Effective Field Theory of Structure Formation

Lecture 2: Lagrangian Approach

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Outline:

1. Dark Matter Clustering in Lagrangian Framework
2. Resummation of the Long Displacements
3. Baryon Acoustic Oscillations
4. Primordial Features in the Power Spectrum
5. Summary

Selected bibliography:

- Large-Scale Galaxy Bias, Desjacques et al., 2018, 1611.09787
- The EFT approach to gravitational dynamics, Porto, 2016, 1601.04914
- Lectures on EFTofLSS, Senatorel, (online notes)
- Modern Cosmology, Dodelson & Schmidt, 2021
- LSS of the Universe and PT, Bernardeau et al., 2002, astro-ph/0112551

Dark Matter Clustering in Lagrangian Framework

Eulerian:

Lagrangian:

Coordinate of a (t)racer particle at a given moment in time $\boldsymbol{r}$

$$\boldsymbol{r}(\boldsymbol{q},\tau) = \boldsymbol{q} + \psi(\boldsymbol{q},\tau),$$

is given in terms of Lagrangian displacement.
Continuity equation:

$$(1+\delta(\boldsymbol{r}))\, d^3r = d^3q \quad \text{vs.} \quad 1+\delta(\boldsymbol{r}) = \int_q \delta^D\left(\boldsymbol{r}-\boldsymbol{q}-\psi(\boldsymbol{q})\right),$$

Fourier space

$$(2\pi)^3\delta^D(\boldsymbol{k}) + \delta(\boldsymbol{k}) = \int_q e^{i\boldsymbol{k}\cdot\boldsymbol{q}} \, \exp\left(i\boldsymbol{k}\cdot\psi\right),$$

Lagrangian Perturbation Theory

Density is given by the continuity relation

$$(1+\delta(\boldsymbol{x}))d^3x = d^3q\,.$$

We can combine it with the equation of motion for displacement field:

$$\ddot{\psi} + \mathcal{H}\dot{\psi} = -\nabla\Phi(\boldsymbol{q}+\psi(\boldsymbol{q}))$$

and solve perturbatively

$$\psi = \psi^{(1)} + \psi^{(2)} + \psi^{(3)} + \ldots.$$

Solutions are of the form

$$\psi^{(n)} = \int \frac{d^3p}{(2\pi)^3}\, L_n(\boldsymbol{p}_1,\ldots,\boldsymbol{p}_n)\delta_L(\boldsymbol{p}_1)\ldots\delta_L(\boldsymbol{p}_n)\,.$$

Example of leading kernels solutions

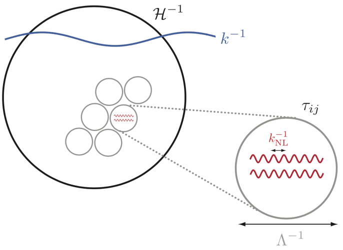
$$L_1(\boldsymbol{p}_1) = \boldsymbol{p}_1/p^2$$
$$L_2(\boldsymbol{p}_1,\boldsymbol{p}_2) = 3/7(\boldsymbol{p}_{12})/p_{12}^2\left(1-(\boldsymbol{p}_1.\boldsymbol{p}_2)/(p_1^2p_2^2)\right)$$

Lagrangian dynamics and EFT

The evolution of ψ is governed by

$$\ddot{\psi} + \mathcal{H}\dot{\psi} = -\nabla\Phi(\boldsymbol{q} + \psi(\boldsymbol{q})).$$

Integrating out short modes (using filter $W_R(\boldsymbol{q}, \boldsymbol{q}'))$ system is splitting tnto $L-$long and $S-$short wavelength modes, e.g.:



$$\psi_L(\boldsymbol{q}) = \int_q W_R(\boldsymbol{q}, \boldsymbol{q}')\psi(\boldsymbol{q}'), \quad \psi_S(\boldsymbol{q}, \boldsymbol{q}') = \psi(\boldsymbol{q}') - \psi_L(\boldsymbol{q}).$$

This defines δ_L as the long-scale component of the density perturbation corresponding to ψ_L and also Φ_L as the gravitational potential $\nabla^2\Phi_L \sim \delta_L$. E.o.m. for long displacement:

$$\ddot{\psi}_L + \mathcal{H}\dot{\psi}_L = -\nabla\Phi_L(\boldsymbol{q} + \psi_L(\boldsymbol{q})) + \boldsymbol{a}_S\big(\boldsymbol{q}, \psi_L(\boldsymbol{q})\big),$$

and $\boldsymbol{a}_S(\boldsymbol{q}) = -\nabla\Phi_S(\boldsymbol{q} + \psi_L(\boldsymbol{q})) - \frac{1}{2}Q_L^{ij}(\boldsymbol{q})\nabla\nabla_i\nabla_j\Phi_L(\boldsymbol{q} + \psi_L(\boldsymbol{q})) + \ldots,$
Similar formalism was also derived in [Porto et al, '14].

Matter power spectrum in Lagrangian EFT

Pair dispacement:

$$\Delta = \psi(\boldsymbol{r}_2) - \psi(\boldsymbol{r}_1).$$

The correlation function and power spectrum can now be defined through the cumulants of the displacement, e.g.

$$P(k) = \int_q e^{i\boldsymbol{q}\cdot\boldsymbol{k}} \left[\left\langle e^{i\boldsymbol{k}\cdot\boldsymbol{\Delta}(\boldsymbol{q})} \right\rangle - 1 \right].$$

For one loop power spectrum results, keeping linear modes resumed:

$$P(k) = \int_q e^{i\boldsymbol{k}\cdot\boldsymbol{q}} \exp\left[-\frac{1}{2} k_i k_j \left\langle \Delta_i \Delta_j \right\rangle_c + \frac{i}{6} k_i k_j k_k \left\langle \Delta_i \Delta_j \Delta_k \right\rangle_c + \cdots \right]$$

Final results equivalent to the Eulerian scheme.

Allows for the insight in the counter term structure and IR resummation schemes (in particular one leads to the scheme in [Senatore+:14]).

Alternative derivation of IR schemes: [Baldauf++:15, Blas++:16].

Lagrangian dynamics and EFT

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For one loop power spectrum results, keeping linear modes resumed:

$$P(k) = \int_q e^{i\boldsymbol{k}\cdot\boldsymbol{q}-(1/2)k_i k_j A_{ij}^{\rm lin}} \left[1 - \frac{1}{2}k_i k_j A_{ij}^{\rm lpt+eft} + \frac{i}{6}k_i k_j k_k W_{ijk}^{\rm lpt+eft} + \cdots\right]$$

with

$$A_{ij}(\boldsymbol{q}) = 2\left\langle \Psi_i(\boldsymbol{0})\Psi_j(\boldsymbol{0})\right\rangle - 2\left\langle \Psi_i(\boldsymbol{q}_1)\Psi_j(\boldsymbol{q}_2)\right\rangle$$

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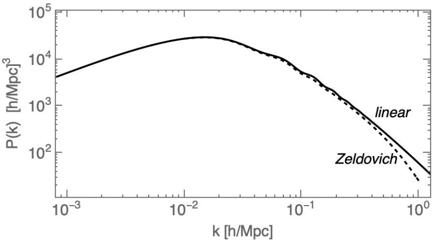
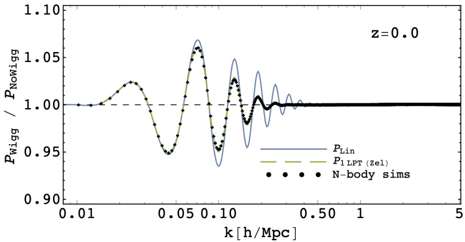
Zeldovich power spectrum

Keeping only linear displacements modes

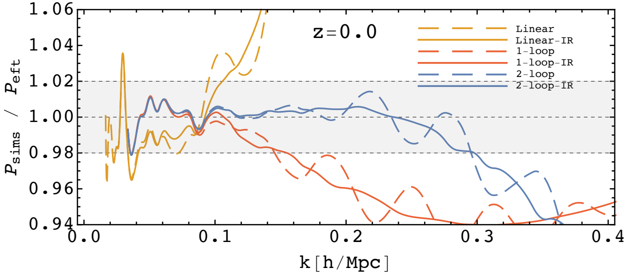
$$P_{\rm Zel}(k) = \int_q \; e^{i\boldsymbol{k}\cdot\boldsymbol{q} - (1/2) k_i k_j A^{\rm lin}_{ij}}$$





Resummation of the Long (IR) Displacements

$$P_{\text{EFT}}(k) = P_0 + P_{\text{1-loop}} + P_{\text{2-loop}} - c_s^2 \frac{k^2}{k_{\text{NL}}^2}(P_{11} + P_{\text{1-loop}}) + \text{c.t.}$$

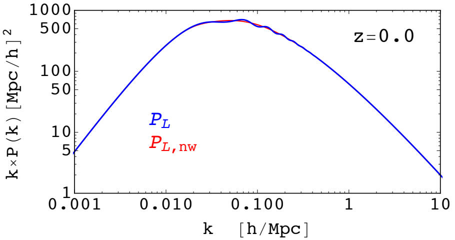


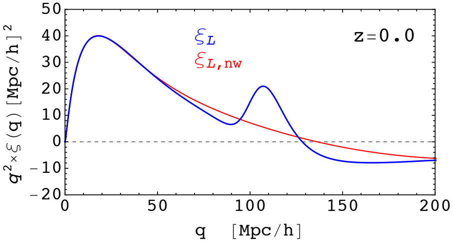
- Well defined/convergent expansion in k/k_{NL}.
- IR divergence and IR safely of equal time correlators (ψ_L).
- IR resummation (Lagrangian approach) - well described BAO

Linear power spectrum, correlation function & BAO

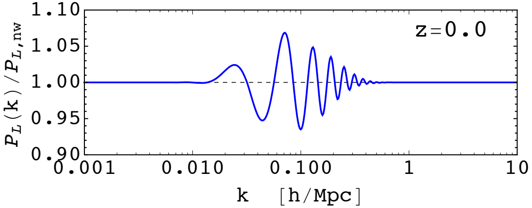
Linear power spectrum P_L: form Boltzmann codes (CAMB, Class).

Can be divided: smooth $P_{L,nw}$ + wiggle part $P_{L,w}$: $P_L = P_{L,nw} + P_{L,w}$





Wiggle power spectrum: $P_{\mathrm{L,w}}$.

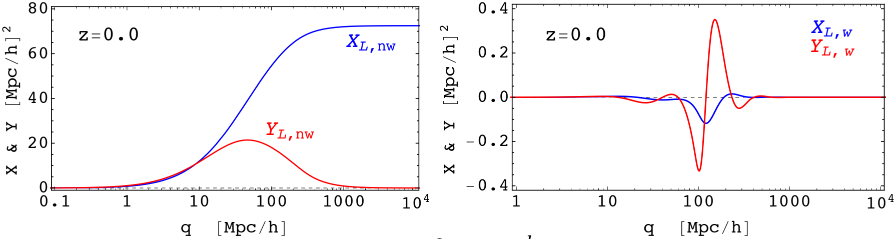


Resummation of IR modes: simple scheme

Separating the wiggle and non-wiggle part

$$A^{ij}_{\rm L}(\boldsymbol{q}) = A^{ij}_{\rm L,nw}(\boldsymbol{q}) + A^{ij}_{\rm L,w}(\boldsymbol{q});$$

$$P = P_{\rm nw} + \int_q e^{i\boldsymbol{k}\cdot\boldsymbol{q} - (1/2)k_ik_jA^{ij}_{\rm L,nw}} \left[-\frac{k_ik_j}{2}\mathcal{A}^{ij}_{\rm L,w} + \cdots \right] \simeq P_{\rm nw} + e^{-k^2\Sigma^2}P_{\rm L,w} + \ldots$$

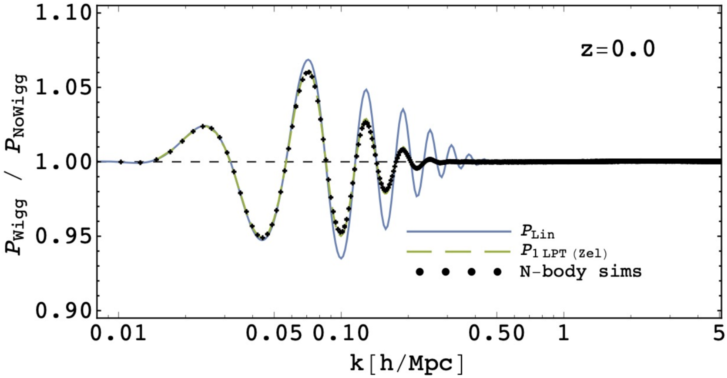


IR-SPT resummation model with $\Sigma^2 = \int \frac{dp}{3\pi^2}(1 - j_0(q_{\rm max}k))P_L(p)$:

$$P_{\sf EFT}(k) = \Big[P_0 + P_{\textsf{1-loop}} - c_s^2 \frac{k^2}{k^2_{\sf NL}} P_{11} + e^{-k^2\Sigma^2}\left(P_{\rm w,0} + \Delta P_{\rm w,1-loop} + (c_s^2/k^2_{\rm NL} + \Sigma^2)k^2 \right) P_{\rm w,0} \Big)$$

Alternative derivation in: [Baldauf:2015]

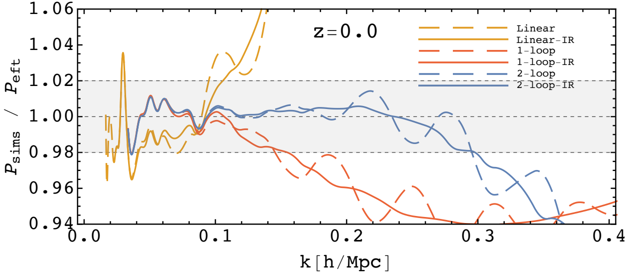
BAO wiggles



z=0.0
1.10
1.05
1.00
0.95
0.90
P_Wigg / P_NoWigg
P_Lin
P_1 LPT (Zel)
N-body sims
0.01
0.05 0.10
0.50
1
5
k[h/Mpc]

Power spectrum, loops & BAO

$$P_{\text{EFT}}(k) = P_0 + P_{\text{1-loop}} + P_{\text{2-loop}} - c_s^2 \frac{k^2}{k_{\text{NL}}^2}(P_{11} + P_{\text{1-loop}}) + \text{c.t.}$$



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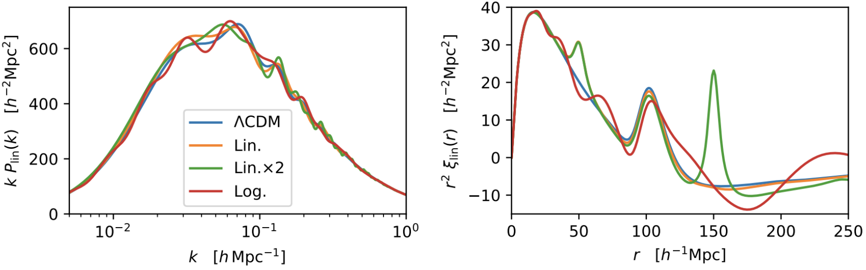
Primordial Features in the Power Spectrum

Phenomenological models of primordial features:

$$\text{(linear)}: \quad P_L = P_{\Lambda\text{CDM}}\left(1 + A\sin(\omega k)\exp(-(kr_d)^2\right)$$

$$\text{(logarithmic)}: \quad P_L = P_{\Lambda\text{CDM}}\left(1 + A\sin\left(\omega\ln(k/k^*)\right)\exp(-(kr_d)^2\right)$$



Primordial Features in the Power Spectrum

Power spectrum after resummation of the IR modes.