Effective Field Theory of Structure Formation

Lecture 1: Dark Matter Clustering

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Tonale Winter School on Cosmology 2023

Outline:

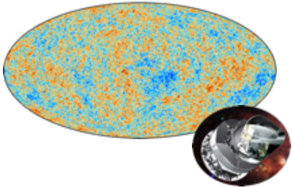
1. Quick Intro to Galaxy Surveys
2. Dark Matter as a Fluid
3. Perturbative Solution
4. Coarse Graining the EoM
5. One & Two Loop Matter Power Spectrum
6. Higher-Order Solutions
7. Summary

Selected bibliography:

- Large-Scale Galaxy Bias, Desjacques et al., 2018, 1611.09787
- Lectures on EFTofLSS, Senatorel, (online notes)
- Modern Cosmology, Dodelson & Schmidt, 2021
- LSS of the Universe and PT, Bernardeau et al., 2002, astro-ph/0112551

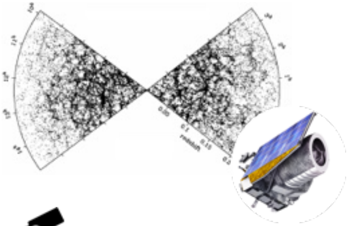
Quick Intro to Galaxy Surveys: Motivation

microwave background radiation



$\Longrightarrow$

structure formation



Physics motivation:

- Origin of structures & tests of gravity
- Expansion & composition of the universe
- Nature of dark energy and dark matter
- Neutrino mass and number of species

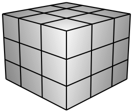
Observations:

- DESI, Rubin, Euclid, DES, SKA, SPHEREx, CMB-S4, ...

Information: CMB $\sim k^2$

$\Longrightarrow$

LSS $\sim k^3$

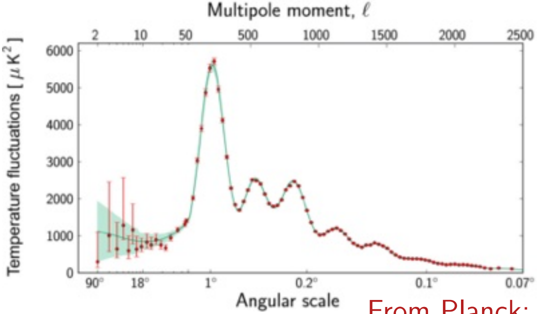


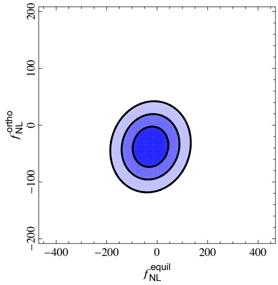
What are the challenges?

Nonlinear gravitational evolution, complex system (galaxies), multiscale dynamics, ...

Motivation and physics: Inflation

Origin of fluctuations in the universe: (slow role inflaton?)





From Planck:

primordial, scale invariant, gaussian, have a tilet $n_s \approx -0.965$

Current constraints on the Effective Lagrangian of inflation

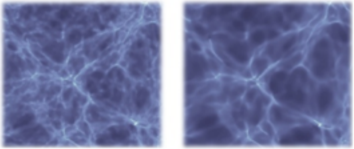
$$S_\pi = \int d^4\sqrt{-g}\left[-\frac{\dot{H}}{c_s^2}\left(\dot{\pi}^2 - c_s^2(\partial_i\pi)^2\right) + \frac{\dot{H}}{c_s^2}\left(\dot{\pi}(\partial_i\pi)^2 + \bar{c}_3\dot{\pi}^3\right)\right]$$

[Cheung++:08]

Is this the best we can do on inflation? Does having more modes help?

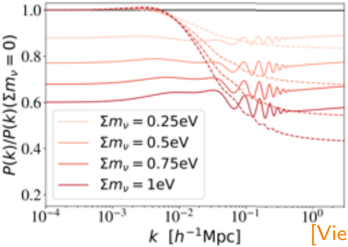
Motivation and physics: Neutrinos

Massive neutrinos $\implies$ affect dark matter structure on small scales

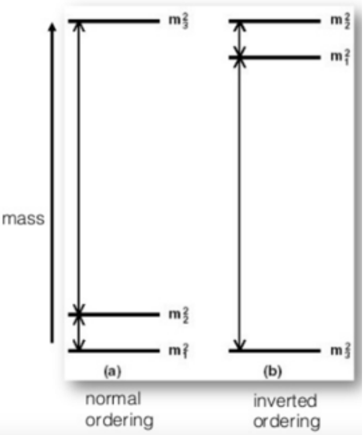


Neutrino Mass Negligible

Neutrino Mass Really Large (qualitative)



[Viel++:13]

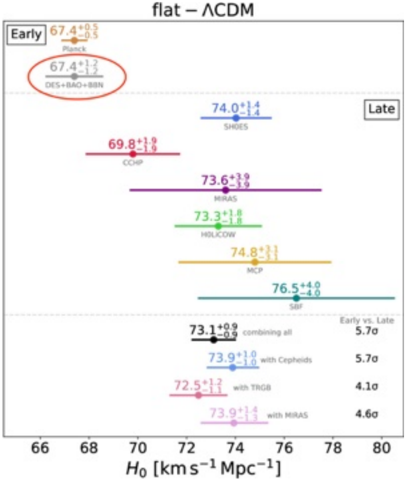


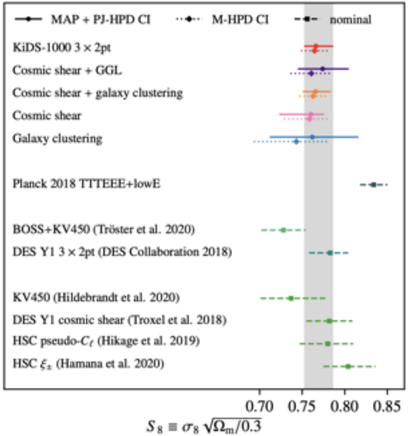
Measuring mass sum $\sum_\alpha m_\alpha$ gives insights on scale, mass ordering & type.

Target mass: > 60meV

Current mass: < 0.24eV (Planck) & < 0.12eV (Planck+LSS)

Tensions: Hubble parameter H_0, fluctuation variance σ_8.



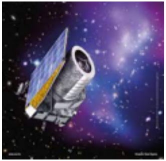


[HOLiCOW/Bonvin] [Snowmass2021]

Are these n-σ discrepancies a sign of new physics?

Galaxy Surveys

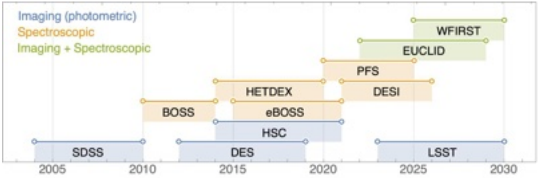
EUCLID

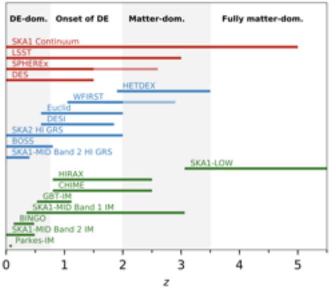


DESI

Rubin

Past, current and upcoming LSS surveys:





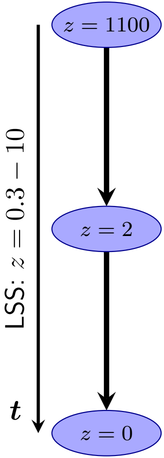
[Weltman++:19]

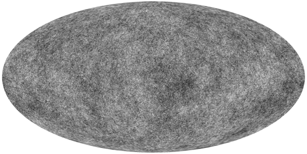
Structure Formation and Evolution

CMB: $\Delta\rho/\rho \sim 10^{-6}$

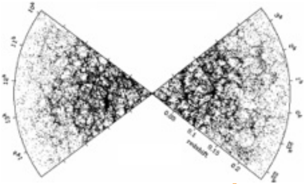
LSS: $\Delta\rho/\rho \sim 10^{0}$

Galaxies: $\Delta\rho/\rho \sim 10^{6}$

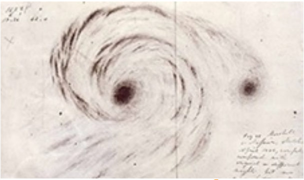




[Planck,2013]

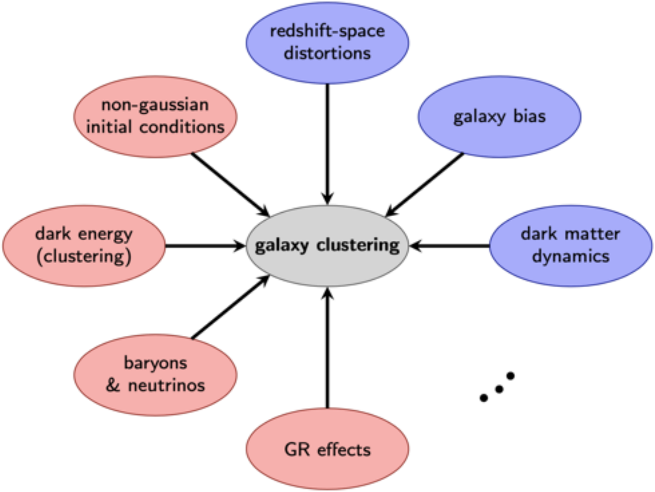


[2dF, 2002]

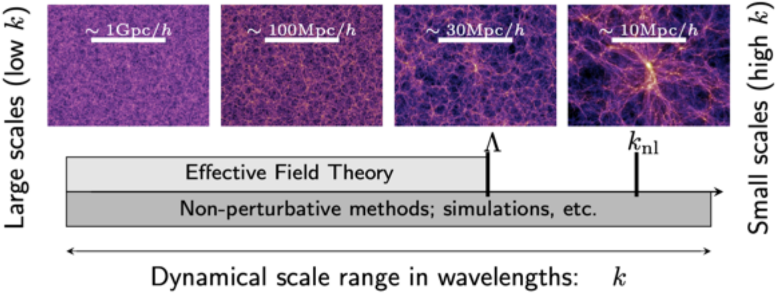


[Parsons, 1845]

Scope: Application to Galaxy Clustering



EFT applied to Structure Formation



Describe the matter density on large-scales (small fluctuations).

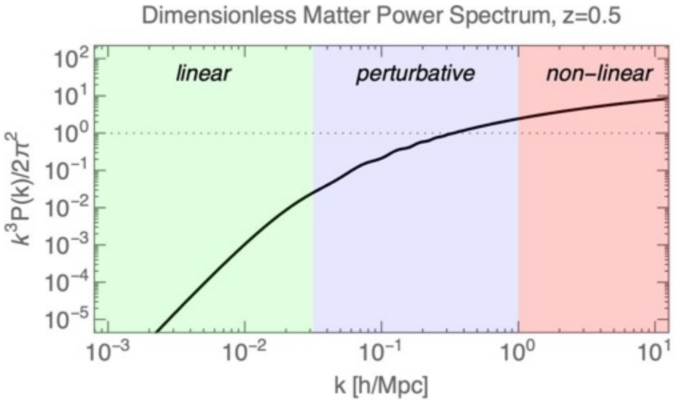
EFT methods:

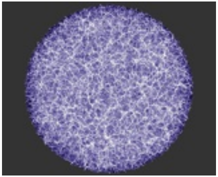
a) UV physics unknown, and we have scale separation (inflation, baryonic fluids, dielectrics)

b) UV physics known, but long-wavelengths are of interest (phonons, QCD (CPT))

Bias coefficients incorporate complicated galaxy formation physics: halo formation, merger history, feedback (SN, AGN), ...

Gravitational clustering of dark matter





Dark Matter as a Fluid

Cosmological fluid in the Newtonian limit, i.e. $r \ll H^{-1}$ and $v \ll 1$.

Evolution of collisionless particles - Vlasov equation:

$$\frac{df}{d\tau} = \frac{\partial f}{\partial \tau} + \frac{1}{ma}\boldsymbol{p} \cdot \boldsymbol{\nabla}_x f - am\boldsymbol{\nabla}\phi \cdot \boldsymbol{\nabla}_p f = 0\,,$$

and $\nabla^2\phi = 3/2\mathcal{H}^2\Omega_m\delta$.

Integral moments of the distribution function:

Mass density field:

$$\rho(\boldsymbol{x}) = ma^{-3}\int d^3p\ f(\boldsymbol{x},\boldsymbol{p})\,,$$

Streaming velocity field:

$$v_i(\boldsymbol{x}) = \int d^3p\ \frac{p_i}{am} f(\boldsymbol{x},\boldsymbol{p}) \Big/ \int d^3p\ f(\boldsymbol{x},\boldsymbol{p})\,.$$

Velocity dispersion field:

$$\sigma_{ij}(\boldsymbol{x}) = \int d^3p\ \frac{p_i p_j}{(am)^2} f(\boldsymbol{x},\boldsymbol{p}) \Big/ \int d^3p\ f(\boldsymbol{x},\boldsymbol{p}) - v_i v_j\,.$$

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and $\nabla^2\phi = 3/2\mathcal{H}^2\Omega_m\delta$.

Eulerian framework - fluid approximation:

$$\frac{\partial \delta}{\partial \tau} + \boldsymbol{\nabla} \cdot [(1+\delta)\boldsymbol{v}] = 0$$

$$\frac{\partial v_i}{\partial \tau} + \mathcal{H}v_i + \boldsymbol{v} \cdot \nabla v_i = -\nabla_i \phi - \frac{1}{\rho}\nabla_i(\rho\sigma_{ij}),$$

where σ_{ij} is the velocity dispersion.

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and $\nabla^2\phi = 3/2\mathcal{H}^2\Omega_m\delta$.

Eulerian framework - **pressureless perfect fluid** approximation:

$$\frac{\partial \delta}{\partial \tau} + \boldsymbol{\nabla} \cdot [(1+\delta)\boldsymbol{v}] = 0$$
$$\frac{\partial v_i}{\partial \tau} + \mathcal{H} v_i + \boldsymbol{v} \cdot \nabla v_i = -\nabla_i \phi.$$

Irrotational fluid: $\theta = \nabla \cdot \boldsymbol{v}$.
Manny variations of PT: SPT, RPT, ClosurePT, RG, TimeRG, GRPT . . .

Gravitational clustering of dark matter

Perturbative solution ansatz (SPT):

$$\delta(\boldsymbol{k}) = \sum_{n=1}^{\infty} D^n \int_q F_n(\boldsymbol{q}_1 \ldots \boldsymbol{q}_n)\delta_L(\boldsymbol{q}_1)\ldots\delta_L(\boldsymbol{q}_n)\delta^D(\boldsymbol{k}-\boldsymbol{q}|_1^n),$$

$$\theta(\boldsymbol{k}) = -f\mathcal{H}\sum_{n=1}^{\infty} D^n \int_q G_n(\boldsymbol{q}_1 \ldots \boldsymbol{q}_n)\delta_L(\boldsymbol{q}_1)\ldots\delta_L(\boldsymbol{q}_n)\delta^D(\boldsymbol{k}-\boldsymbol{q}|_1^n).$$

Kernels

$$F_2(\boldsymbol{k}_1,\boldsymbol{k}_2) = \frac{5}{7} + \frac{1}{2}\frac{\boldsymbol{k}_1\cdot\boldsymbol{k}_2}{k_1 k_2}\left(\frac{k_1}{k_2}+\frac{k_2}{k_1}\right) + \frac{2}{7}\frac{(\boldsymbol{k}_1\cdot\boldsymbol{k}_2)^2}{k_1^2 k_2^2}$$

and in configuration space

$$\delta^{(2)}(\boldsymbol{x}) = \underbrace{\frac{17}{21}\delta^2(\boldsymbol{x})}_{\text{growth}} - \underbrace{\boldsymbol{\psi}(\boldsymbol{x})\cdot\boldsymbol{\nabla}\delta(\boldsymbol{x})}_{\text{displacement}} + \underbrace{\frac{2}{7}s^2(\boldsymbol{x})}_{\text{tidal}}$$

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$$\theta(\boldsymbol{k}) = -f\mathcal{H} \sum_{n=1}^{\infty} D^n \int_q G_n(\boldsymbol{q}_1 \dots \boldsymbol{q}_n)\delta_L(\boldsymbol{q}_1) \dots \delta_L(\boldsymbol{q}_n)\delta^D(\boldsymbol{k} - \boldsymbol{q}|_1^n).$$

Total one-loop SPT solution for power spectrum:

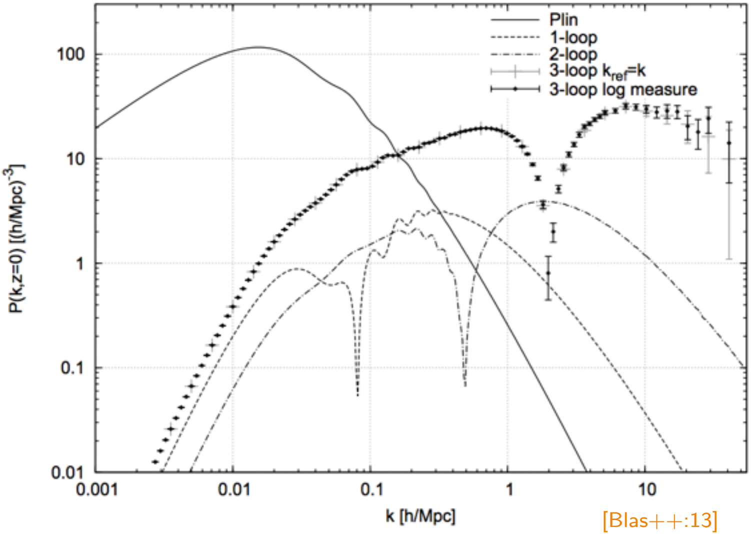
$$\langle\delta|\delta\rangle = \left\langle \delta^{(1)}|\delta^{(1)} \right\rangle + 2\left\langle \delta^{(1)}|\delta^{(3)} \right\rangle + \left\langle \delta^{(2)}|\delta^{(2)} \right\rangle$$
$$\Rightarrow P_{\mathsf{1loop}}(k) = P_{\mathsf{L}}(k) + 2P_{13}(k) + P_{22}(k),$$

with the one-loop correction terms (UV not under control $\delta \sim k/k_{NL}$):

$$P_{22}(k) = 2\int_q \left[F_2\left(\boldsymbol{q}, \boldsymbol{k} - \boldsymbol{q}\right)\right]^2 P_L(q) P_L\left(|\boldsymbol{k} - \boldsymbol{q}|\right)$$

$$P_{13}(k) = 3P_L(k)\int_q F_3\left(\boldsymbol{k}, \boldsymbol{q}, -\boldsymbol{q}\right) P_L(q)$$

SPT results



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Are we done? What is wrong with SPT?

- standard perturbation theory is not well defined!
- standard field solution

$$\delta^{(n)} \propto \int \mathrm{Kernel}^{(n)}\left(\delta^{(1)} \dots \delta^{(n-1)}\right)$$

- two point solution has loops

$$\left\langle \delta^{(2)}\delta^{(2)} \right\rangle \propto \int_{k'} \left\langle \delta^{(1)}\delta^{(1)} \right\rangle_{k'-k} \left\langle \delta^{(1)}\delta^{(1)} \right\rangle_{k'}$$

- Pure perturbative solution brakes in the UV limit: $\delta \propto k/k_{NL} \gg 1$ for $k > k_{NL}$
- Truncations brake the fluid picture.

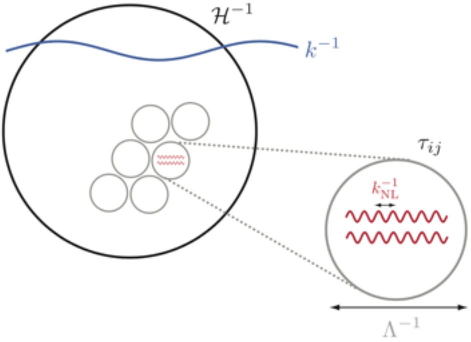
Gravitational clustering of dark matter

Evolution of collisionless particles - Vlasov equation:

$$\frac{df}{d\tau} = \frac{\partial f}{\partial \tau} + \frac{1}{m}\boldsymbol{p}\cdot\boldsymbol{\nabla} f - am\boldsymbol{\nabla}\phi\cdot\boldsymbol{\nabla}_p f = 0, \quad \text{and} \quad \nabla^2\phi = \tfrac{3}{2}\mathcal{H}\Omega_m\delta$$

PT solution for long modes:

$$\delta(\boldsymbol{x}) = \delta_\Lambda^{(1)}(\boldsymbol{x}) + \delta_\Lambda^{(2)}(\boldsymbol{x}) + \delta_\Lambda^{(3)}(\boldsymbol{x}) - c_s^2(\Lambda)\nabla^2/k_{\rm NL}^2\delta_\Lambda^{(1)}(\boldsymbol{x})$$



Gravitational clustering in EFT

The resulting equations are equivalent to Eulerian fluid equations

$$\partial_t \rho + H\rho + \partial_i(\rho v^i) = 0$$
$$\partial_t v^i + H v^i + v^j \partial_j v^i = -\partial_i \phi + \frac{1}{\rho}\partial_j \tau^{ij}$$
$$\partial^2 \phi = H^2 \delta\rho/\rho$$

Integrating out the short modes

$$[f_s g_s]_\Lambda = \langle f_s g_s \rangle_\Lambda + (\partial \langle f_s g_s \rangle / \partial \delta_l)|_0 \delta_l + \dots$$

EFT introduces a non-trivial stress tensor for the long-distances:

$$[\tau_{ij}]_\Lambda = p_0 \delta_{ij} + c_s^2 \delta\rho\delta_{ij} + O(\partial^2, \delta, \dots)$$

- Two approaches: one is obtained by smoothing the short scales in the fluid with the smoothing filter $W(\Lambda)$.
- Alternative in the Lagrangian picture: integrating out the short scales int the Lagrangian of the displacement fields.

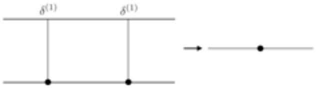
Eulerian PT results for the power spectrum

One loop Power Spectrum results in the Eulerian PT:

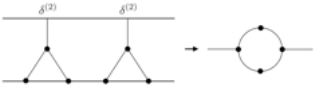
$$P(k) = \underbrace{P_0(k)}_{\text{LO}} + \underbrace{P_{22}(k,\Lambda) + 2P_{13}(k,\Lambda) - 2c_s^2(\Lambda)\frac{k^2}{k_{\rm NL}^2}P_0(k,\Lambda)}_{\text{NLO}}$$

Renormalization leads to the theory that is under control in the UV:

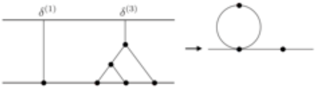
$$P_0(k) =$$



$$P_{22}(k) = \int_q F_{22}(q, k-q)^2 P_0(q) P_0(k-q) =$$



$$P_{13}(k) = 3P_0(k)\int_q F_{13}(k,q,-q)P_0(q) =$$



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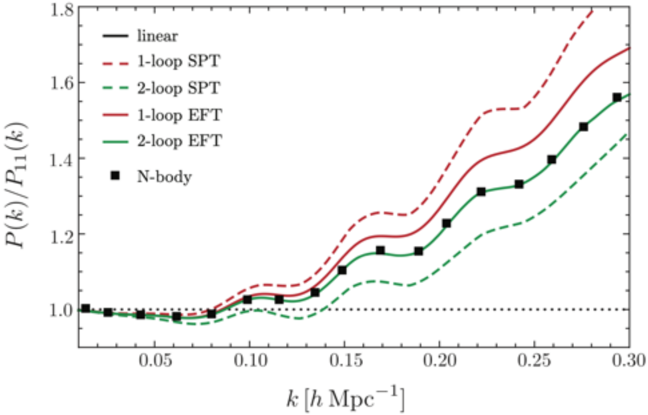
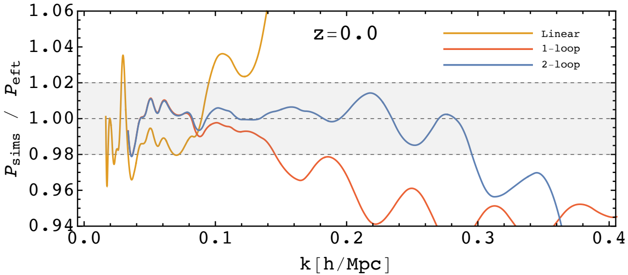


Power spectrum, loops & BAO

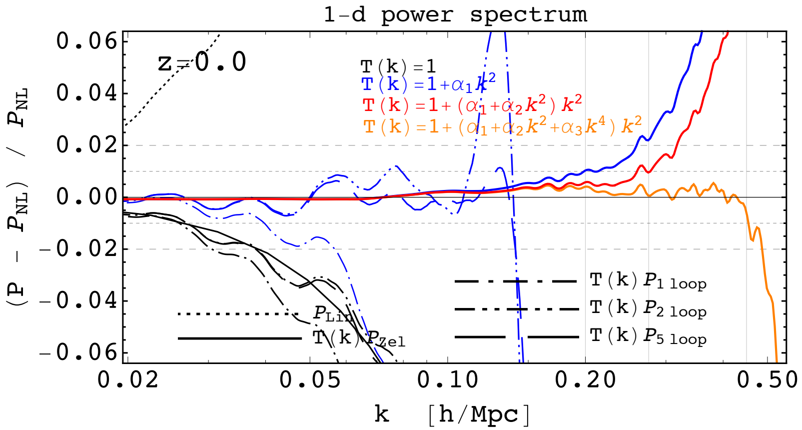
$$P_{\mathsf{EFT}}(k) = P_0 + P_{\text{1-loop}} + P_{\text{2-loop}} - c_s^2 \frac{k^2}{k_{\mathsf{NL}}^2}(P_{11} + P_{\text{1-loop}}) + \text{c.t.}$$



- Well defined/convergent expansion in k/k_{NL}.
- IR divergence and IR safely of equal time correlators (ψ_L).
- IR resummation (Lagrangian approach) - well described BAO

Clustering in 1D

1D case: Exists closed analytic solution, i.e. any n-order



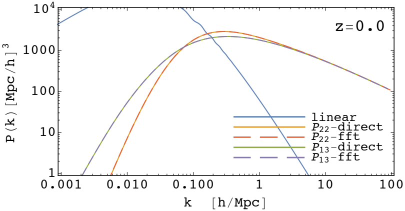
Efficient Evolution of Loops

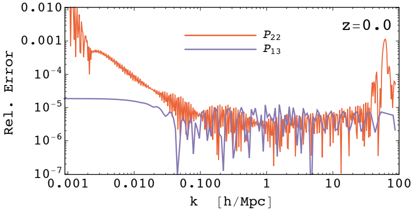
Problem: How do we get the i.c. parameters - MCMC runs - SLOW!

$$P_{1-\text{loop}} = P_{\text{lin}} + P_{22} + 2P_{13} + P_{\text{c.t.}}$$

$$P_{22} \sim \int_q f(q)g(k-q)P_q^{\text{lin}}P_{k-q}^{\text{lin}} = \int_0^\infty r^2 j_0(rk)\left[\xi^{\text{lin}}(r)\right]^2$$

Solution: Mellin transform used to reduce the problem to Hankel/Bessel!





Very fast to evaluate - useful is FFTLog

Why perturbative approach?

- Goal is the high precision at large scales (in scope of next gen. surveys), as well as to push to small scales.
- This problem is also amenable to direct simulation.
  - Though the combination of volume, mass and force resolution and numerical accuracy is very demanding - in scope of next gen. surveys.
  - PT is a viable alternative as well as a guide what range of k, M_h, scales are necessary and what statistics are needed.
  - N-body can be used to test PT for 'fiducial' models.
- However, PT can be used to search a large parameter space efficiently, and find what kinds of effects are most important.
  - Can be much more flexible/inclusive, especially for biasing schemes.
  - It is much easier to add new physics, especially if the effects are small (e.g. neutrinos, clustering dark energy, non-Gaussianity)
- Gaining insights
- Complementarity reason; if we can, we should.