A NEW LOOK AT HYDRODYNAMIC ATTRACTORS

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Hydrodynamic attractor: **1503.07514**

Review of hydrodynamization in heavy-ion collisions: **1707.02282**

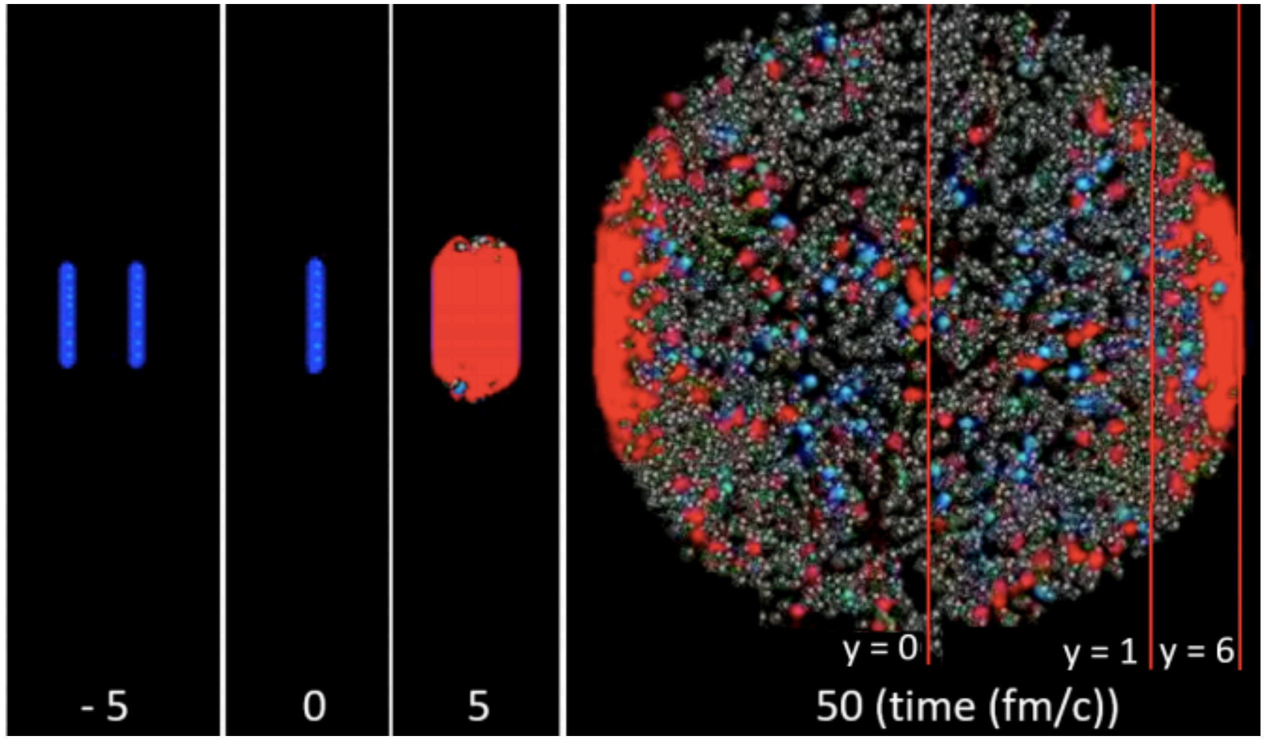
New results: an upcoming paper with Viktor Svensson  et al.

Quantum systems in extreme conditions

Ultrarelativistic heavy-ion collisions at RHIC (LHC) have provided for the past 20 (10) years a unique experimental window on quantum fields of the strong force under extreme conditions existing otherwise only in the early universe

see Alexander Kalweit's talk

fig. taken from 1802.04801



The pheno paradigm for these experiments is based on creating a collective medium described hydrodynamically from very early times $\mathcal{O}(0.5 \text{ fm/c})$

see Silvia Masciocchi's talk

AdS/CFT: the strong coupling approach to HIC

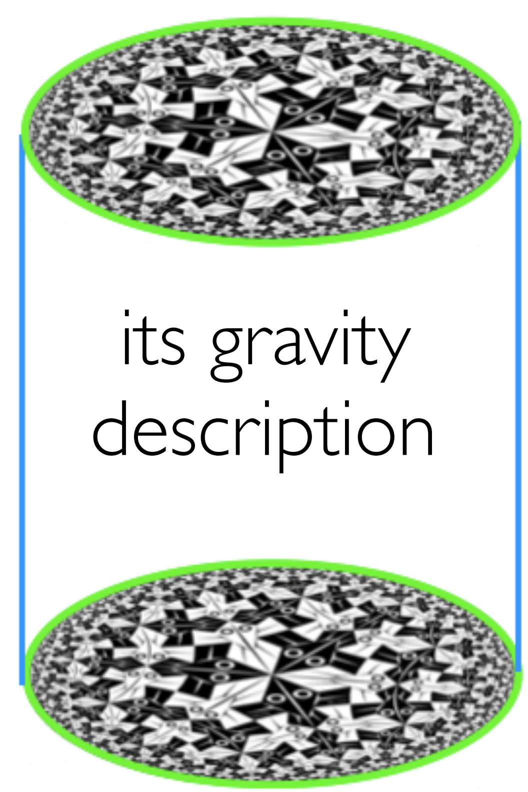
We do not have yet a technology to describe all aspects of HIC using QCD

This talk: replace QCD with certain strongly-coupled quantum field theories,

$SU(3) \longrightarrow SU(N_c \gg 1)$ and adding matter to make $g^2 N_c \gg 1$ a constant

, and study the pheno paradigm with their ab initio AdS/CFT description*:

certain QFT
(here: certain conformal field theory)



see also Paul Wittmer's talk

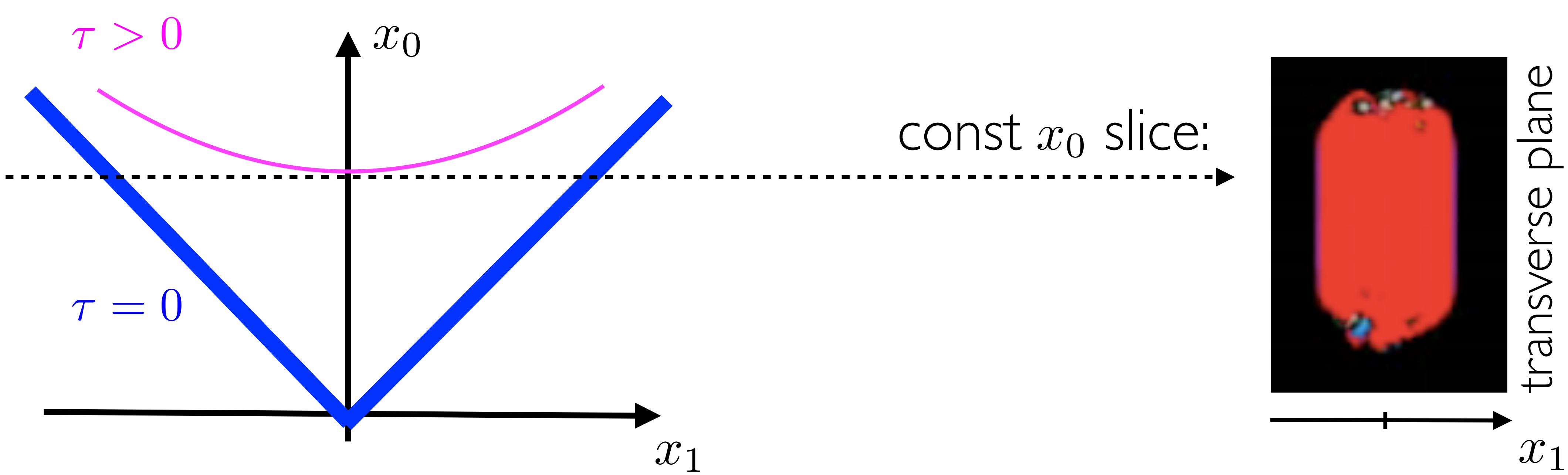
A model for a heavy-ion collision

Bjorken 1982

Idea: to be able to deal with many different initial non-equilibrium states, focus on the simplest setup that keeps the essence of the problem

Expansion only along the collision axis, no dynamics in transverse directions

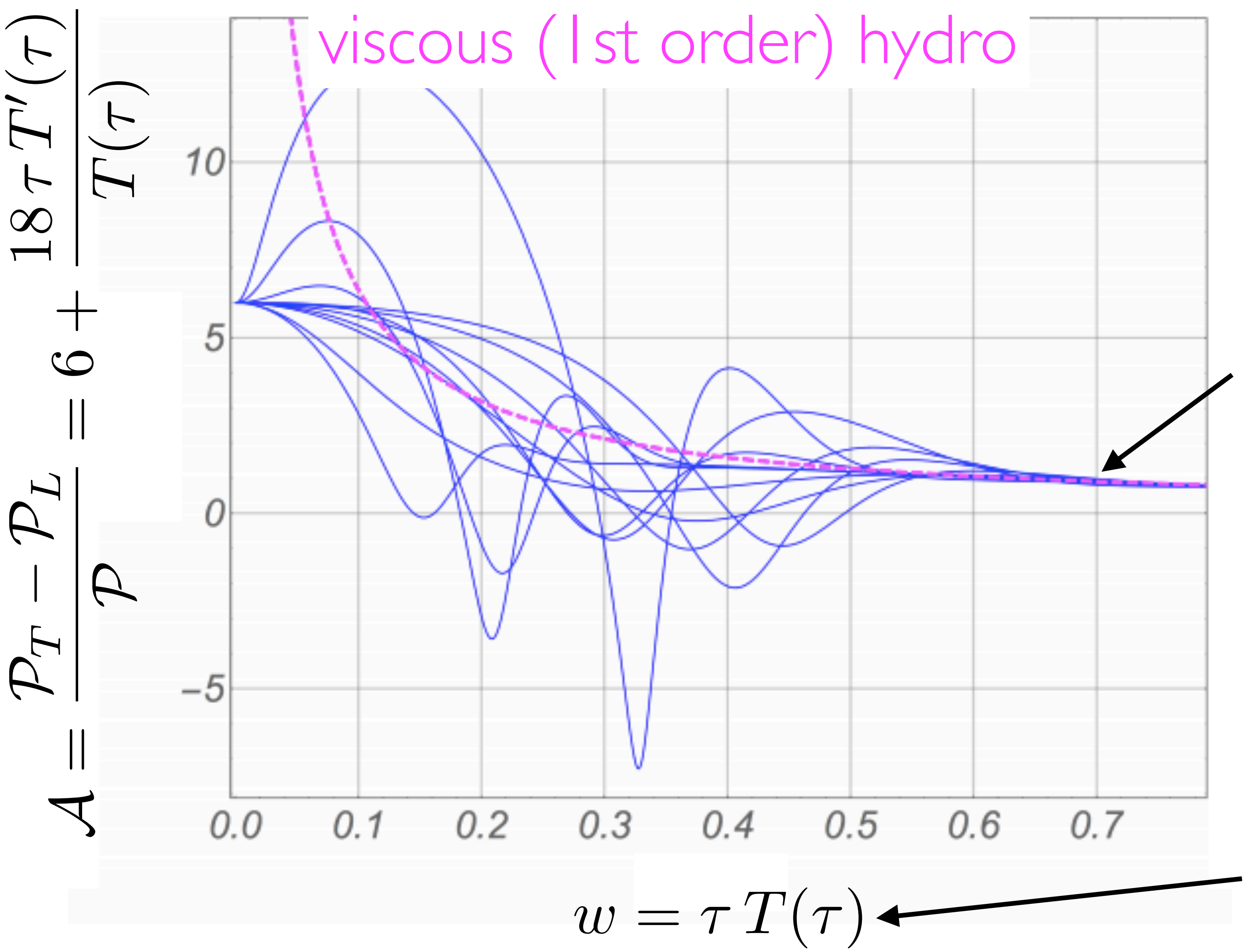
Still difficult: employ the boost-invariance ("rotation" symmetry between time and expansion axis) $\longrightarrow$ physics depends only on proper time $\tau = \sqrt{x_0^2 - x_1^2}$



Ab initio solution in strongly-couled QFTs

1103.3452 with Janik & Witaszczyk (29 states);
1411.1969 by Jankowski, Plewa & Spaliński (> 600 states);
parametrization and plot from 1707.02282 with Florkowski & Spaliński

The object of central interest is $\langle T^{\mu\nu} \rangle (x^{\alpha})$ since it provides initial conditions for hydro in the pheno paradigm. Sample AdS/CFT n-eq states:



key surprise: simple hydro applies when the system is extremely $\mathcal{O}(100\%)$ anisotropic (i.e. far from equilibrium)

the effective temperature

Motivations for hydrodynamic attractors

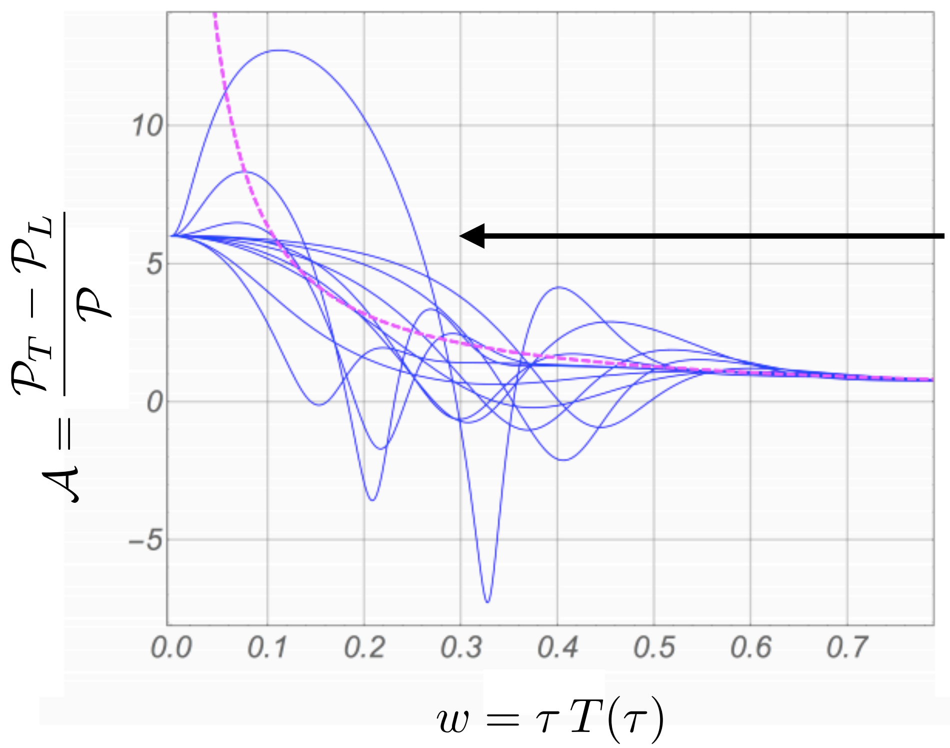
1302.0697 with Janik and Witaszczyk

Textbook hydro: $\langle T^{\mu\nu} \rangle = \mathcal{E}\, u^\mu u^\nu + \mathcal{P}(\mathcal{E})\,(g^{\mu\nu} + u^\mu\, u^\nu) - \eta(\mathcal{E})\, \sigma^{\mu\nu} + \mathcal{O}(\nabla^2 u)$ $= \pi^{\mu\nu}$

We showed $\mathcal{A}_{\text{textbook hydro}} = a_1\, w^{-1} + a_2\, w^{-2} + \ldots$ with $a_n \sim n!$

see also Gerald Dunne's talk

Furthermore:



$\sim\; e^{-\gamma_j / w^{-1}} \cos\left(\omega_j / w^{-1} + \text{phase}_j\right)$

Key question: how to properly describe far-from equilibrium hydrodynamics?

Replacing AdS/CFT by a hydrodynamic theory

1503.07514 with Spaliński

At least naively, the core of the problem lies in having a divergent hydro series and exponentially suppressed contribution that controls equilibration

AdS/CFT is complicated since there are ∞-many exponential contributions

Idea: a similar structure arises in theories of relativistic dissipative hydro:

$$(\tau_\pi u^\alpha \mathcal{D}_\alpha + 1)\left[\pi^{\mu\nu} - (-\eta\,\sigma^{\mu\nu})\right] = 0 \longrightarrow \pi^{\mu\nu} = -\eta\,\sigma^{\mu\nu} - \tau_\pi\, u^\alpha \mathcal{D}_\alpha\, \pi^{\mu\nu} - \tau_\pi\, u^\alpha \mathcal{D}_\alpha\, (\eta\,\sigma^{\mu\nu})$$

decay timescale

Müller 1967, Israel 1976, Israel & Stewart 1976
0712.2451 by Baier, Romatschke, Son, Starinets & Stephanov

EOM for boost-invariant flow: 2nd order ODE for $T(\tau)$ or, equivalently, the following master eqn for $\mathcal{A}(w)$: $C_{\tau_\pi}\, w\,(1 + \frac{1}{12}\mathcal{A})\,\mathcal{A}' + \left(\frac{1}{3}C_{\tau_\pi} + \frac{1}{8}\frac{C_{\lambda_1}}{C_\eta} w\right)\mathcal{A}^2 + \frac{3}{2}\,w\,\mathcal{A} - 12\,C_\eta = 0$

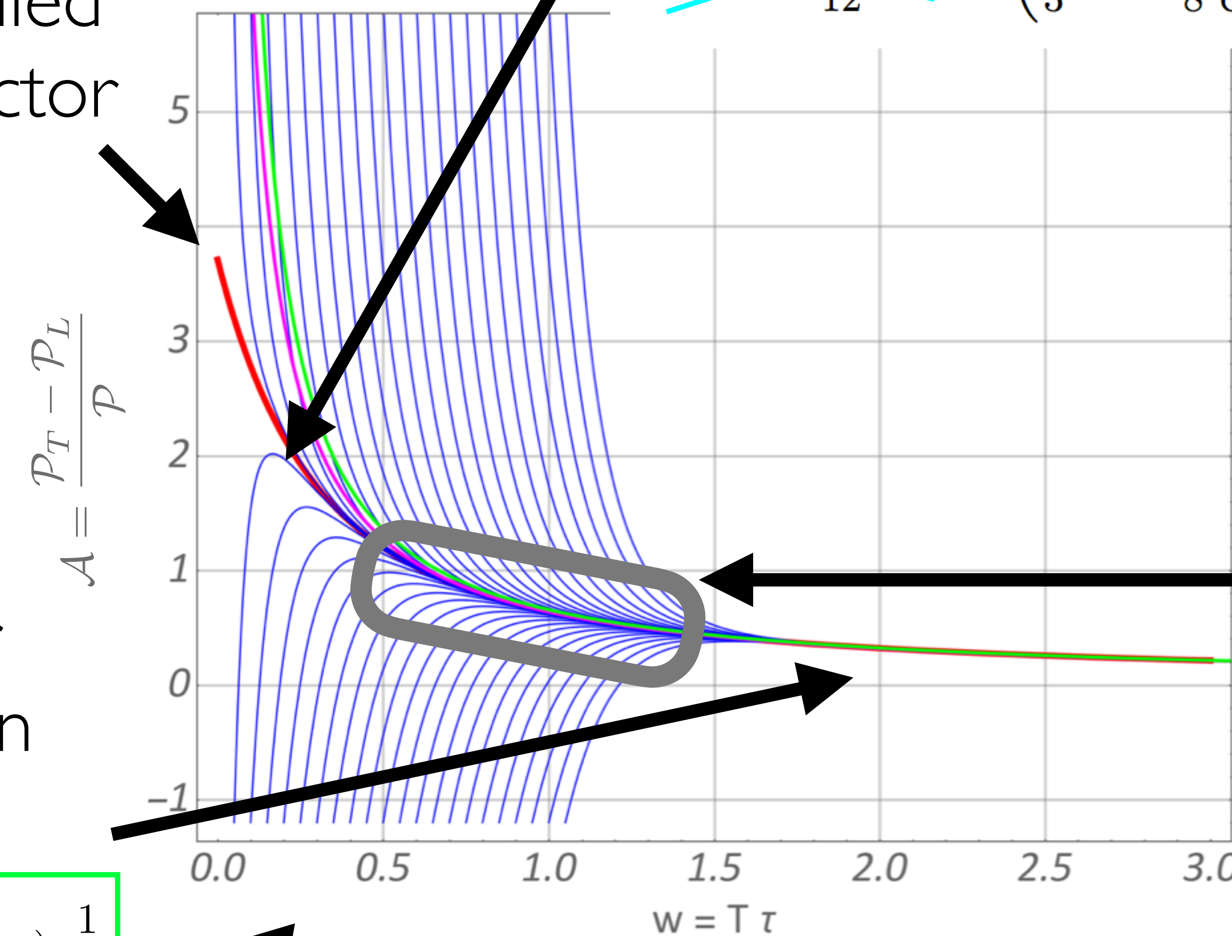
The (hydro) attractor

1503.07514 with Spaliński

At w = 0 there is only 1 sensible solution with finite $\mathcal{A}$ and we called it the (hydro) attractor

"slow roll" approximates it well:

$$C_{\tau_\pi}\, w\, (1 + \frac{1}{12}\mathcal{A})\, \mathcal{A}' + \left(\frac{1}{3}C_{\tau_\pi} + \frac{1}{8}\frac{C_{\lambda_1}}{C_\eta} w\right)\mathcal{A}^2 + \frac{3}{2} w\,\mathcal{A} - 12\, C_\eta = 0$$



built-in exponential decay

1st or 2nd order gradient expansion works well:

$$\mathcal{A}_H(w) = 8\, C_\eta \frac{1}{w} + \frac{16}{3} C_\eta \left(C_{\tau_\pi} - C_{\lambda_1}\right)\frac{1}{w^2} + \ldots$$

divergent series, see also 1509.05046 by Basar & Dunne

attractor = a particular resummation

Since 2015: many studies of finite / slow-roll solutions (= hydro attractors)

A new look at hydrodynamic attractors

1910.xxxxx with Ro Jefferson, Michal Spaliński and Viktor Svensson

The idea of attractors arose as a way of providing a preferred resummation of divergent hydrodynamic series and, eventually, of doing hydro better

It is not clear at this level if solutions having a finite $A(w=0)$/ slow-roll solutions are phenomenologically interesting. In particular, for a generic solution, hydro attractor of 1503.07514 becomes relevant only from some time onwards

New idea: make use of expansion and dissipation to argue for a reduction of effective degrees of freedom in heavy-ion collision setups (~ local "attractors")

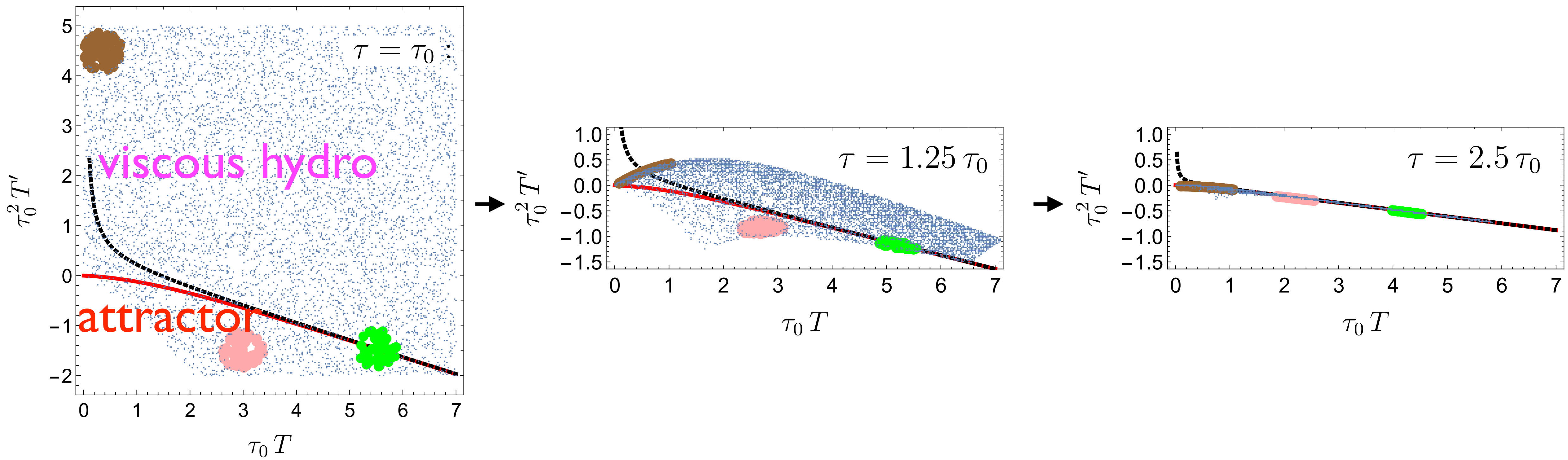
Dimensionality reduction in phase space

1910.xxxxx with Ro Jefferson, Michal Spaliński and Viktor Svensson

Let's revisit the theory of hydrodynamics: EOMs are 2nd order ODEs for $T(\tau)$

The phase space* therefore is 2D: for each τ a state is parametrized by (T, T')

Time evolution turns out to dimensionally reduce regions of phase space:



We expect this phenomenon to be generic, i.e. apply also e.g. in AdS/CFT

Comments on the new idea

1910.xxxxx with Ro Jefferson, Michal Spaliński and Viktor Svensson

Our present expectation is that after some short time some directions in phase space become locally irrelevant

As a result, all initial states localized in a finite phase space volume end up on a lower D manifold

A pheno potential of this idea might lie in the fact that in heavy-ion collisions we might then not care about evolving all initial states till late times, but rather just representatives from different points of this lower D manifold

From what we explored so far, a good way of detecting and parametrizing dimensional reduction is the principal component analysis. This can help in the really interesting cases (AdS/CFT, kinetic theory) when phase space is ∞ D

Summary

AdS/CFT allowed to understand the transition to hydrodynamics in a class of strongly-coupled quantum field theories in a completely ab initio manner

Connections to other disciplines: numerical relativity, heavy-ion collisions pheno, theory of resurgence, theories of relativistic hydro, dynamical systems

hydro attractors $=$ a surprising spin-off of AdS/CFT studies having to do with a discovery that hydro works much better than expected

A new look on attractors: instead of thinking about globally special solutions, focus on dimensionality reduction in the phase space picture

Its outlook: any macroscopic solution is a resummation of hydrodynamics, but various groups of solutions tend to become indistinguishable very early on

THANK YOU
AND PLEASE STAY TUNED,
IN PARTICULAR:

see **Viktor Svensson's talk on Wed 12:00-12:30** on ab initio modelling of equilibration processes in (1+1)-dim. QFTs using tensor networks

have a look at **1910.xxxxx with Ro Jefferson, Michal Spaliński and Viktor Svensson** when it appears on the arXiv next month