Prescaling and far-from-equilibrium hydrodynamics in the quark-gluon plasma

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AM and Jürgen Berges, *Phys. Rev. Lett.* 122 (2019) (arXiv:1810.10554)

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Isolated quantum systems and universality in extreme conditions

The Universal Law That Aims Time's Arrow

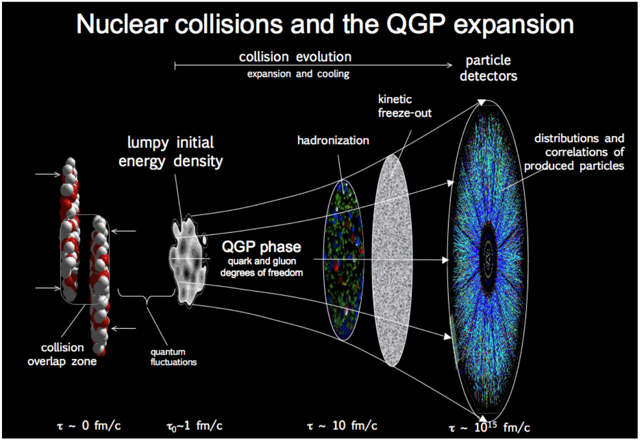
Article by Natalie Wolchover, *Quanta Magazine* (2019)

Thermalization—the end stage of non-equilibrium phenomena.

How do far-from-equilibrium quantum systems begin to thermalize?

Heavy ion collision experiments at RHIC and LHC

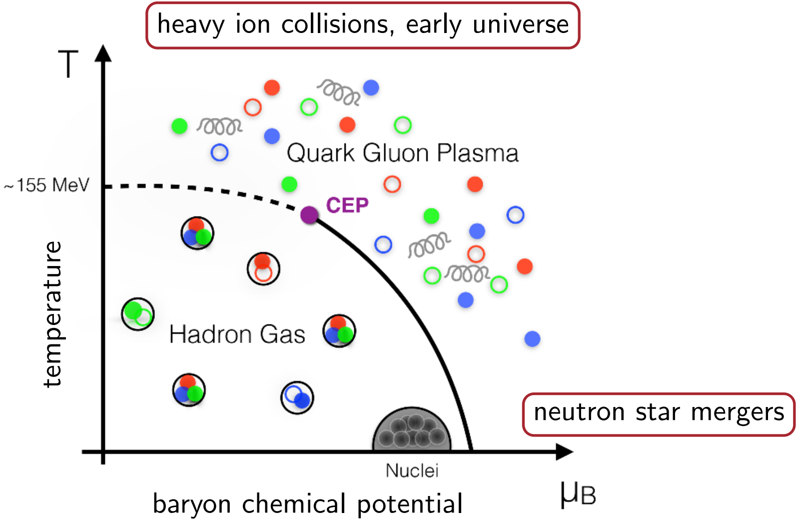
study new phenomena *"by distributing high energy or high nucleon density over a relatively large volume."* – T.D. Lee, 1974



Sorensen, Quark-gluon plasma 4, 2010

Creation of dense QCD matter at extreme temperatures $T \sim 10^{12}\,\mathrm{K}$.

Phase diagram of thermalized QCD matter



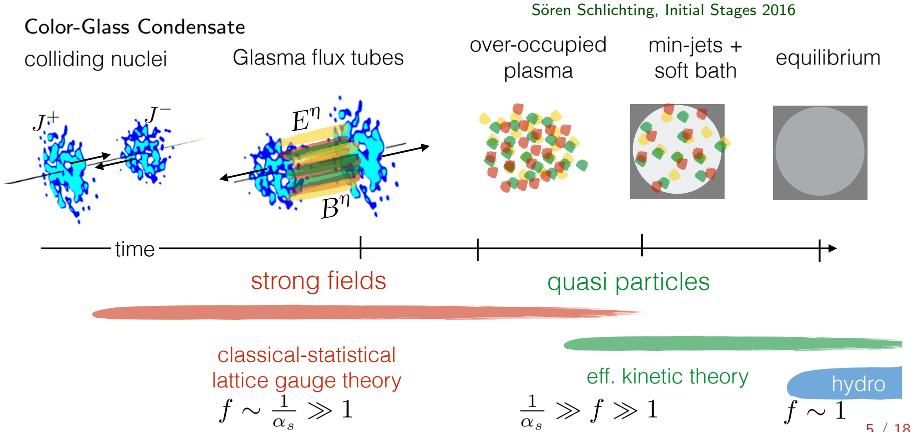
Ding, Karsch, Mukherjee (2015)

Quark-Gluon Plasma—the state with fundamental QCD constituents.

Non-equilibrium QCD descriptions at weak coupling $\alpha_s \to 0$

At high energies mid-rapidity is dominated by small Bjorken-x gluons

- $p \sim Q_s$ saturation scale $\gg \Lambda_{QCD}$, strong gluon fields $A_\mu \sim \frac{1}{\alpha_s} \gg 1$
  $\Longrightarrow$ classical-statistical simulations
- decoherence of classical fields at $\tau Q_s \gg 1$
  $\Longrightarrow$ kinetic evolution of gluon phase space distribution f



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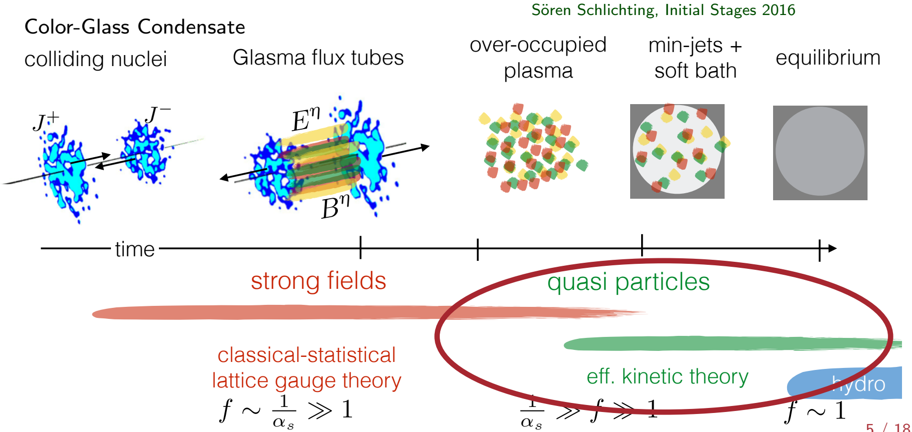


High temperature gauge kinetic theory

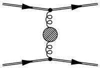
Boltzmann equation for distribution f of quark and gluon quasi-particles.

Arnold, Moore, Yaffe (2003)[1]

$$\partial_\tau f_{g,q} - \frac{p_z}{\tau}\partial_{p_z} f_{g,q} = -\mathcal{C}_{2\leftrightarrow 2}[f] - \mathcal{C}_{1\leftrightarrow 2}[f]$$

Leading order processes in the coupling constant $\lambda = 4\pi\alpha_s N_c$:

- $2 \leftrightarrow 2$ elastic scatterings: $gg \leftrightarrow gg$, $qq \leftrightarrow qq$, $qg \leftrightarrow gq$, $gg \leftrightarrow q\bar{q}$



$$= |\mathcal{M}^{gg}_{qq}|^2 = \lambda^2\, 16 \frac{d_F C_F}{C_A^2}\left[C_F\left(\frac{u}{t}+\frac{t}{u}\right) - C_A\left(\frac{t^2+u^2}{s^2}\right)\right]$$

Hard Thermal Loop resummed propagators, screening mass $m_D \sim gT$

Braaten, Pisarski (1990) [2]

Microscopic studies of QGP equilibration with QCD kinetic theory.

Kurkela and Zhu (2015) [3], Keegan, Kurkela, AM and Teaney (2016) [4], Kurkela, AM, Paquet, Schlichting and Teaney (2018)[5, 6], A. Kurkela, AM (2018) [7, 8], AM, J. Berges (2018)[9]

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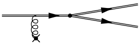
Leading order processes in the coupling constant $\lambda = 4\pi\alpha_s N_c$:

- $1 \leftrightarrow 2$ medium induced colinear radiation: $g \leftrightarrow gg$, $q \leftrightarrow qg$, $g \leftrightarrow q\bar{q}$



$$= |\mathcal{M}^g_{qq}|^2 = \frac{k'^2 + p'^2}{k'^2 p'^2 p^3} \underbrace{\mathcal{F}_q(k'; -p', p)}_{\text{splitting rate}}$$

  Resummed multiple scatterings with the medium (Landau–Pomeranchuk–Migdal suppression).

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"Bottom-up" thermalization scenario

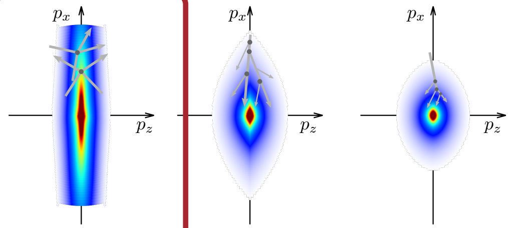
Baier, Mueller, Schiff, and Son (2001)[11]

Evolution of initially over-occupied hard gluons $p \sim Q_s \gg \Lambda_{\mathrm{QCD}}$

I) over-occupied $\quad p_z \sim \frac{Q_s}{(Q_s\tau)^{1/3}} \quad 1 \ll Q_s\tau \ll \alpha_s^{-3/2}$

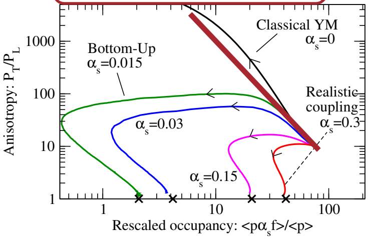
II) under-occupied $\quad p_z \sim \sqrt{\alpha_s} Q_s \quad \alpha_s^{-3/2} \ll Q_s\tau \ll \alpha_s^{-5/2}$

III) mini-jet quenching $\quad p_z \sim \alpha_s^3 Q_s (Q_s\tau) \quad \alpha_s^{-5/2} \ll Q_s\tau \ll \alpha_s^{-13/5}$



$2 \leftrightarrow 2$ broadening — collinear cascade — mini-jet quench

nonthermal attractor



Berges, Boguslavski, Schlichting, Venugopalan (2014) [10]

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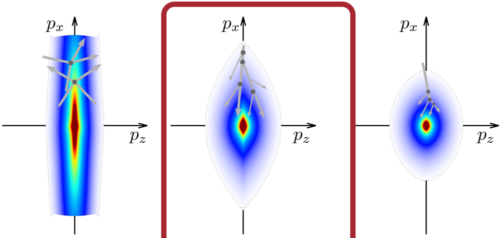
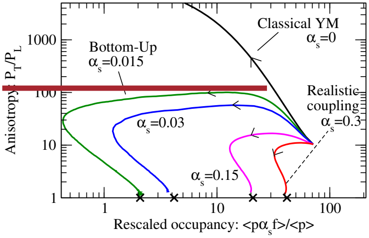


$2 \leftrightarrow 2$ broadening | collinear cascade | mini-jet quench



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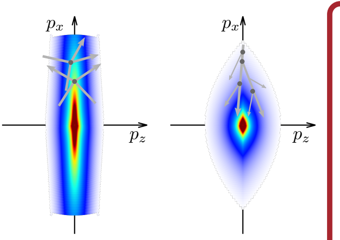
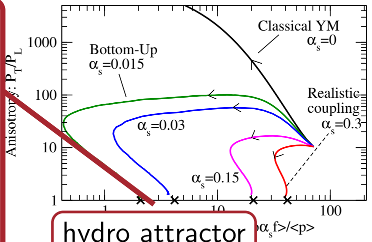


$2 \leftrightarrow 2$ broadening   collinear cascade   mini-jet quench



hydro attractor

Berges, Boguslavski, Schlichting, Venugopalan (2014) [10]

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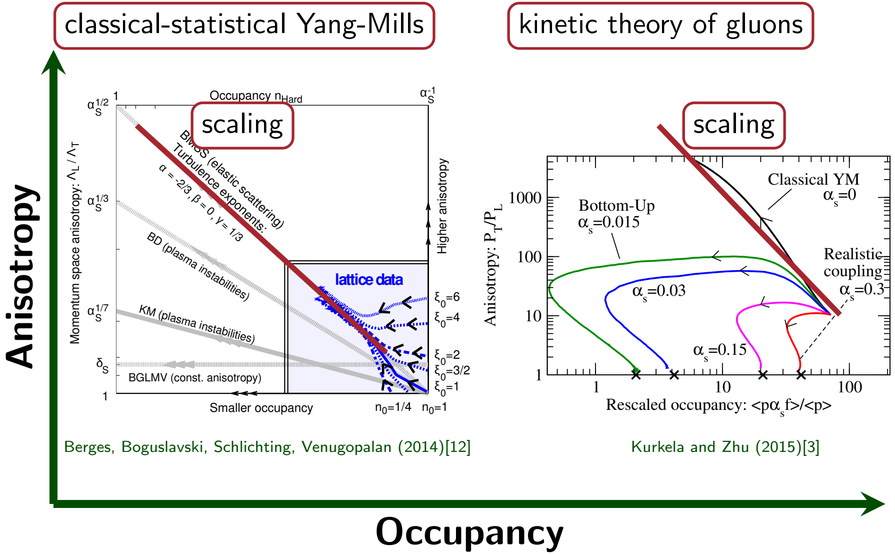


Berges, Boguslavski, Schlichting, Venugopalan (2014)[12]

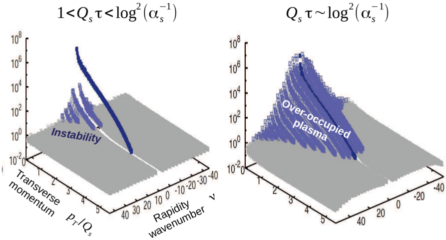
Kurkela and Zhu (2015)[3]

Non-thermal fixed point (NTFP) for gauge theories

For $f \sim A^2 \gg 1$ classical-statistical Yang-Mills describes gluon evolution

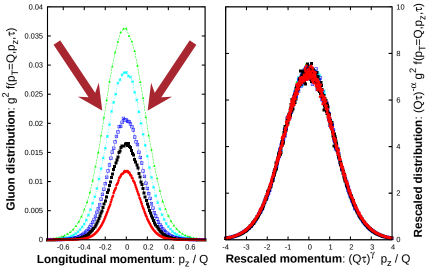
Aarts, Berges (2002), Mueller, Son (2004), Jeon (2005)

initial plasma instabilities



later evolution

scaling   rescaled



Berges, Schenke, Schlichting, Venugopalan (2014) [13] Berges, Boguslavski, Schlichting, Venugopalan (2014) [10]

Self-similar scaling $\Longrightarrow$ simplification of non-equilibrium physics

$$f_g(p_\perp, p_z, \tau) = \tau^\alpha f_S(\tau^\beta p_\perp, \tau^\gamma p_z), \quad \tau = \sqrt{t^2 - z^2}$$

Universal exponents: $\alpha \approx -\frac{2}{3}, \quad \beta \approx 0, \quad \gamma \approx \frac{1}{3}$

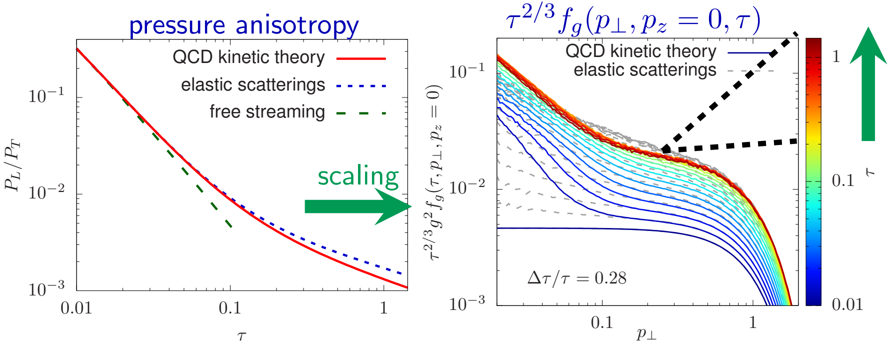
scaling in other systems: Orioli et al. (2015) [14], Mikheev et al. (2018) [15], Prüfer et al. (2018) [16], Erne et al. (2018) [17]

Scaling in leading order QCD kinetic theory

Initial conditions $f_g = \frac{\sigma_0}{g^2} e^{-(p_\perp^2 + \xi^2 p_z^2)}$, $\sigma_0 = 0.1$, $g = 10^{-3}$, $\xi = 2$

Scaling regime is reached at late times

$$f_g(p_\perp, p_z, \tau) = \tau^{-2/3} f_S(p_\perp, \tau^{1/3} p_z), \quad \tau \to \tau/\tau_{\text{ref}}$$



Approach to a non-thermal fixed point in full QCD kinetic evolution.

Pre-scaling regime in QCD kinetic theory

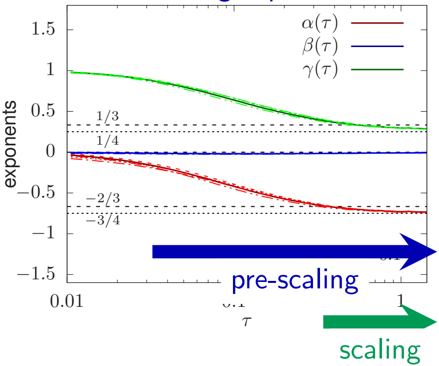
Non-equilibrium dynamics undone by self-similar renormalization

$$f_g(p_\perp, p_\perp, \tau) = \tau^{\alpha(\tau)} f_S(\tau^{\beta(\tau)} p_\perp, \tau^{\gamma(\tau)} p_z)$$

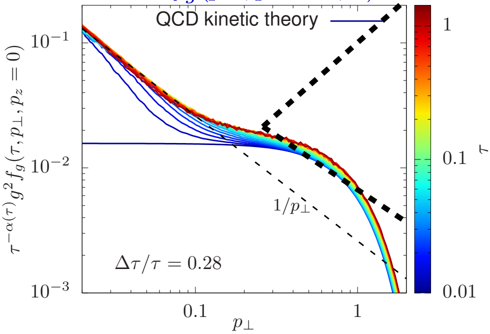
AM and Berges (2018) [9]

Scaling exponents $\alpha(\tau)$, $\beta(\tau)$, $\gamma(\tau)$ can be time dependent!

scaling exponents

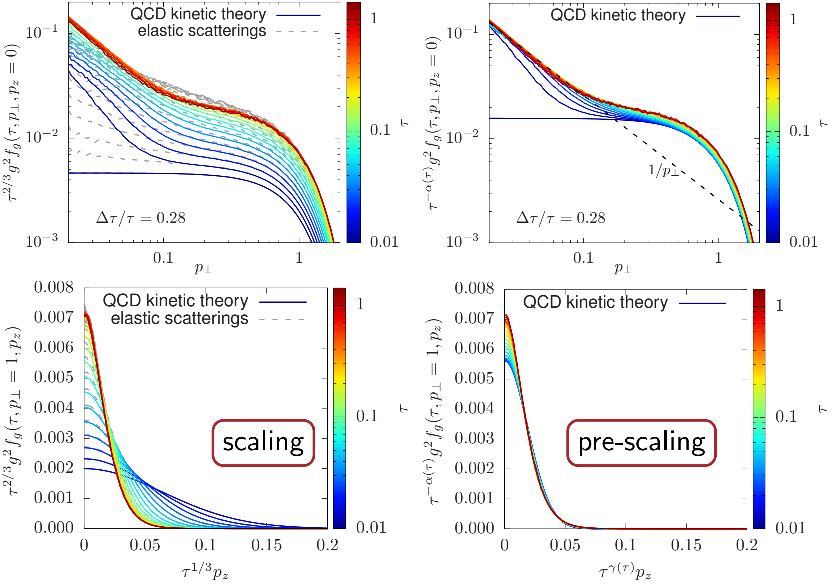


$\tau^{-\alpha(\tau)} f_g(p_\perp, p_z = 0, \tau)$

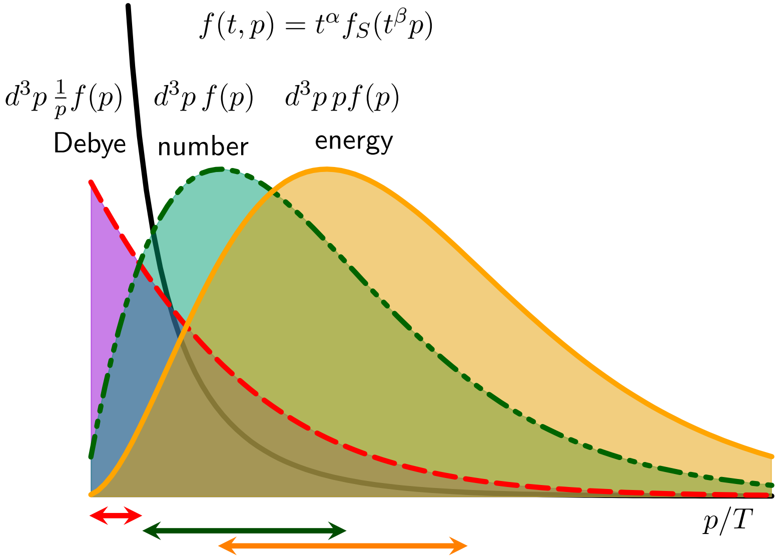


Much earlier collapse to scaling solution f_S — pre-scaling regime.

Comparison between constant and time dependent exponents



Weighted momentum distribution



Moments of distribution function probe different momentum scales.

Extracting scaling exponents from integral moments

Pre-scaling evolution

$$f_g(p_\perp, p_z, \tau) = \tau^{\alpha(\tau)} f_S(\tau^{\beta(\tau)} p_\perp, \tau^{\gamma(\tau)} p_z).$$

imposes relations between integral moments

$$n_{m,n}(\tau) \equiv \int_{\mathbf{p}} p_\perp^m |p_z|^n f_g(p_\perp, p_z, \tau) \sim \tau^{\alpha(\tau)-(m+2)\beta(\tau)-(n+1)\gamma(\tau)}$$

Momentum range of scaling solution ⇔ range of moments obeying scaling.

Integrals of Boltzmann equation ⇒ equations of motion for moments

$$\partial_\tau f - \frac{p_z}{\tau} \partial_{p_z} f = -C[f]$$

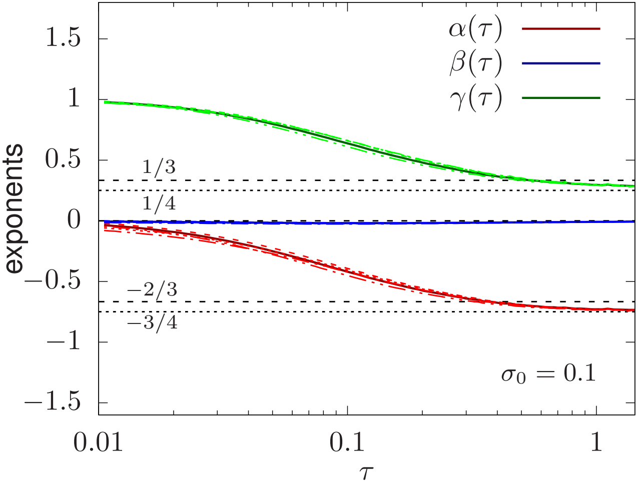
Relaxation to hydrodynamic solution / relaxation to scaling solution

$$\tau_\pi \dot{\pi}^{\mu\nu} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} \qquad / \qquad \tau \log\tau\, \dot{\alpha} + \alpha = \alpha_\infty \tau^{\mu(\tau)-\alpha(\tau)+1}.$$

Time evolution encoded into few hydrodynamic degrees of freedom

Time dependent exponents

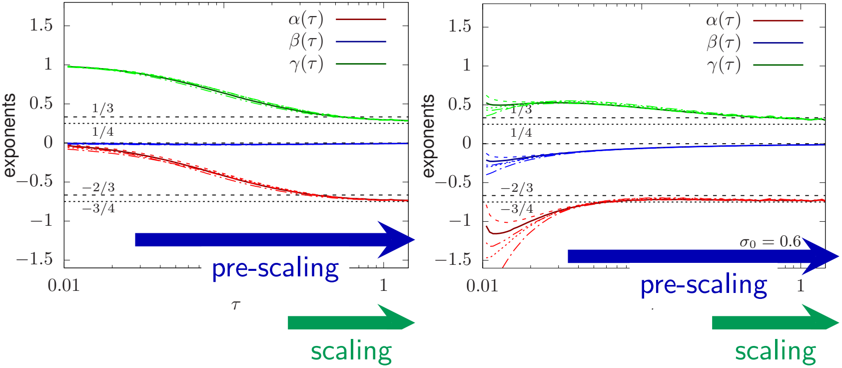
$$f_g(p_\perp, p_\perp, \tau) = \tau^{\alpha(\tau)} f_S(\tau^{\beta(\tau)} p_\perp, \tau^{\gamma(\tau)} p_z)$$



Closely related evolution of moments $n_{m,n}$ *with* $0 \leq n, m \leq 3$

Dependence on initial conditions

Vary initial gluon occupation $\sigma_0 = 0.1, 0.6$: $f_g = \frac{\sigma_0}{g^2} e^{-(p_\perp^2 + \xi^2 p_z^2)}$

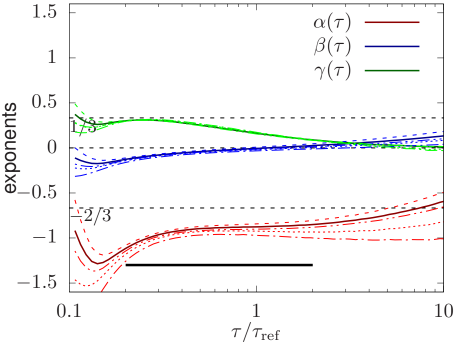


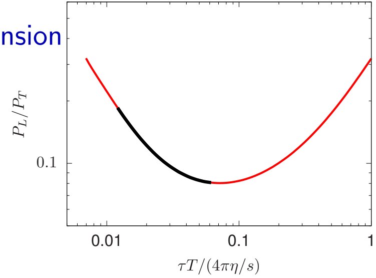
New hydrodynamic degrees of freedom $\alpha(\tau)$, $\beta(\tau)$, $\gamma(\tau)$ far away from equilibrium.

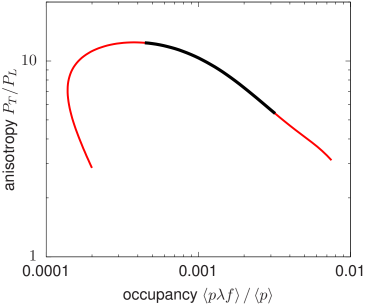
The onset of thermalization

Later initialization times $\Longrightarrow$ slower expansion

larger coupling $\Longrightarrow$ stronger interactions







Pre-scaling useful in studying the intermediate regimes of thermalization

Summary and Outlook

$$f_g(p_\perp, p_z, \tau) = \tau^{\alpha(\tau)} f_S(\tau^{\beta(\tau)} p_\perp, \tau^{\gamma(\tau)} p_z)$$

- Scaling and pre-scaling present in full QCD kinetic theory evolution.

  AM and Berges (2019)

- Pre-scaling in classical-statistical Yang-Mills/non-relativistic scalars?

  see work of Schmied, Mikheev, Gasenzer (2018)[18], poster of Chatrchyan, Geier, Berges

- $\alpha(\tau)$, $\beta(\tau)$, $\gamma(\tau)$—new hydrodynamic-like degrees of freedom for evolution not around equilibrium.
- Extending pre-scaling description towards the onset of thermalization.

  Berges, AM, Mikheev, *work in progress*

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