Effective Field Theory of Structure Formation

Lecture 3: Biased Tracers

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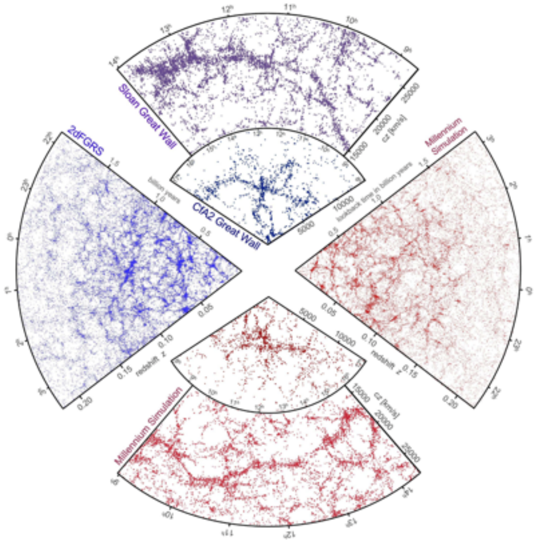
Outline:

1. Biased tracer in Eulerian picture
2. Non-locality in time and the equivalence of basis
3. Renormalization of bias coefficients
4. Bootstrap approach to galaxy bias
5. Biasing of shapes
6. Summary

Selected bibliography:

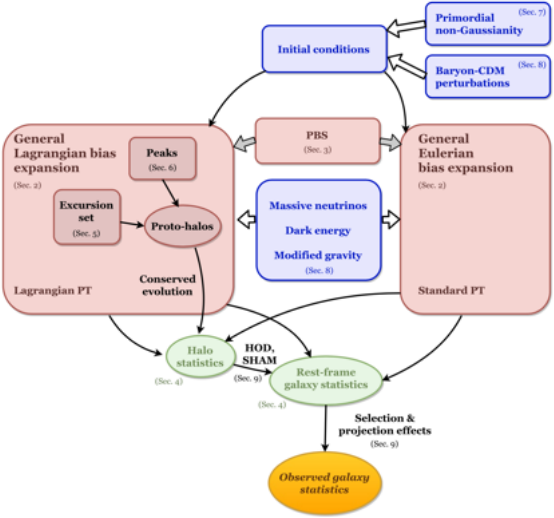
- Large-Scale Galaxy Bias, Desjacques et al., 2018, 1611.09787
- Lectures on EFTofLSS, Senatorel, (online notes)
- Modern Cosmology, Dodelson & Schmidt, 2021
- LSS of the Universe and PT, Bernardeau et al., 2002, astro-ph/0112551

Galaxies and galaxy halos



[Desjacques++:18]

Galaxies and galaxy halos

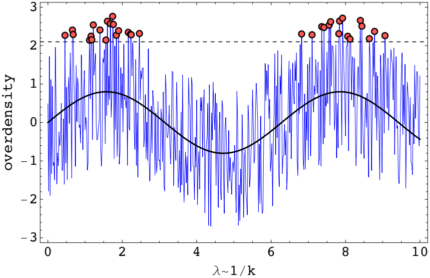


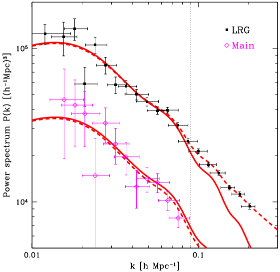
[Desjacques++:18]

Galaxies and their relation to dark matter distribution

Galaxies form at high density peaks of matter density:

rare peaks $\Longrightarrow$ higher clustering!





Tracer detriments the amplitude: $P_g(k) \sim b^2 P_m(k) + \ldots$ on large scales.
Understanding galaxy bias is crucial for understanding the galaxy clustering.
Coefficients incorporate complicated small scale (UV) physics:

- dark matter halo formation & merger history
- chemistry and cooling processes & background radiation
- feedback processes (SN, AGN, …)
- (and more ...)

Canonical approaches to galaxy biasing

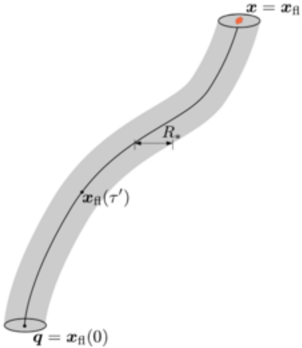
Local biasing model: relation to dark matter

$$\delta_{\mathsf{h}} = c_\delta \delta + c_{\delta^2}\delta^2 + c_{\delta^3}\delta^3 + \ldots$$ [Fry+:93]

Quasi-local (in space): [McDonald+:09]

$$\delta_{\mathsf{h}}(\boldsymbol{x}) = c_\delta \delta(\boldsymbol{x}) + c_{\delta^2}\delta^2(\boldsymbol{x}) + c_{\delta^3}\delta^3(\boldsymbol{x}) + c_{s^2}s^2(\boldsymbol{x}) + c_{\delta s^2}\delta(\boldsymbol{x})s^2(\boldsymbol{x}) + c_\epsilon \epsilon + \ldots,$$

with effective (bias) coefficients c_l and operators:



[from Desjacques++:18]

$$s_{ij}(\boldsymbol{x}) = \partial_i\partial_j\phi(\boldsymbol{x}) - \frac{1}{3}\delta^{\mathrm{K}}_{ij}\delta(\boldsymbol{x}), \quad \ldots$$

where ϕ is the gravitational potential, and white noise (stochasticity) ϵ.

Complete set set of operators including non-locality in time effects!

[Senatore:14,Angulo++:15, Desjacques++:18, ...]

Scalar field biasing: effective approach

[Desjacques++:18, ...]

Alternative systematisation in terms of derivatives of potential ϕ :

$$\Pi^{[1]}_{ij} = \frac{2}{3\Omega_m \mathcal{H}^2} k_i k_j \phi,$$

with higher operators O_h:

$$\begin{aligned}
&(1) \qquad \mathrm{tr}\left[\Pi^{[1]}\right], \\
&(2) \qquad \mathrm{tr}\left[\left(\Pi^{[1]}\right)^2\right], \quad \left(\mathrm{tr}\left[\Pi^{[1]}\right]\right)^2, \\
&(3) \qquad \mathrm{tr}\left[\left(\Pi^{[1]}\right)^3\right], \quad \mathrm{tr}\left[\left(\Pi^{[1]}\right)^2\right]\mathrm{tr}\left[\Pi^{[1]}\right], \quad \left(\mathrm{tr}\left[\Pi^{[1]}\right]\right)^3, \quad \mathrm{tr}\left[\Pi^{[1]}\Pi^{[2]}\right],
\end{aligned}$$

and additional derivative operators $R_*^2 \nabla^2 \mathrm{tr}\left[\Pi^{[1]}\right], \ldots$.

– series allows one to estimate the higher order (theory) errors

– coefficients - physics from the R_* scale (some degeneracies)

Tracer field is then given

$$\delta_{\mathrm{s}}(\boldsymbol{x}) = \sum_O b_O^{(s)} \mathrm{tr}[O_{ij}](\boldsymbol{x}),$$

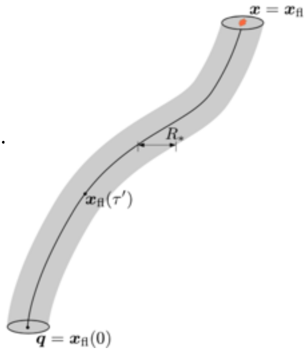
Effective field theory of biasing

Non-local (time) and quasi-local (spece) relation of tracers to the dark matter [Senatore 2014, Mirbabayi et al, 2014]

$$\delta_h(\boldsymbol{x},t) \simeq \int^t dt' \; H(t') \; \Big[\bar{c}_\delta(t,t')\,\delta(\boldsymbol{x}_{\rm fl},t') + \bar{c}_{\delta^2}(t,t')\,\delta(\boldsymbol{x}_{\rm fl},t')^2 + \bar{c}_{s^2}(t,t')\,s^2(\boldsymbol{x}_{\rm fl},t') + .. + \bar{c}_{\partial^2\delta}(t,t')\,\frac{\partial^2_{x_{\rm fl}}}{k_M^2}\delta(\boldsymbol{x}_{\rm fl},t') + \ldots\Big]$$

Fields evaluated on a past path:

$$\boldsymbol{x}_{\rm fl}(\boldsymbol{x},\tau,\tau') = \boldsymbol{x} - \int_{\tau'}^{\tau} d\tau''\;\boldsymbol{v}(\tau'',\boldsymbol{x}_{\rm fl}(\boldsymbol{x},\tau,\tau''))$$



[from Desjacques++:18]

Alternative - all effects chaptered in Lagrangian approach.

Assembly bias effects captured in the scheme.

Effective field theory of biasing

New physical scale $k_M \sim 2\pi \left(\frac{4\pi}{3}\frac{\rho_0}{M}\right)^{1/3}$.

Can be different then k_{NL}. Interesting case $k_{NL} \gg k_M$!

We look at the correlations at $k \ll k_M$.
Each order in perturbation theory we get new bias coefficients:

$$\begin{aligned}\delta_{\rm h}(k,t) &= \int_t \tilde{c}_{\delta,1}\left[D_t\delta^{(1)}(k) + \text{flow terms}\right] + \int_t \tilde{c}_{\delta,2}\left[D_t^2\delta^{(2)}(k) + \text{flow terms}\right] + \ldots \\ &= c_{\delta,1}\left[\delta^{(1)}(k) + \text{flow terms}\right] + c_{\delta,2}\left[\delta^{(2)}(k) + \text{flow terms}\right] + \ldots\end{aligned}$$

Emergence of degeneracy: choice of most convenient basis

Renormalization! (takes care of short distance effects at long distances)
In practice, $\tilde{c}_{\delta,1}$ is a bare parameter, the sum of a finite part and a counterterm:

$$\tilde{c}_{\delta,1} = \tilde{c}_{\delta,1,\,\text{finite}} + \tilde{c}_{\delta,1,\,\text{counter}},$$

After renormalization we end up with using 7 finite bias parameters b_i.

Observables: $P_{\rm hm}$, $P_{\rm hh}$, $B_{\rm hmm}$, $B_{\rm hhm}$, $B_{\rm hhh}$

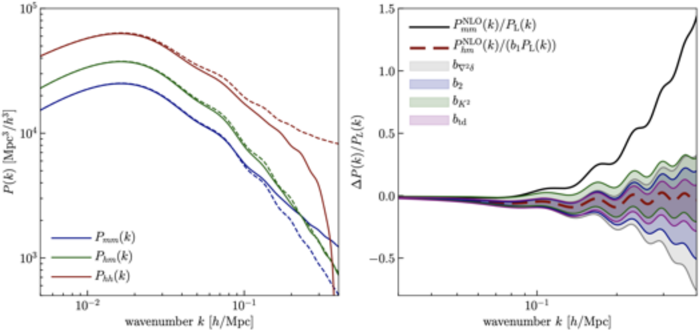
Power Spectrum

2-point observables:

$$P_{gm} = b_1 P_{mm} + b_{\delta^2} P_{\delta^{(2)}\delta^2} + b_{s^2} P_{\delta^{(2)}s^2} + (\text{3rd order}) - b_{\nabla^2} k^2/k_M^2 P_L + (\text{noise})\,,$$

$$P_{gg} = b_1^2 P_{mm} + \sum_{O\in\{\delta^2,s^2\}} b_O b_{O'} P_{OO'} + (\text{3rd order}) - b_1 b'_{\nabla^2} k^2/k_M^2 P_L + (\text{noise})\,.$$

We also now know how to add also the IR-resummation! (Long displacement)



[Desjacques++:18]

Adding baryonic effects

Baryons at large distances described as additional fluid component
(short distance physics is encoded in an effective stress tensor)

$$\delta_h(\boldsymbol{x},t) = \int^t dt'\ H(t')\ \Big[\bar{c}_{\partial^2\phi}(t,t')\ \frac{\partial^2\phi(\boldsymbol{x}_{\rm fl},t')}{H(t')^2} + \bar{c}_{\delta_b}(t,t')\, w_b\, \delta_b(\boldsymbol{x}_{{\rm fl},b)}$$
$$+\, \bar{c}_{\partial_i v_c^i}(t,t')\ w_c\ \frac{\partial_i v_c^i(\boldsymbol{x}_{{\rm fl},c},t')}{H(t')} + \bar{c}_{\partial_i v_b^i}(t,t')\ w_b\ \frac{\partial_i v_b^i(\boldsymbol{x}_{{\rm fl},b},t')}{H(t')}$$
$$\ldots\Big]$$

where ϕ is defined by Poisson equation and:

$$\boldsymbol{x}_{\rm fl,b}(\boldsymbol{x},\tau,\tau') = \boldsymbol{x} - \int_{\tau'}^{\tau} d\tau''\ \boldsymbol{v}_b(\tau'',\boldsymbol{x}_{\rm fl}(\boldsymbol{x},\tau,\tau''))\ ,$$
$$\boldsymbol{x}_{\rm fl,c}(\boldsymbol{x},\tau,\tau') = \boldsymbol{x} - \int_{\tau'}^{\tau} d\tau''\ \boldsymbol{v}_c(\tau'',\boldsymbol{x}_{\rm fl}(\boldsymbol{x},\tau,\tau''))$$

Similar expressions valid when including neutrinos, clustering dark energy ...

Adding Non-Gaussianities

For non-Gaussian fluctuations present only in the initial conditions

and effect described by the squeezed limit, $k_L \ll k_S$ of correlations.

After horizon re-rentry, but still early enough to neglect all gravitational non-linearities, the primordial density fluctuation are given by

$$\delta^{(1)}(\boldsymbol{k}_S, t_{\rm in}) \simeq \delta_g(\boldsymbol{k}_S) + f_{\rm NL}\tilde{\phi}(\boldsymbol{k}_L, t_{\rm in})\delta_g(\boldsymbol{k}_S - \boldsymbol{k}_L, t_{\rm in}) \ ,$$

where $\tilde{\phi}(\boldsymbol{k}_L, t_{\rm in}) \sim \frac{1}{k_S^2\, T(k)} \left(\frac{k_L}{k_S}\right)^{\alpha} \delta_g(\boldsymbol{k}_L, t_{\rm in})$ with a transfer function $T(k)$.

In the presence of primordial non-Gaussianities, additional components:

$$\begin{aligned} \delta_h(\boldsymbol{x}, t) \simeq {} & f_{\rm nl}\, \tilde{\phi}(\boldsymbol{x}_{\rm fl}(t, t_{\rm in}), t_{\rm in}) \\ & \times \int^t dt'\ H(t') \left[\bar{c}^{\,\tilde{\phi}}(t, t') + \bar{c}^{\,\tilde{\phi}}_{\partial^2\phi}(t, t')\, \frac{\partial^2\phi(\boldsymbol{x}_{\rm fl}, t')}{H(t')^2} + \ldots\right] \\ & + {f_{\rm nl}}^2\, \tilde{\phi}(\boldsymbol{x}_{\rm fl}(t, t_{\rm in}), t_{\rm in})^2 \int^t dt'\ H(t') \left[+ \ldots \right. \end{aligned}$$

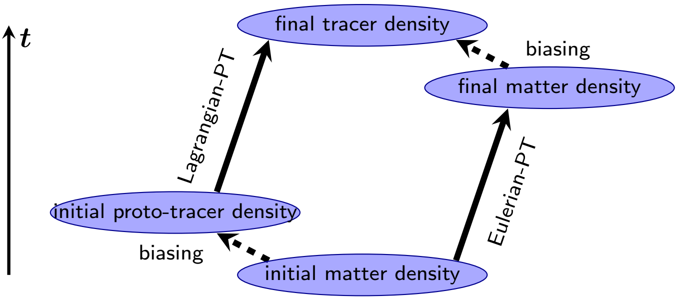
Non-linear dynamics and galaxy bias

- Eulerian bias: relates final d.m. density field and the final halo density

$$\delta_{\rm g}(\boldsymbol{x}) = c^{\rm e}_{\delta}\delta(\boldsymbol{x}) + c^{\rm e}_{\delta^2}\delta^2(\boldsymbol{x}) + c^{\rm e}_{s^2}s^2(\boldsymbol{x}) + \ldots + c^{\rm e}_{\partial^2\delta}\frac{\partial_q^2}{k_*^2}\delta(\boldsymbol{x}) + \text{“stochastic”} + \ldots$$

- Lagrangian bias: relates initial d.m. density field and the proto-halo density

$$\delta_{\rm g}(\boldsymbol{q}) = c^{\ell}_{\delta}\delta_L(\boldsymbol{q}) + c^{\ell}_{\delta^2}\delta_L^2(\boldsymbol{q}) + c^{\ell}_{s^2}s_L^2(\boldsymbol{q}) + \ldots + c^{\ell}_{\partial^2\delta}\frac{\partial_q^2}{k_*^2}\delta_L(\boldsymbol{q}) + \text{“stochastic”} + \ldots,$$

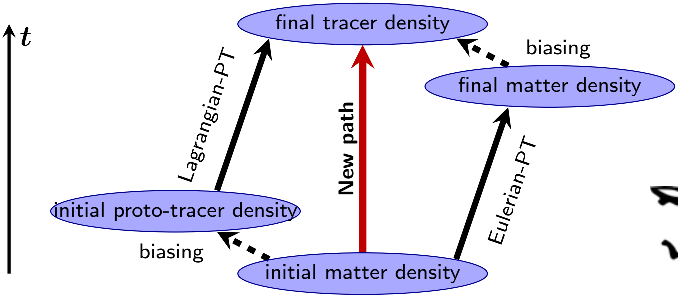


A new look at bias expansion: "Bootstrap in LSS"

A new idea: [Fujita+:2020]

(I.) construct a bias of operators from linear density - as a Monkey would,

(II.) impose physical constraints - consistency relations in LSS



A similar approach also done in [D'Amico++:2021] .

A new look at bias expansion: "Bootstrap in LSS"

A new idea:

(I.) construct a bias of operators from linear density - as a Monkey would,

(II.) impose physical constraints - consistency relations in LSS

How do we describe the system for a tracer?

Balance equations:

$$\partial_\tau \delta_\alpha(\boldsymbol{x}) + \boldsymbol{\nabla} \cdot \left([1+\delta_\alpha] \, \boldsymbol{u}_\alpha \right)(\boldsymbol{x}) = S_\delta[\delta](\boldsymbol{x}),$$

$$\partial_\tau \boldsymbol{u}_\alpha(\boldsymbol{x}) + \mathcal{H}\boldsymbol{u}_\alpha(\boldsymbol{x},\tau) + \boldsymbol{u}_\alpha(\boldsymbol{x},\tau)\cdot\boldsymbol{\nabla}\boldsymbol{u}_h(\boldsymbol{x},\tau) = -\boldsymbol{\nabla}\phi(\boldsymbol{x},\tau) + S_u[\delta](\boldsymbol{x}),$$

The lhs. terms are:

$$\nabla^2\phi(\boldsymbol{x}) \propto \delta_m(\boldsymbol{x}),$$

and small scale sources $S_\delta(\boldsymbol{x})$, $S_u(\boldsymbol{x})$ typically suppressed by some scale k_*.

The key notion is the separation of scales in the system, i.e. gravity dominates on large scales.

I. Specifying the non-linear terms

This is the "Monkey part":

$$\text{Continuity eq.}: \quad \partial_\tau \delta + (\text{linear terms}) = -\delta\theta - \partial_i \delta \frac{\partial_i}{\partial^2}\theta,$$

$$\text{Euler eq.}: \quad \partial_\tau \theta + (\text{linear terms}) = -\frac{\partial_i \partial_j}{\partial^2}\theta \frac{\partial_i \partial_j}{\partial^2}\theta,$$

where δ is the density and θ is the velocity divergence.
Solution is constructed by the iterative "Monkey" process

$$\left\{ XY, \quad \partial_i X \frac{\partial_i}{\partial^2} Y, \quad \frac{\partial_i \partial_j}{\partial^2} X \frac{\partial_i \partial_j}{\partial^2} Y \right\},$$

where X and Y are drown from the list of the lower order operators.

New bias basis:

$$\delta_\alpha = a_1 \delta_L$$
$$+ b_1 \delta_L^2 + b_2 \partial_i \delta_L \frac{\partial_i}{\partial^2}\delta_L + b_3 \frac{\partial_i \partial_j}{\partial^2}\delta_L \frac{\partial_i \partial_j}{\partial^2}\delta_L + \dots$$

In the paper we keep terms up to the third order terms in PT.

II. Constraining the coefficients

Consistency relations of LSS are direct consequence of the equivalence principle and adiabatic initial conditions:

$$\langle \delta^m_{\boldsymbol{k}}(\tau)\delta^{\rm g}_{\boldsymbol{q}_1}(\tau_1)\ldots\delta^{\rm g}_{\boldsymbol{q}_n}(\tau_n)\rangle' \sim -P_{\rm g}(\boldsymbol{k},\tau)\sum_\alpha \frac{D(\eta_\alpha)}{D(\eta)}\frac{\boldsymbol{k}\cdot\boldsymbol{q}_\alpha}{k^2}\langle \delta^{\rm g}_{\boldsymbol{q}_1}(\eta_1)\ldots\delta^{\rm g}_{\boldsymbol{q}_n}(\eta_n)\rangle', \quad \text{as } k\to 0.$$

Tree-level statistics is the simplest way to impose the constraints:

$$\lim_{k\to 0}\langle \delta_{\boldsymbol{k}}\delta^\alpha_{\boldsymbol{q}_1}\delta^\beta_{\boldsymbol{q}_2}\rangle' = \left(a_1{}^{(\alpha)}b_2{}^{(\beta)} - a_1{}^{(\beta)}b_2{}^{(\alpha)}\right)\frac{\boldsymbol{k}\cdot\boldsymbol{q}_1}{2k^2}P_\ell(k)P_\ell(q_1) + \mathcal{O}(k^0),$$

By requiring the IR-divergent term to vanish we get:

$$\frac{b_2{}^{(\alpha)}}{a_1{}^{(\alpha)}} = \frac{b_2{}^{(\beta)}}{a_1{}^{(\beta)}} = \mathcal{C}(\tau).$$

The $\mathcal{C}(\tau)$ is universal, tracers independent, function of time.

Fixing the dynamical degrees of freedom

New bias expansion:

$$\delta_g = a_1\left[\delta_L + \mathcal{C}\ \partial_i\delta_L\frac{\partial_i}{\partial^2}\delta_L\right] + b_1\delta_L^2 + b_3\left(\frac{\partial_i\partial_j}{\partial^2}\delta_L\frac{\partial_i\partial_j}{\partial^2}\delta_L\right) + (\text{3rd order})$$

How to determine the universal coefficients $\mathcal{C}(\tau)$?

Easy way is to fix it to the simples 'tracer' of dark matter: dark matter!

$$\mathcal{C} = 1.$$

In general these coefficients reflect dynamics and modifications of GR!
Example: clustering quintessence

$$\mathcal{C} = 1 - \epsilon(\tau),$$

where ϵ depends on the quintessence field and τ.
This motivates the construction on the near-optimal estimators for $\mathcal{C}$.

Ellipsoids, 2-tensors, galaxy shapes

How can we describe the field of ellipsoids?

Ellipsoid – 3 parameters;

$$T^0_{ij} = \begin{pmatrix} 1/a^2 & 0 & 0 \\ 0 & 1/b^2 & 0 \\ 0 & 0 & 1/c^2 \end{pmatrix}$$

Rotation matrix – 3 Euler angles;

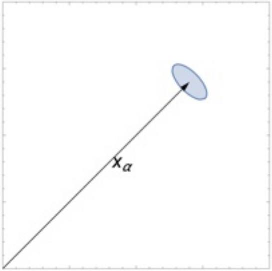
$$\mathcal{R}_{ij}(\psi, \theta, \phi) \implies \boldsymbol{T} = \boldsymbol{\mathcal{R}} \boldsymbol{T}^0 \boldsymbol{\mathcal{R}}^T$$

Ellipsoid equation;

$$(\boldsymbol{x} - \boldsymbol{x}_\alpha) \cdot \boldsymbol{T}^{(\alpha)} \cdot (\boldsymbol{x} - \boldsymbol{x}_\alpha) = 1$$

Tensor field:

$$T_{ij}(\boldsymbol{x}) = \sum_\alpha T^{(\alpha)}_{ij}(\boldsymbol{x}_\alpha) \delta^{\mathrm{D}}(\boldsymbol{x} - \boldsymbol{x}_\alpha)$$



Biasing of shapes in 3D: effective approach

Expansion of the field of galaxy shapes:

$$g_{ij}(\boldsymbol{x}) = \sum_O b_O^{(g)} \mathrm{TF}[O_{ij}](\boldsymbol{x}).$$

where the list of operators (up to higher derivatives and stochastic contributions) is

$$
\begin{aligned}
&(1) \qquad \mathrm{TF}\left[\Pi^{[1]}\right]_{ij}, \qquad\qquad \text{[Hirata\&Seljak : 04]}\\
&(2) \qquad \mathrm{TF}\left[\Pi^{[2]}\right]_{ij}, \quad \mathrm{TF}\left[\left(\Pi^{[1]}\right)^2\right]_{ij}, \quad \mathrm{TF}\left[\Pi^{[1]}\right]_{ij}\mathrm{tr}\left[\Pi^{[1]}\right],\\
&(3) \qquad \mathrm{TF}\left[\Pi^{[3]}\right]_{ij}, \quad \mathrm{TF}\left[\Pi^{[1]}\Pi^{[2]}\right]_{ij}, \quad \mathrm{TF}\left[\Pi^{[2]}\right]_{ij}\mathrm{tr}\left[\Pi^{[1]}\right],\\
&\qquad\quad \mathrm{TF}\left[\left(\Pi^{[1]}\right)^3\right]_{ij}, \quad \mathrm{TF}\left[\left(\Pi^{[1]}\right)^2\right]_{ij}\mathrm{tr}\left[\Pi^{[1]}\right], \quad \mathrm{TF}\left[\Pi^{[1]}\right]_{ij}\left(\mathrm{tr}\left[\Pi^{[1]}\right]\right)^2 \ldots
\end{aligned}
$$

Derivative operators relevant for leading power spectrum corrections

$$R_*^2 \nabla^2 \mathrm{TF}\left[\Pi^{[1]}\right]_{ij}.$$

Tensor fields and galaxy intrinsic alignments

Expansion of the field of galaxy shapes:

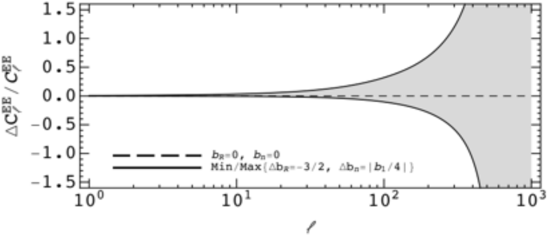
$$g_{ij}(\boldsymbol{x}) = \sum_{O} c_o O_{ij}(\boldsymbol{x}),$$

with biasing operator basis

$$(1) \qquad \mathrm{TF}\left[\Pi^{[1]}\right]_{ij},$$

$$(2) \qquad \mathrm{TF}\left[\Pi^{[2]}\right]_{ij}, \quad \mathrm{TF}\left[\left(\Pi^{[1]}\right)^2\right]_{ij}, \quad \mathrm{TF}\left[\Pi^{[1]}\right]_{ij}\mathrm{tr}\left[\Pi^{[1]}\right]$$

Relevant for: galaxy intrinsic alignment and galaxy lensing.



Formalism can be used as a probe of cosmological collider physics.