Unraveling universal phenomena with unequal-time correlations

TECHNISCHE UNIVERSITÄT WIEN

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In collaboration with: **Asier Piñeiro Orioli** (in prep.)

Talk also based on:

Piñeiro Orioli, KB, Berges, *PRD 92, 025041 (2015)*

KB, Kurkela, Lappi, Peuron, *PRD 98, 014006 (2018)*

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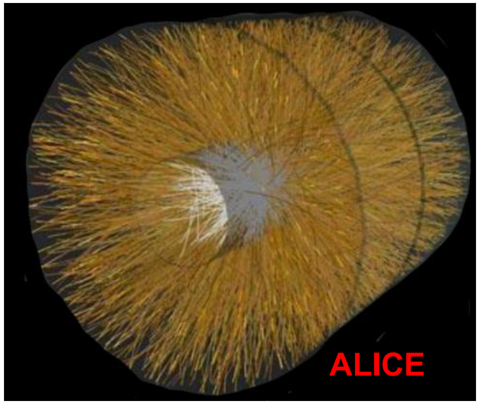
Motivation

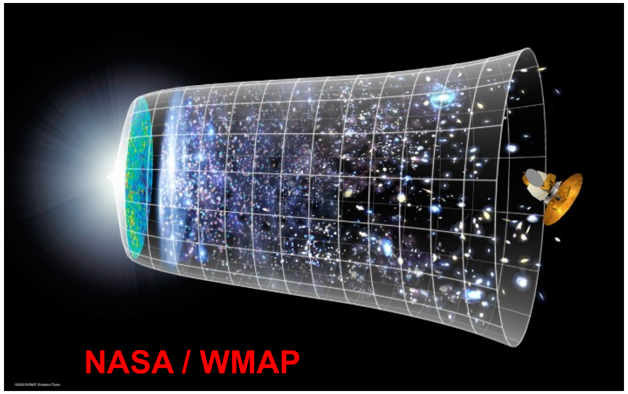
Far from equilibrium dynamics in

Heavy-ion collisions



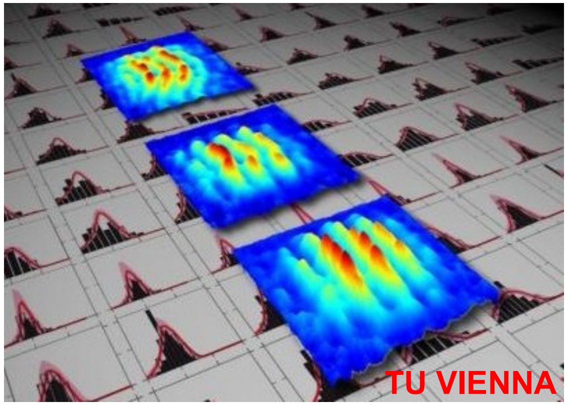
Longitudinally expanding non-Abelian plasmas

Early Universe



Relativistic scalar systems (O(N), $\lambda(\phi_a\phi_a)^2$)

Ultracold atoms

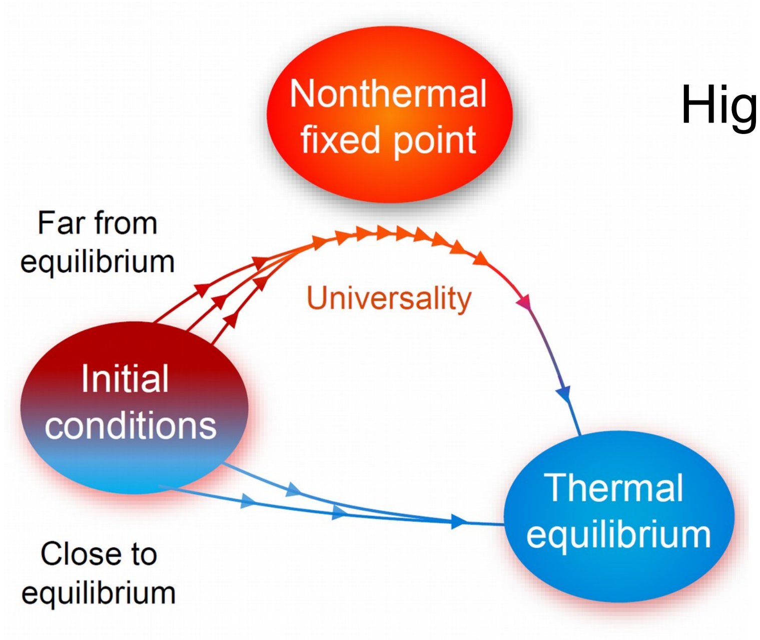


Non-relativistic scalar systems (GPE)

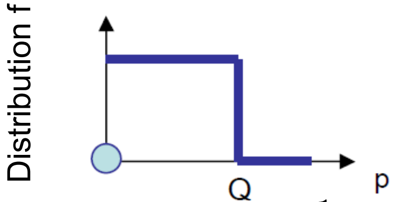
- When couplings $g, \lambda \ll 1$ weak, closed system highly occupied $f \gg 1$
- Methods: class.-stat. simulations (TWA) for $f \gg 1$, effective / kinetic descriptions, ...

Aarts, Berges (2002); Mueller, Son (2004); Jeon (2005), ...

Motivation: Nonthermal fixed points



Highly occupied:



$$f(t_0, p \lesssim Q) \sim \frac{1}{\lambda}$$

Nonthermal fixed point (NTFP)

- ✓ Partial memory loss
- ✓ Time scale independence
- ✓ Self-similar dynamics

Distribution function: $f(t,p) = t^{\alpha} f_s(t^{\beta} p)$

First experimental observations:
Prüfer et al., *Nature 563, 217 (2018)*
Erne et al., Nature 563, 225 (2018)

NTFP: *Micha, Tkachev (2004); Berges, Rothkopf, Schmid (2008)*

Universality far from equilibrium: *Berges, KB, Schlichting, Venugopalan (2015); Piñeiro Orioli, KB, Berges (2015)*

What phenomena underlie the universality?

More generally, what is the description?

Example:

2. Universality in scalar systems

(equal-time distribution functions)

- Relativistic O(N)-symmetric scalar theories

$$S = \int d^{d+1}x \left[\frac{1}{2} \partial^\mu \varphi_a \partial_\mu \varphi_a - \frac{m^2}{2} \varphi_a \varphi_a - \frac{\lambda}{4!N} \left(\varphi_a \varphi_a \right)^2 \right]$$

- Non-relativistic U(1) (Gross-Pitaevskii)

$$S = \int d^{d+1}x \left[\psi^* \left(i\partial_0 + \frac{\nabla^2}{2m} \right) \psi - \frac{g}{2} \left(\psi \psi^* \right)^2 \right]$$

Equal-time distribution: Universality in scalar systems

Self-similar evolution in IR

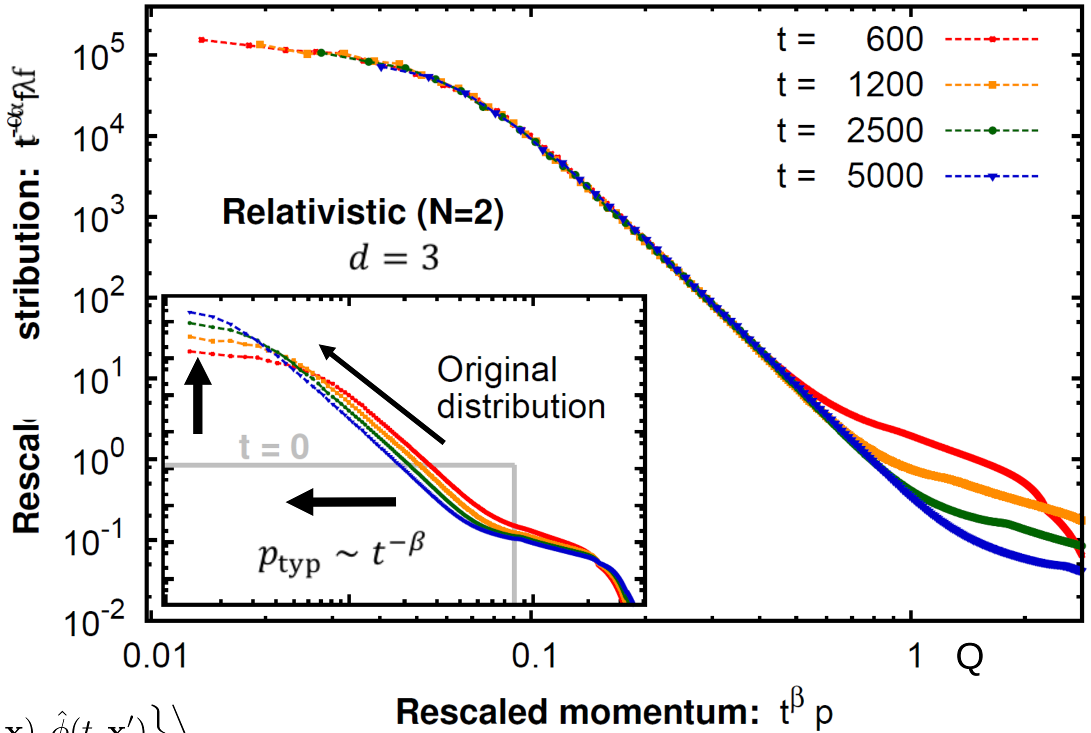
Piñeiro Orioli, KB, Berges,
PRD 92, 025041 (2015)

Self-similarity $f(t,p) = t^{\alpha} f_s(t^{\beta} p)$

Universal scaling exponents $\alpha \approx \frac{d}{2}\,,\ \beta \approx \frac{1}{2}$

$\int d^d p\, f(t,p) \approx const$ (Particle number conserved)

Bose-Einstein condensation



Equal-time correlation, $F(t, \mathbf{x} - \mathbf{x}') = \frac{1}{2} \left\langle \left\{ \hat{\phi}(t, \mathbf{x}), \hat{\phi}(t, \mathbf{x}') \right\} \right\rangle_C$

Distribution function $f(t,p) = \sqrt{F(t,p)\, \ddot{F}(t,p)} \approx \omega(t,p)\, F(t,p)$

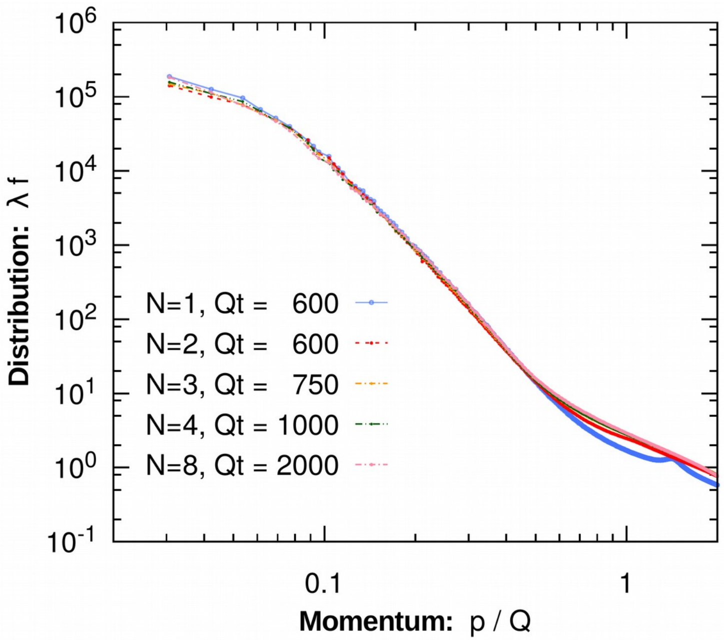
Berges, Rothkopf, Schmidt (2008); Piñeiro Orioli, KB, Berges (2015); Berges, KB, Schlichting, Venugopalan (2015); Moore (2016); Karl, Gasenzer (2016); Walz, KB, Berges (2017); Berges, KB, Chatrchyan, Jäckel (2017); Chantesana, Piñeiro Orioli, Gasenzer (2018); Schmied, Mikheev, Gasenzer (2018) ...

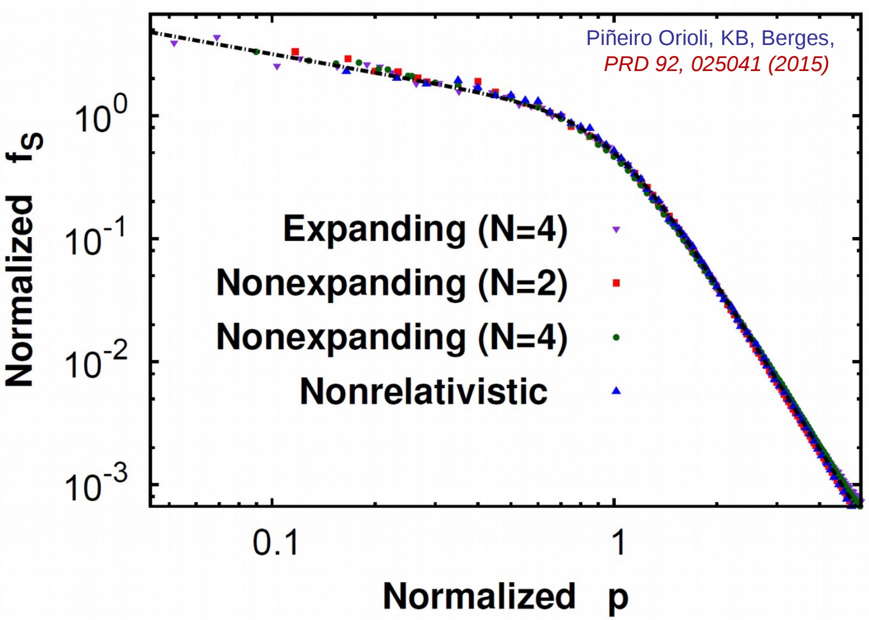
Self-similar evolution

$$f(t,p) = t^{\alpha} f_s(t^{\beta} p)$$

Universality in scalar systems

Universal attractor for O(N) and nonrelativistic





Piñeiro Orioli, KB, Berges (2015); Berges, KB, Schlichting, Venugopalan (2015); Moore (2016); Schmied, Mikheev, Gasenzer (2018); ...

- Same $\alpha, \beta, f_s(p)$
- Conjecture: *Universality class*

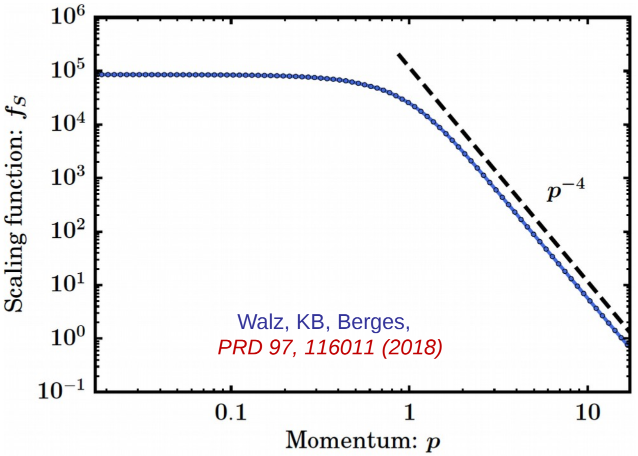
Universality in scalar systems

What are the relevant excitations?

- An explanation: *Large-N kinetic theory*
  - → Non-perturbative 1/N expansion at NLO
  - → Effectively vertex resummation $\lambda_{\text{eff}}(t,\omega,p)$
  - → Quasiparticles with $\omega(p) \sim m + p^2/(2m)$
  - → Leads to observed exponents

$$\alpha = \frac{d}{2}\,,\ \beta = \frac{1}{2}$$

  - → And to observed scaling function (s. Fig.)



Berges (2002); Aarts et al. (2002); Berges, Rothkopf, Schmidt (2008); Scheppach, Berges, Gasenzer (2009); Berges, Sexty (2011); Piñeiro Orioli, KB, Berges (2015); Walz, KB, Berges (2017); Chantesana, Piñeiro Orioli, Gasenzer (2018); Schmied, Mikheev, Gasenzer (2018); ...

- Why should large-N theory work for small N?
- Alternative explanation: *Vortex dynamics (or other defects)*
  - → Often used for U(1), O(1), O(2)

Different papers by Gasenzer et al.; see also Deng et al. (2018)

3. Unraveling universality classes

(with unequal-time correlations)

Unequal time correlation functions

Statistical function

$$F(t,t',\mathbf{x}-\mathbf{x}') = \frac{1}{2N}\left\langle\left\{\hat{\phi}_a(t,\mathbf{x}),\hat{\phi}_a(t',\mathbf{x}')\right\}\right\rangle_C$$

- Occupancy of excitations
- Computation in classical limit

$$F(t,t',\mathbf{p}) = \frac{1}{VN}\langle\phi_a(t,\mathbf{p}),\phi_a^*(t',\mathbf{p})\rangle_C$$

- Fourier transform:

$$\tau \equiv \frac{t+t'}{2} \approx t\,, \quad \Delta t \equiv t-t' \to \omega\,, \quad \mathbf{x}-\mathbf{x}' \to \mathbf{p}$$

- Connection to distribution (low p)

$$F(\tau,\Delta t=0,\mathbf{p}) = \int\frac{d\omega}{2\pi}\,F(\tau,\omega,\mathbf{p}) = \frac{f(t,p)}{\omega_{\text{eff}}(t,p)}$$

Spectral function

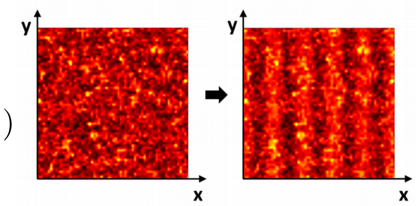
$$\rho(t,t',\mathbf{x}-\mathbf{x}') = \frac{1}{N}\left\langle\left[\hat{\phi}_a(t,\mathbf{x}),\hat{\phi}_a(t',\mathbf{x}')\right]\right\rangle$$

- Excitation spectrum of system
- Computation via linear response

$$\phi_a \to \phi_a + \delta\phi_a$$

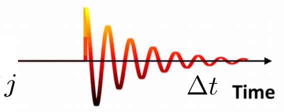
$$j_a(t,x) = j_a^0\,\delta(t-t_w)$$

$$(\text{with } \langle j_a^0(p)(j_b^0(q))^*\rangle_j = V\delta_{p,q}\delta_{ab})$$



- Response fct. $t \geq t_w$

$$\rho_{ab}(t,t_w,p) = \frac{1}{V}\langle\langle\delta\phi_a(t,p)\rangle(j_b^0(p))^*\rangle_j$$



Method:

KB, Kurkela, Lappi, Peuron, *PRD 98, 014006 (2018);*
Piñeiro Orioli, Berges, *PRL 122, 150401 (2019)*

Unequal time correlation functions

Relation between F and ρ

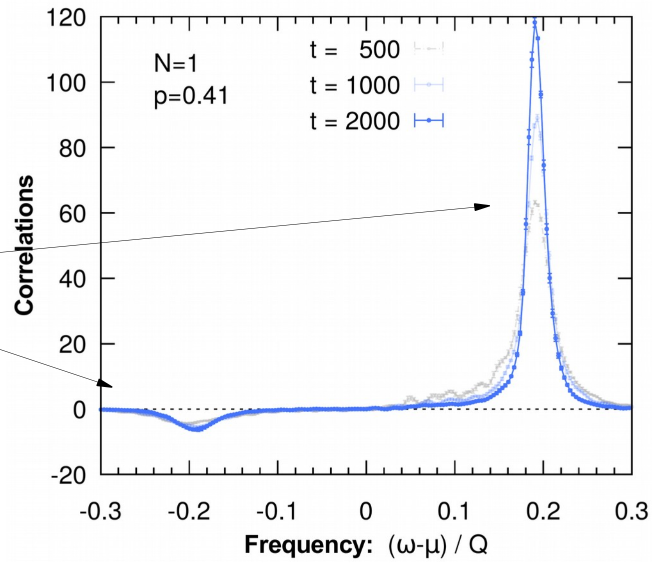
- Connected in thermal equilibrium via fluctuation-dissipation relation (FDR)

$$F_{(\mathrm{eq})}(\omega,p) = \left[f_{BE}(\omega) + 1/2\right]\rho_{(\mathrm{eq})}(\omega,p) \approx \frac{T}{\omega-\mu}\,\rho_{(\mathrm{eq})}(\omega,p)$$

- Independent out of equilibrium! – ***Example in O(1)***

Spectral ρ

Bogolyubov-Peak

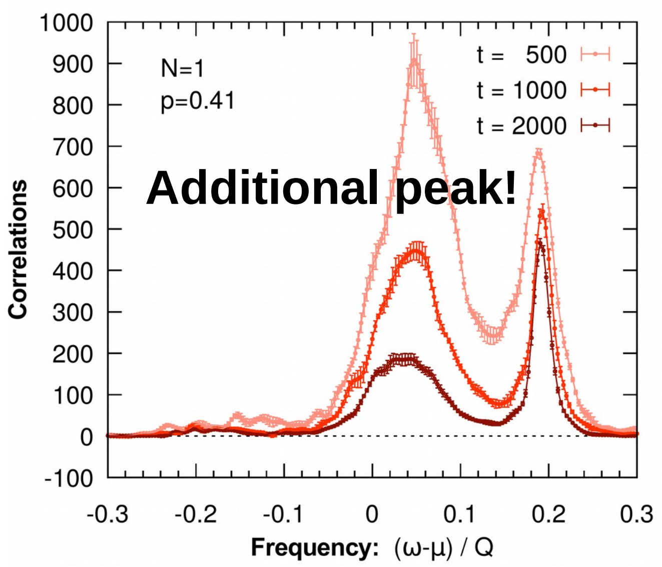


Unequal time correlation functions

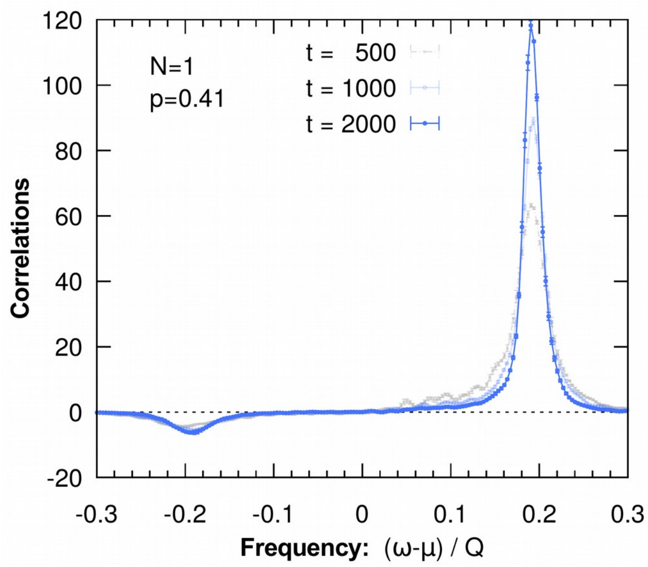
Relation between F and ρ

- Independent out of equilibrium! – ***Example in O(1)***

Statistical F



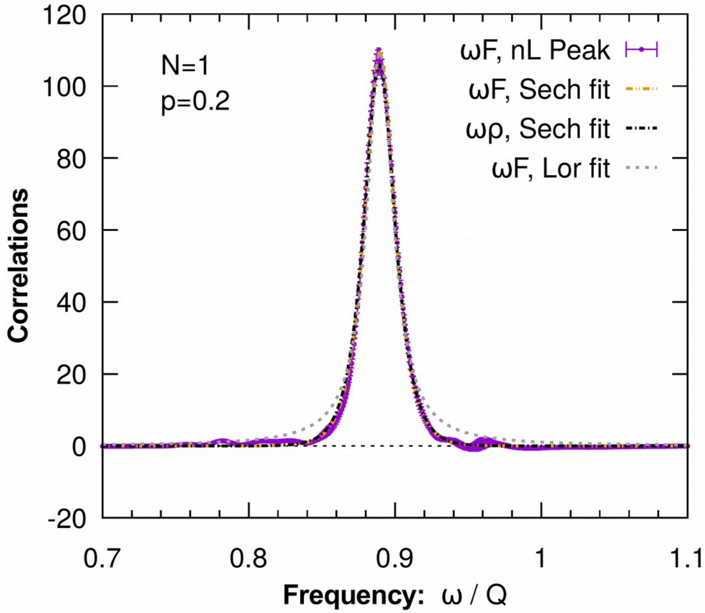
Spectral ρ



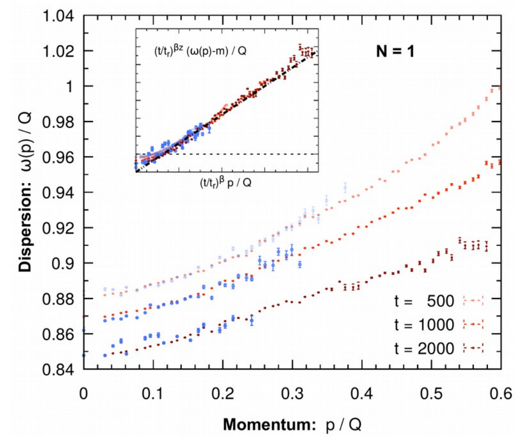
- Extra peak *dominates* at low momenta (IR)
- Reminder: $f(\tau, p) = \omega_{\text{eff}}(\tau, p) \int d\omega\, F(\tau, \omega, \mathbf{p})$

O(1): non-Lorentzian (nL) peak

Statistical F



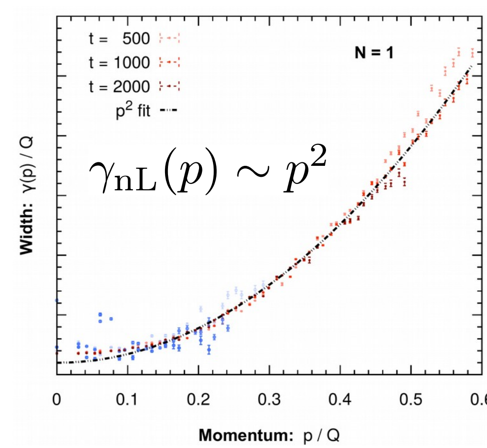
Dispersion



$$\omega_{\rm nL}(p) - \mu \sim p$$

$$(\mu(t) \equiv m_{\rm eff}(t))$$

Width



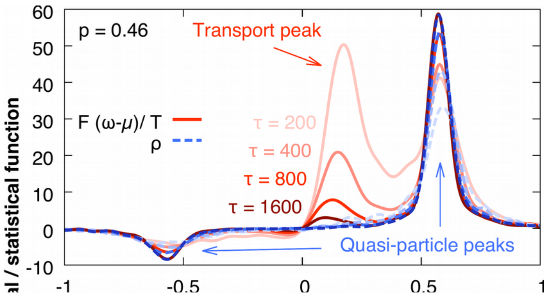
$$F_{\rm nL}(\tau,\omega,p) \simeq \frac{f(\tau,p)}{\omega_{\rm nL}(p)} \frac{\pi}{2\gamma_{\rm nL}(p)} \boxed{\operatorname{sech}\left[\frac{\pi}{2}\,\frac{\omega - \omega_{\rm nL}(\mathrm{p})}{\gamma_{\rm nL}(\mathrm{p})}\right]}$$

As opposed to $F_{\rm L}(\tau,\omega,p) \simeq \frac{f(\tau,p)}{\omega_{\rm L}(p)} \frac{1}{\gamma_{\rm L}(p)} \frac{1}{1+((\omega-\omega_{\rm L}(p))/\gamma_{\rm L}(p))^2}$

O(1): Universality with U(1)

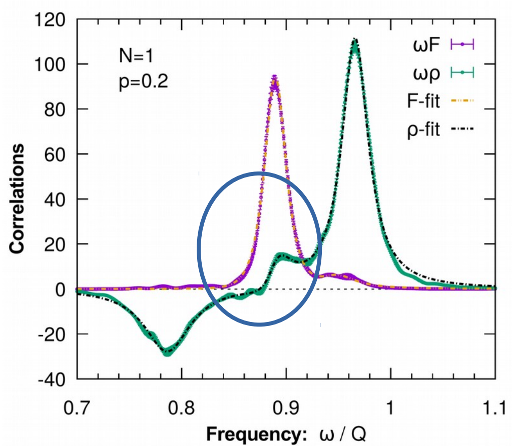
- The properties of nL peak are very similar to U(1)
- Genuine universality class: O(1) and U(1)
- Explanation: at low momenta, O(1) becomes effectively non-relativistic U(1)

U(1)



Piñeiro Orioli, Berges,
PRL 122, 150401 (2019)

O(1)

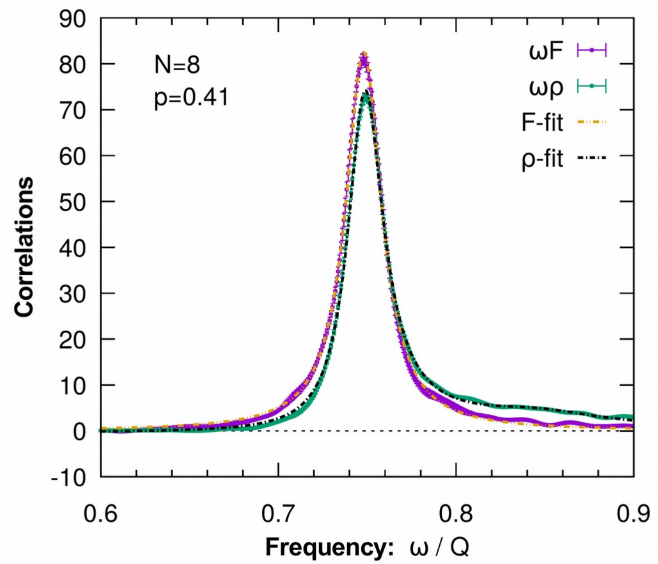


- At low p, nL Peak also visible in ρ
- Generalized FDR valid

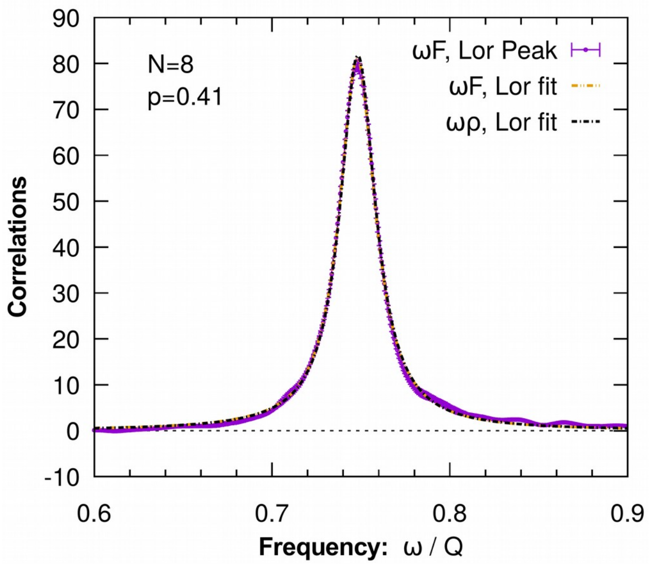
$$F_{\mathrm{nL}}(\tau, \omega, p) = \frac{T_{\mathrm{nL}}(\tau, p)}{\omega - \mu} \rho_{\mathrm{nL}}(\tau, \omega, p)$$

O(8): Large-N peak (Lorentzian)

F and ρ



Filtered
(other peaks subtracted)

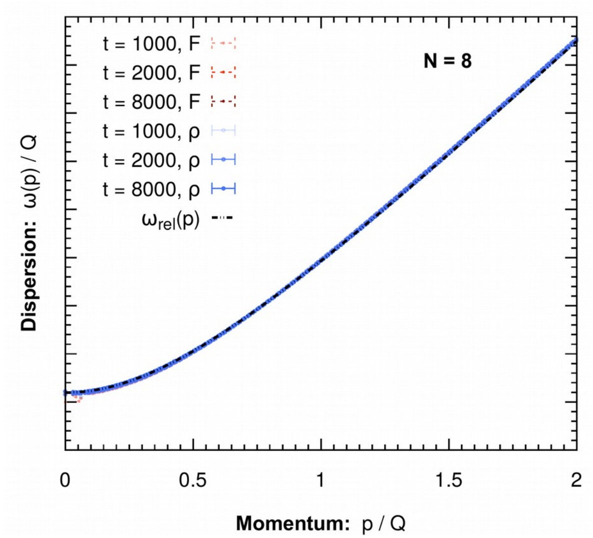


- The same (Lorentzian) peak dominates in both correlation functions (for all p)
- Generalized FDR:

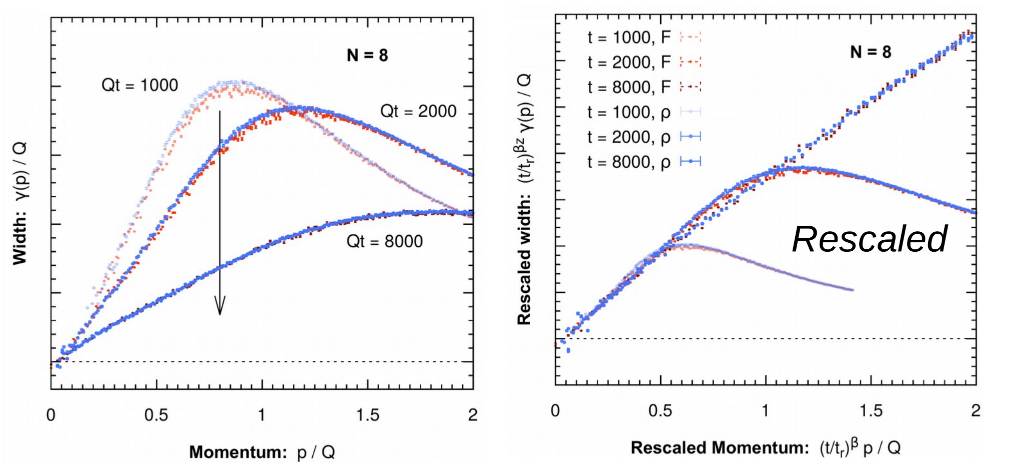
$$F_{\mathrm{L}}(\tau,\omega,p) = \frac{T_{\mathrm{L}}(\tau,p)}{\omega-\mu}\,\rho_{\mathrm{L}}(\tau,\omega,p) \approx \frac{f(\tau,p)}{\omega_{\mathrm{L}}(p)}\,\frac{\omega_{\mathrm{L}}(p)-\mu}{\omega-\mu}\,\rho_{\mathrm{L}}(\tau,\omega,p)$$

O(8): Dispersion and width

Dispersion relation



Peak width (damping rate)



$$\omega_{\rm L}(p) = \sqrt{m_{\rm eff}^2 + p^2} \simeq \mu + \frac{p^2}{2m_{\rm eff}}$$

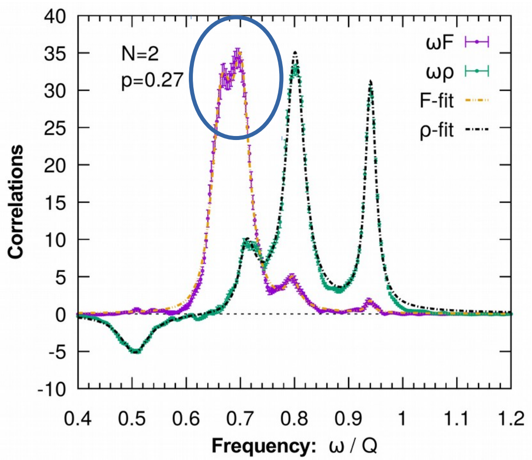
$$\gamma_{\rm L}(p) \sim p$$

$$F_{\rm L}(\tau,\omega,p) \simeq \frac{f(\tau,p)}{\omega_{\rm L}(p)} \frac{1}{\gamma_{\rm L}(p)} \frac{1}{1+((\omega-\omega_{\rm L}(p))/\gamma_{\rm L}(p))^2}$$

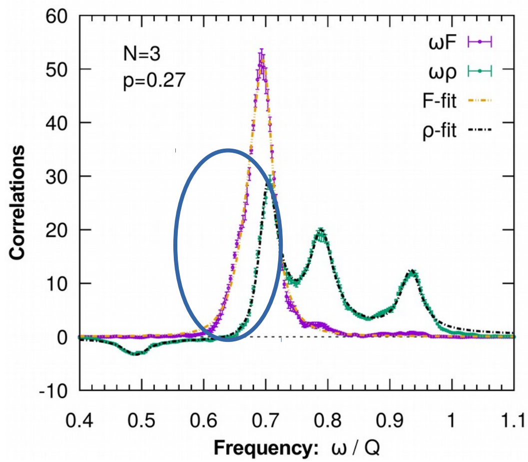
- Peak becomes more narrow with time, Delta peak in late-time limit
- Peak more dominant with larger N

Intermediate N

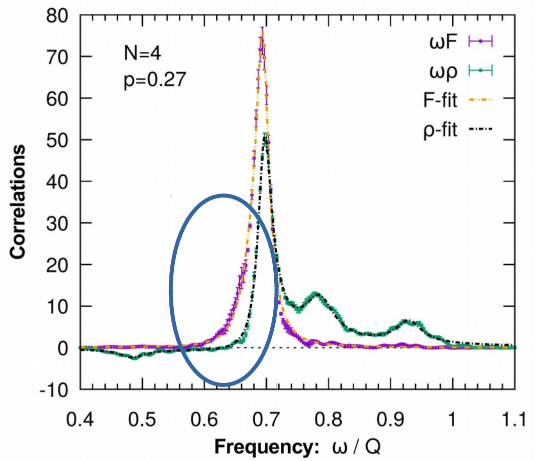
N = 2

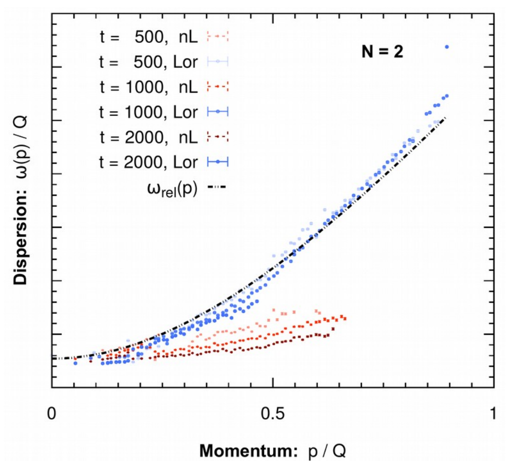


N = 3



N = 4



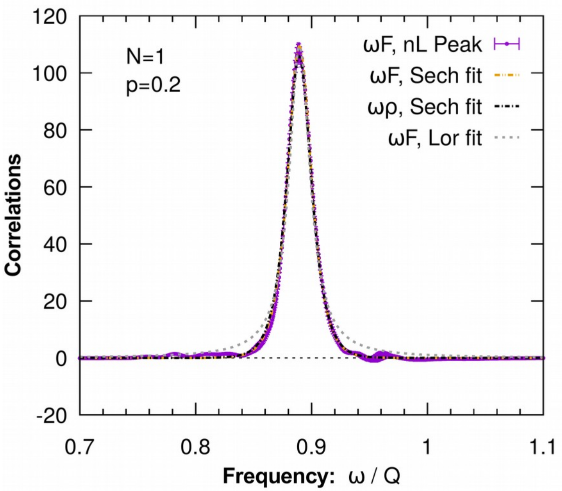


- In general both contributions visible
- Large-N Peak dominates for $N \geq 3$
- Unclear what dominates for $N = 2$

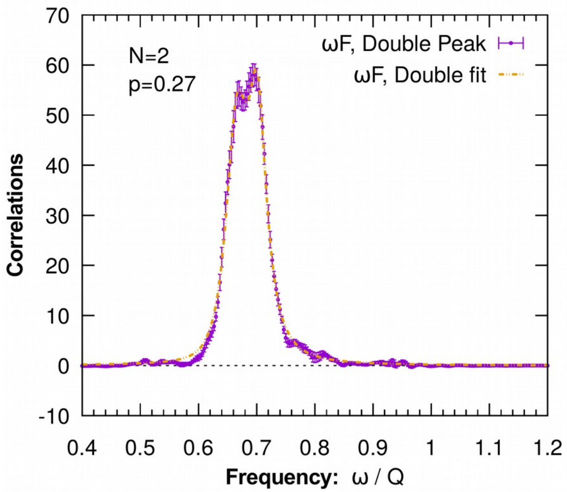
Unraveling universality classes

Relevant phenomena at low p

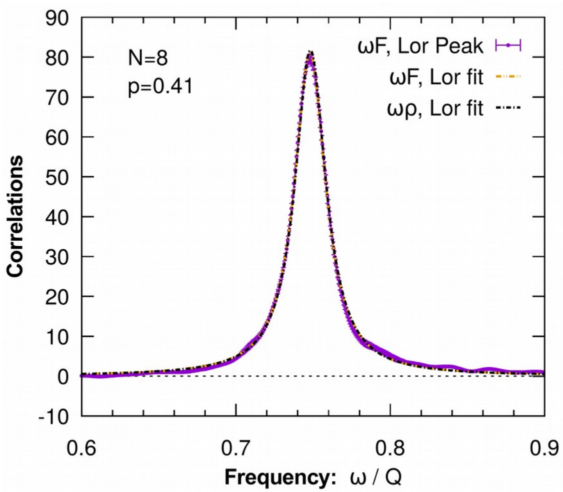
N = 1
Non - Lorentzian



N = 2
Mixed



N ≥ 3
Lorentzian



- As in nonrel. U(1)
- Vortex / defects dynamics?

- Large-N kinetic theory works!
- Rotational phase excitations

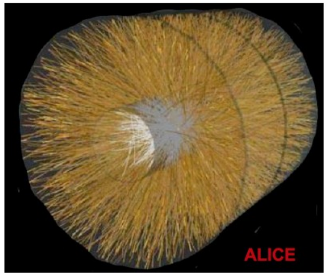
Conclusion

- ✔ Equal-time observables insufficient to understand underlying dynamics

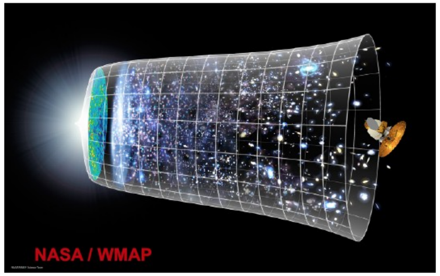
- ✔ With unequal-time correlations, we disentangled two universality classes in O(N)

- ✔ **Outlook:** These techniques important also in other systems and experiments to disentangle competing phenomena

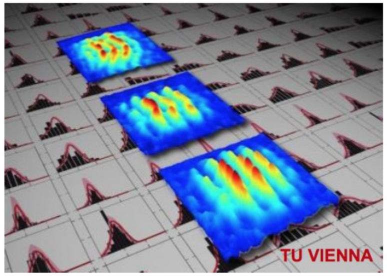
Heavy-ion collisions

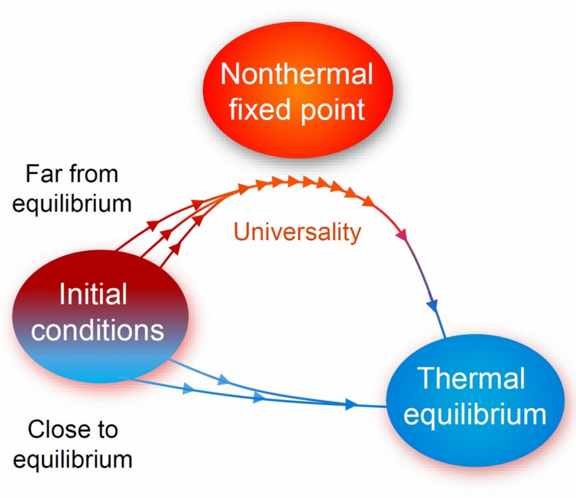


Early Universe



Ultra-cold atoms





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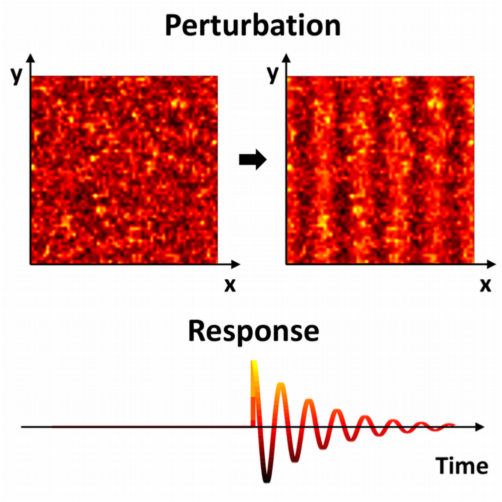
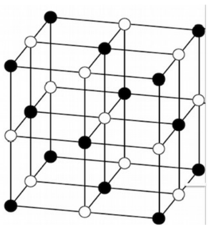


BACKUP SLIDES

Motivation

Reminder: CS simulations / Truncated Wigner (TWA)

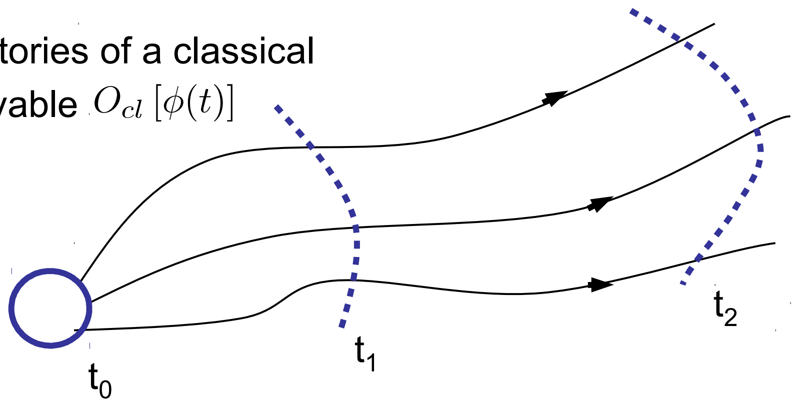


- Initial time t_0, quantum (Gaussian) initial conditions (IC):

  Choose $\langle\phi\rangle$, $\langle\phi\phi\rangle$, which defines weight functional $W\left[\phi(t_0)\right]$

- Dynamics given by classical field equations (e.g., GPE)
- Evolve fields classically, obtain observable at time t by averaging over IC

$$O(t) = \langle O_{cl}[\phi(t)]\rangle = \int D\phi\, W\left[\phi(t_0)\right] O_{cl}\left[\phi(t)\right]$$



Distribution function:

$$f(p) = \sqrt{\langle\phi\phi\rangle\langle\partial_t\phi\partial_t\phi\rangle} \approx \langle\phi\phi\rangle\,\omega(p) \approx \frac{\langle\partial_t\phi\partial_t\phi\rangle}{\omega(p)}$$

Motivation: Nonthermal fixed points

Emerge in such physical systems:

- *Heavy-ion collisions*

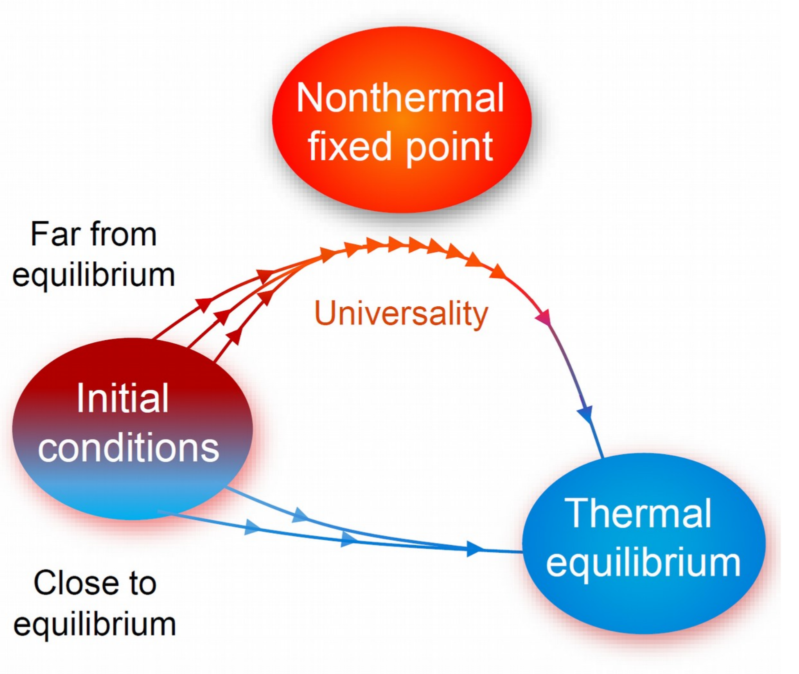
$$S = -\frac{1}{4}\int d^{d+1}x\; F_a^{\mu\nu} F^a_{\mu\nu}$$

- O(N) Scalar models of *inflation, dark matter*

$$S = \int d^{d+1}x \left[\frac{1}{2}\partial^\mu \varphi_a \partial_\mu \varphi_a - \frac{m^2}{2}\varphi_a\varphi_a - \frac{\lambda}{4!N}\left(\varphi_a\varphi_a\right)^2\right]$$

- *Ultra-cold atoms* (experimental observation, Gross-Pitaevskii U(1) simulations)

$$S = \int d^{d+1}x \left[\psi^* \left(i\partial_0 + \frac{\nabla^2}{2m}\right)\psi - \frac{g}{2}\left(\psi\psi^*\right)^2\right]$$



Berges, KB, Schlichting, Venugopalan, *PRL 114, 061601 (2015)*

Self-similar evolution

$$f(t,p) = t^{\alpha} f_s(t^{\beta} p)$$

Universality in scalar systems

Experimental observation in ultra-cold atoms

Prüfer, Kunkel, Strobel, Lanning, Linnemann, Schmied, Berges, Gasenzer, Oberthaler, *Nature 563, 217 (2018)*

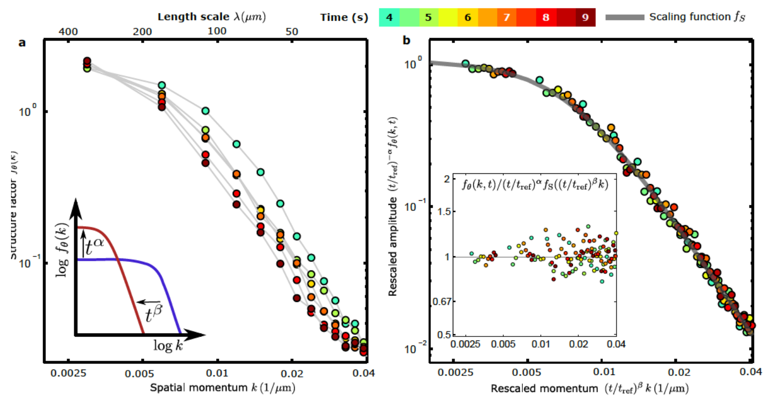
$$\alpha \approx \beta\,,\ \beta \approx \frac{1}{2}$$

(for effectively d = 1 dimensions)

Scaling exponents as found in:

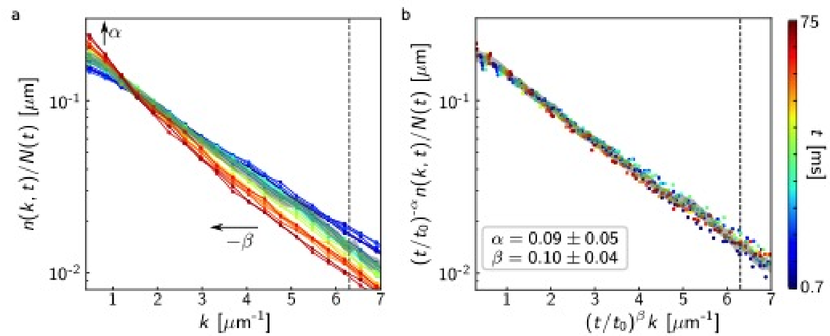
Piñeiro Orioli, KB, Berges, *PRD 92, 025041 (2015)*

Original **Rescaled**



Erne, Bücker, Gasenzer, Berges, Schmiedmayer, *Nature 563, 225 (2018)*

$$\alpha \approx \beta\,,\ \beta \approx \frac{1}{10}$$



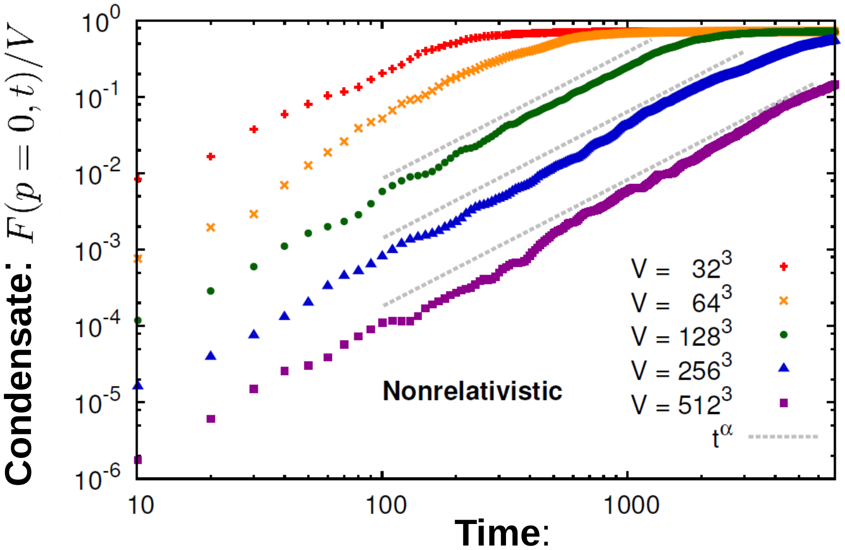
Self-similar evolution

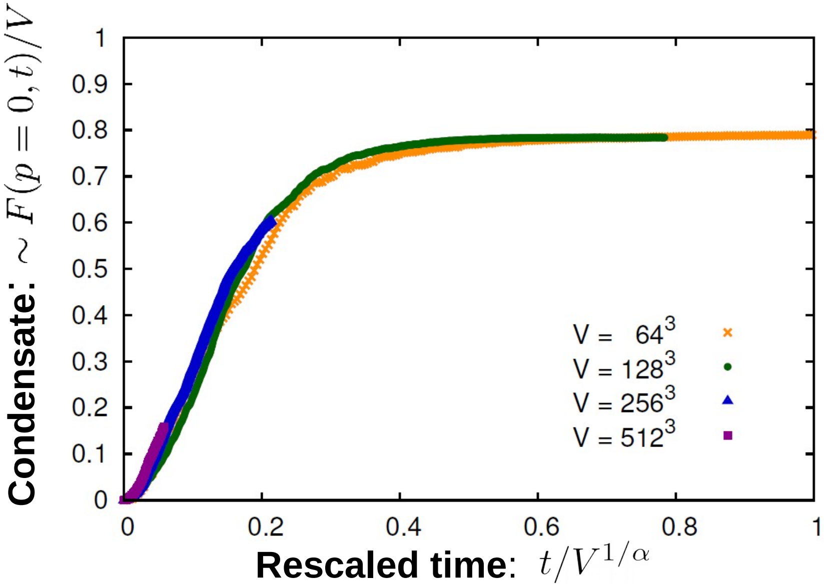
$f(t,p) = t^{\alpha} f_s(t^{\beta} p)$

Universality in scalar systems

Condensation dynamics

Pinerio Orioli, KB, Berges,
PRD 92, 025041 (2015)





- Bose-Einstein condensation, a function of condensation time $t_c \sim V^{1/\alpha} \sim L^{1/\beta}$
- Similar condensation phenomenon for non-Abelian gauge theories, gauge-invariant condensate from Wilson loop

Berges, KB, Mace, Pawlowski,
arXiv:1909.06147

Piñeiro Orioli, Berges,

PRL 122, 150401 (2019)

Unraveling universality classes

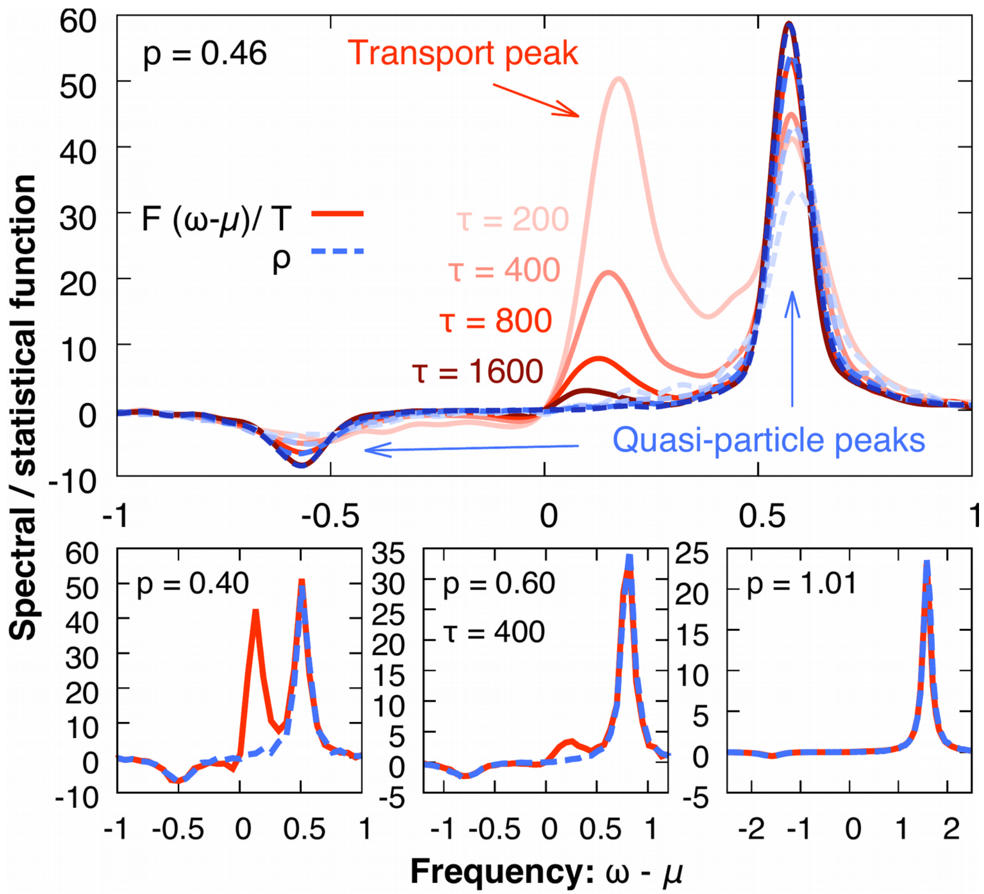
Peak forms in U(1) (non-relativistic)

Observations:

- F and ρ both have Bogolyubov peak (quasiparticle, Lorentzian)
- Fluctuation-dissipation relation (FDR) for them as in equilibrium:

$$F_{\text{Bogol}}(\tau,\omega,p) = \frac{T}{\omega-\mu}\,\rho_{\text{Bogol}}(\tau,\omega,p)$$

- F has extra peak ("transport", non-Lorentzian), *dominates at low p*



$$\rho(t,t',x,x') = \left\langle \left[\hat\psi(t,x), \hat\psi^\dagger(t',x')\right] \right\rangle$$

$$F(t,t',x,x') = \frac{1}{2}\left\langle \left\{\hat\psi(t,x), \hat\psi^\dagger(t',x')\right\} \right\rangle_C$$

Unraveling universality classes

Scalar O(2): Dispersion relations