889 \(2020\) 5](#)
- $2^{\text{nd}}$  rung – Surface brightness fluctuations (SBFs) as SNeIa calibrators:  $H_0 = 70.50 \pm 2.37 \pm 3.38$  [Khetan et al., A&A 647 \(2021\) A72](#)
- $2^{\text{nd}}/3^{\text{rd}}$  rung – Cepheid- and TRGB-calibrated SBFs:  $H_0 = 73.3 \pm 0.7 \pm 2.4$  [Blakeslee et al., ApJ 911 \(2021\) 65](#)
- $2^{\text{nd}}/3^{\text{rd}}$  rung – Cepheid- and TRGB-calibrated SNeIa:  $H_0 = 75.4 \pm 3.7$  [de Jaeger et al., MNRAS 514 \(2022\) 4620](#)
- $2^{\text{nd}}/3^{\text{rd}}$  rung – Cepheid- and TRGB-calibrated baryonic Tully-Fisher relation:  $H_0 = 75.1 \pm 2.5 \pm 1.5$  [Schombert et al., AJ 160 \(2020\) 71](#)
- *Only 2 rungs* –  $d$ - $z$  relation for  $z \lesssim 0.01$  Cepheids:  $H_0 = 73.1 \pm 2.4$  [Kenworthy et al., ApJ 935 \(2022\) 83](#)
- *No rungs* – Water megamasers (stimulated emission from water rotational transition levels):  $H_0 = 73.9 \pm 3.0$  [Pesce et al., ApJ Lett. 891 \(2020\) L1](#)
- *Other possibilities* – GW standard sirens (with or without EM counterpart),  $\gamma$ -ray attenuation, HII galaxies, BH shadows,...



## Strong lensing time-delay cosmography

Completely independent of the local distance ladder (but not completely cosmology model-independent, depends on  $\Omega_m$ ,  $w$ ,  $\Omega_K$ , etc.)



Perivolaropoulos & Skara, *New Astron. Rev.* 95 (2022) 101659

$$\Delta t = D_{\Delta t} \Delta \phi_L \propto \frac{1 + z_L}{c} \frac{d_A(OL) d_A(OS)}{d_A(LS)} \propto \frac{1}{H_0}$$

Main difficulty: mass-sheet degeneracy!

## Strong lensing time-delay cosmography

$$H_0 = 73.3 \pm 1.8$$

*(TDCOSMO, seven quasar  
time-delay lenses)*

Birrer et al., A&A 643 (2020) A165

Attempting to break the mass-sheet degeneracy:

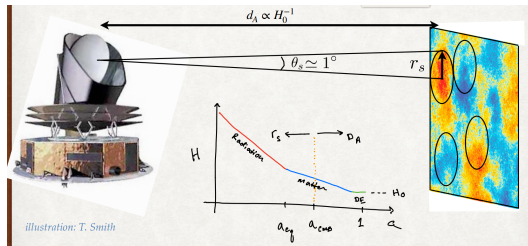
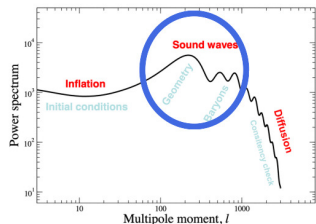
$$H_0 = 67.4 \pm 3.7$$

*(TDCOSMO+SLACS)*

Birrer et al., A&A 643 (2020) A165



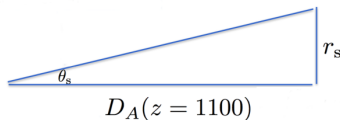
# The CMB as a (self-calibrated) standard ruler



Credits: Planck collaboration and Silvia Galli (left); Tristan Smith and Vivian Poulin (right)

$$\theta_s = \frac{r_s}{D_A(z_*)} = 0.010411 \pm \underline{\underline{\underline{0.000003}}} \quad (!!!)$$

Note:  $\theta_s$  measured exquisitely, but  $r_s$  and  $d_A$  are model-dependent!



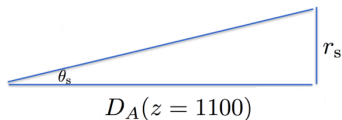
Credits: Silvia Galli

## Steps to apply the CMB ruler

Within  $\Lambda$ CDM:

$$\theta_s = \frac{r_s}{d_A(z_*)}, \quad r_s \simeq \int_{z_*}^{\infty} dz \frac{c_s(z, \omega_b, \omega_r)}{\sqrt{(\omega_c + \omega_b)(1+z)^3 + \omega_r(1+z)^4}}$$

- $\omega_r$ : exquisitely measured from  $T_{\text{CMB}}$  (e.g. COBE)
- $c_s(z) = (1 + 3\rho_b/4\rho_\gamma)^{-1}$
- $\omega_b$ : infer from relative height of odd and even peaks, further improvement from damping tail
- $\omega_c$ : infer from early ISW effect (first peak height), potential envelope, further improvement from lensing-induced peak smoothing

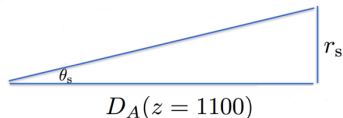


## Steps to apply the CMB ruler

Within  $\Lambda$ CDM:

$$\theta_s = \frac{r_s}{d_A(z_*)}, \quad d_A(z_*) \simeq 3 \int_0^{z_*} dz \frac{1}{\sqrt{\omega_\Lambda + \omega_m(1+z)^3 + \omega_r(z)}} \text{ Gpc}$$

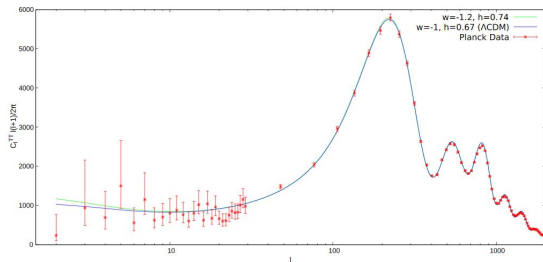
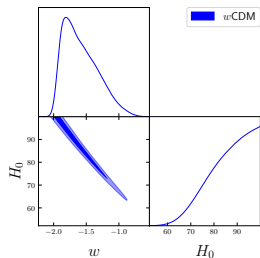
- $\omega_r(z)$ : already known as before
- $\omega_m = \omega_c + \omega_b$ : both terms already known as before
- $\theta_s$ : inferred from peak spacing,  $\theta_s \simeq \pi/\Delta\ell = \pi/(\ell_{p+1} - \ell_p)$
- $\omega_\Lambda$ : only remaining free parameter, to fix from  $d_A(z_*) = r_s \Delta\ell/\pi$
- Once  $\omega_\Lambda$  is known, the whole evolution of  $H(z)$  is known, including  $H(z=0) = H_0!$



## Applying the CMB ruler: some important observations

- In  $\Lambda$ CDM, with all other *physical* densities fixed by early-Universe considerations,  $H_0$  controls only the physical amount of dark energy
- In  $\Lambda$ CDM there is enough information/sufficiently few free parameters to constrain  $H_0$  from the CMB...
- but this is *not* (necessarily) true in extensions of  $\Lambda$ CDM, especially late-time extensions (geometrical degeneracy)

Example:  $w \neq -1$ ,  $H_0$  unconstrained from CMB alone (just lower limit)



## Applying the ruler

$$H_0 = 67.27 \pm 0.60$$

*(Planck 2018 TTTEEE+lowE)*

Planck collaboration, A&A 641 (2020) A6

$$H_0 = 67.9 \pm 1.5$$

*(ACT DR4)*

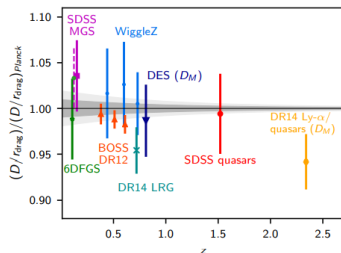
ACT collaboration, JCAP 1212 (2020) 047



## Late-time guard rails

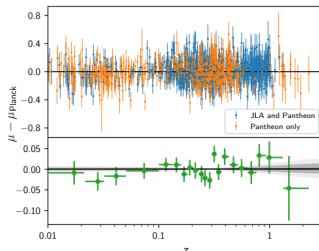
It is important to “stabilize” CMB-only constraints with late-time datasets, *especially when going beyond  $\Lambda$ CDM at late times!*

### BAO



Planck collaboration, A&A 641 (2020) A6

### Cosmological/high- $z$ SNeIa



Planck collaboration, A&A 641 (2020) A6

These are in *very good* agreement with the expansion history inferred from *Planck* within  $\Lambda$ CDM (so in  $\Lambda$ CDM mostly a consistency check)!

## Combining CMB and late-time guard rails

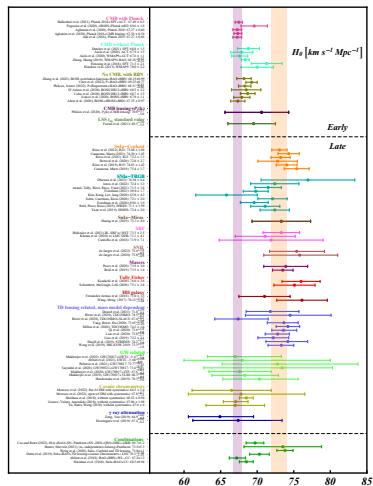
Combination consistent with CMB-only value of  $H_0$  within  $\Lambda$ CDM, important sanity check!

$$H_0 = 67.72 \pm 0.40$$

*(CMB+BAO+uncalibrated SNeIa)*

*Planck* collaboration, A&A 641 (2020) A6

# Hubble tension summary



**CMB with Planck**  
 Balkenhol et al. (2021), Planck 2018+SP4+ACT: 67.49 ± 0.33  
 Aghanim et al. (2020), Planck 2018: 67.27 ± 0.60  
 Aghanim et al. (2020), Planck 2018+CMB lensing: 67.36 ± 0.54

**CMB without Planck**  
 Dutcher et al. (2021), SPT: 68.8 ± 1.5  
 Aida et al. (2020), ACT: 67.9 ± 1.5  
 Aida et al. (2020), WMAP+ACT: 67.6 ± 1.1  
 Zhang, Huang (2019), WMAP+SdS: 68.36<sup>+0.23</sup><sub>-0.22</sub>

**No CMB, with BBN**  
 Colas et al. (2020), BOSS DR12+BBN: 68.7 ± 1.5  
 Philcox et al. (2020), P1+BBO+BBN: 68.6 ± 1.1  
 Ivanoiu et al. (2020), BOSS+BBN: 67.9 ± 1.1  
 Alam et al. (2020), BOSS+eBOSS+BBN: 67.35 ± 0.97

**Cepheids - SH0A**

Riess et al. (2020), R20: 73.2 ± 1.3  
 Breuval et al. (2020): 72.8 ± 2.7  
 Riess et al. (2019), R19: 74.0 ± 1.4  
 Camarena, Marra (2019): 75.4 ± 1.7  
 Burns et al. (2018): 73.2 ± 2.3  
 Pejin, Knox (2017): 73.3 ± 1.7  
 Feeney, Mortlock, Dalmasso (2017): 73.2 ± 1.8  
 Riess et al. (2016), R16: 73.2 ± 1.7  
 Cardona, Kunz, Pettorino (2016): 73.8 ± 2.1  
 Freedman et al. (2012): 74.3 ± 2.1

**TRGB - SH0A**

Soltis, Casertano, Riess (2020): 72.1 ± 2.0  
 Freedman et al. (2020): 69.6 ± 1.9  
 Reid, Pease, Riess (2019), SH0ES: 71.1 ± 1.9  
 Freedman et al. (2019): 69.8 ± 1.9  
 Yuan et al. (2019): 72.4 ± 2.0  
 Jang, Lee (2017): 71.2 ± 2.5

**Masers**

Pease et al. (2020): 73.9 ± 3.0

**Tully - Fisher Relation (TFR)**

Kourkchi et al. (2020): 76.0 ± 2.6  
 Schombert, McGaugh, LeBl (2020): 75.1 ± 2.8

**Surface Brightness Fluctuations**

Blakeslee et al. (2021), B-SF: 73.3 ± 2.5

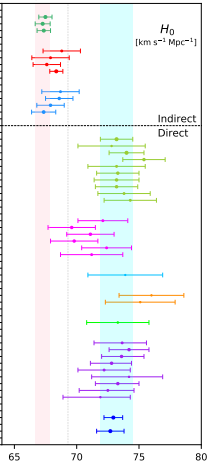
**Lensing related, mass model - dependent**

Yang, Birrer, Hu (2020):  $H_0 = 73.65^{+2.10}_{-1.10}$   
 Milton et al. (2020), TDCOSMO: 74.2 ± 1.6  
 Qi et al. (2020): 73.6 ± 1.1  
 Uzo et al. (2020): 72.2<sup>+1.1</sup><sub>-1.2</sub>  
 Luo et al. (2019): 72.2 ± 2.1  
 Shajib et al. (2019), STRIDES: 74.2 ± 2.1  
 Wong et al. (2019), HOLCOW 2019: 73.2<sup>+1.1</sup><sub>-1.2</sub>  
 Birrer et al. (2018), HOLCOW 2018: 72.5<sup>+1.1</sup><sub>-1.2</sub>  
 Bonvin et al. (2016), HOLCOW 2016: 71.9<sup>+1.1</sup><sub>-1.2</sub>

**Optimistic average**

Di Valentino (2021): 72.94 ± 0.75  
 Ultra - conservative, no Cepheids, no lensing  
 Di Valentino (2021): 72.7 ± 1.1

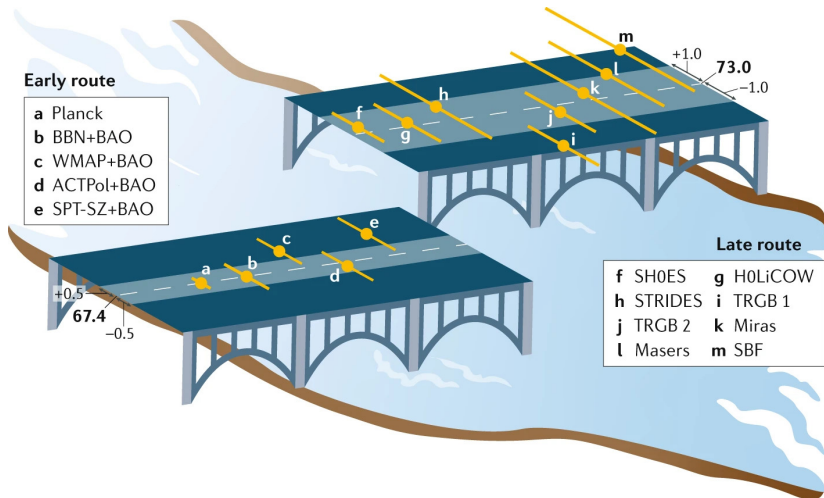
## High Precision Measures of $H_0$



Adapted from Perivolaropoulos & Skara, *New Astron. Rev.* 95 (2022) 101659

Adapted from Di Valentino *et al.*, *Class. Quant. Grav.* 38 (2021) 153001

# Hubble tension summary



# Systematics?

Cepheid-calibrated distance ladder:

- systematics in 1<sup>st</sup> rung distances
- extinction
- metallicity
- crowding/blending
- environmental dependence of Cepheid/SNela properties
- unknown unknowns...

CMB:

- beam systematics
- foregrounds
- instrumental systematics (e.g. half-wave plate systematics)
- atmosphere
- bandpass variability
- unknown unknowns...

If systematics are the answer, why do they conspire to make early-vs-late discrepancy consistent *across so many independent measurements?*

# Inverse distance ladder

BAO measure  $r_s/d \propto r_s H_0 \implies$  BAO can be calibrated with  $H_0$  or  $r_s$ !

## Classical distance ladder

- Determine  $H_0$  from N-rung distance ladder
- Calibrate SNeIa  $d_L$  with  $H_0$
- From BAO  $d_A$  in the same  $z$  range infer  $r_s$

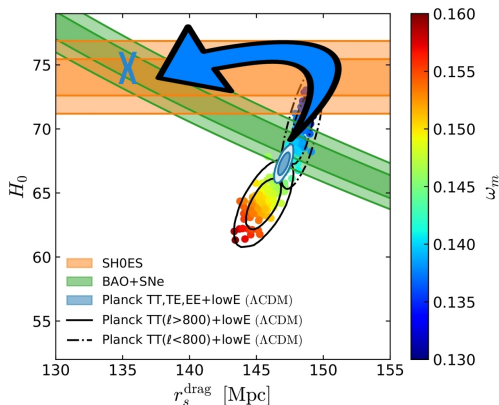
## Inverse distance ladder

- Calibrate BAO with  $r_s$  prior (model-dependent)
- Transfer BAO calibration to SNeIa  $d_L$  in the same  $z$  range
- Extrapolate to  $z = 0$  to infer  $H_0$

If model-dependent  $r_s$  prior (CMB-dependent or not, more later) is correct,  $H_0$  from inverse distance ladder and classical distance ladder should agree!

# What does the tension mean?

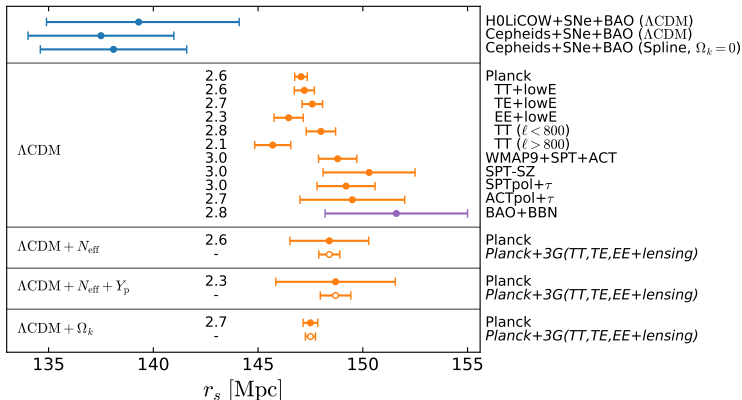
Useful to look at  $r_s$ - $H_0$  plane



Knox & Millea, PRD 101 (2020) 043533

BAO data tell us that  $r_s$  has to decrease by  $\simeq 7\%$ !  $r_s h \sim 100$  Mpc  $\implies$  good fit with  $r_s \sim 147$  and  $H_0 \sim 67$  ( $\Lambda$ CDM) or  $r_s \sim 136$  and  $H_0 \sim 73$

# $H_0$ tension or $r_s$ tension?



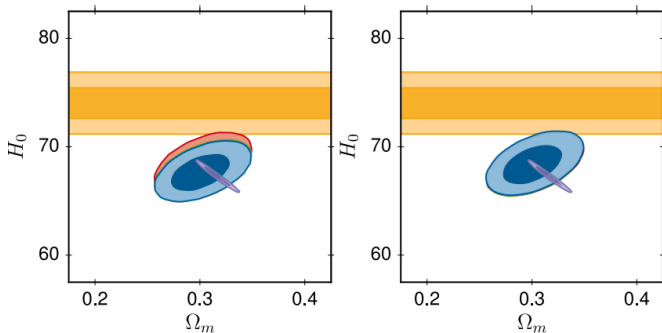
Aylor et al., ApJ 874 (2019) 4

$r_s$  inferred from distance ladder systematically lower than  $\Lambda$ CDM-based inferences for any dataset combination!



## CMB- and SNeIa-free determinations of $H_0$

Can determine  $H_0$  completely free of CMB data: BBN prior on  $\omega_b$  used to calibrate  $r_s$  assuming pre-recombination  $H(z)$ , then infer  $H_0$  from BAO

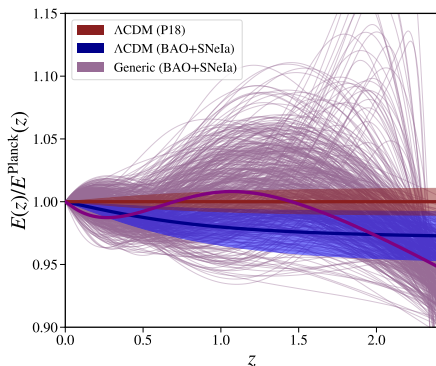


Schöneberg, Lesgourgues & Hooper, JCAP 1910 (2019) 029

Larger error bars, but tension (assuming  $\Lambda$ CDM at early times) remains regardless of BBN model, determinations of  $Y_P$  and  $Y_{DP}$ , and BAO data

# Reconstructing the late-time expansion history

BAO (sparse in redshift) and uncalibrated SNeIa (dense in redshift) highly complementary



Bernal *et al.*, PRD 103 (2021) 103533

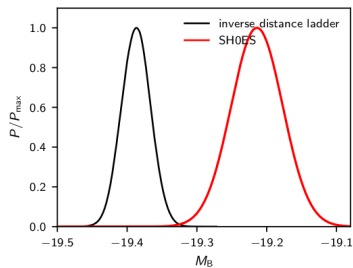
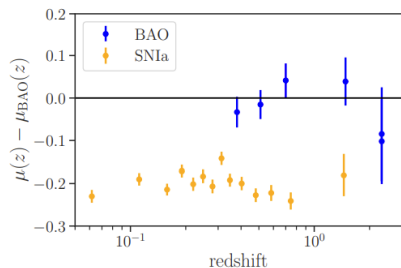
BAO+uncalibrated SNeIa very strongly constrain  $H(z)$  or  $E(z)$ , do not allow more than 10% deviations from  $\Lambda$ CDM at  $z \lesssim 2$

# Tension between calibrators

The tension is between calibrators!

$$\text{BAO: } \theta_s(z) = \frac{r_s}{d_A(z)}$$

$$\text{SN Ia: } \mu(z) = 5 \log_{10} d_L(z) + M_B$$

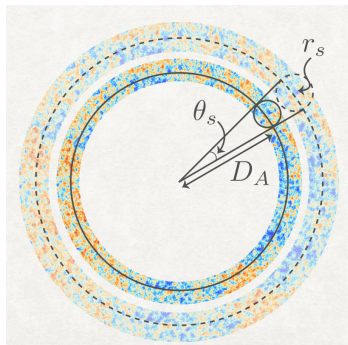


Tutusaus, Kunz & Favre, 2311.16862 (left); Efstathiou, MNRAS 505 (2021) 3866 (right)

Without change in calibration, BAO  $d_A$  and SN Ia  $d_L$  in an overlapping redshift range are incompatible!

## Is the CMB closer to us?

With  $\theta_s$  fixed, lower  $r_s$  implies lower  $d_A$



Credits: Tristan Smith and Vivian Poulin

- Is the CMB closer to us?
- Are the spots in the CMB smaller than what we expect within  $\Lambda$ CDM?

# What is the Hubble tension, really?

3 different interpretations in order of increasing “correctness”

The Hubble tension is the mismatch between:

- 1 CMB vs SH0ES  
→ *“Too wrong”, ignores stabilizing role of late-time datasets (BAO, uncalibrated SNeIa,...)*
- 2 Inverse distance ladder (CMB+BAO+uncalibrated SNeIa) vs SH0ES  
→ *Still wrong, ignores many other local/late-time measurements besides SH0ES (TRGB, strong lensing time delays,...)*  
*(at this level the Hubble tension is best thought of as a  $M_B$  tension)*
- 3 Inverse distance ladder vs **several** low- $z$   $H_0$  measurements  
→ **most correct interpretation of the Hubble tension!**

## Next lecture

7 December, 11:30-12:20

# *How to solve the Hubble tension?*

Early dark energy, varying electron mass, primordial magnetic fields, phantom dark energy,  $M_B$  transitions, and all that!