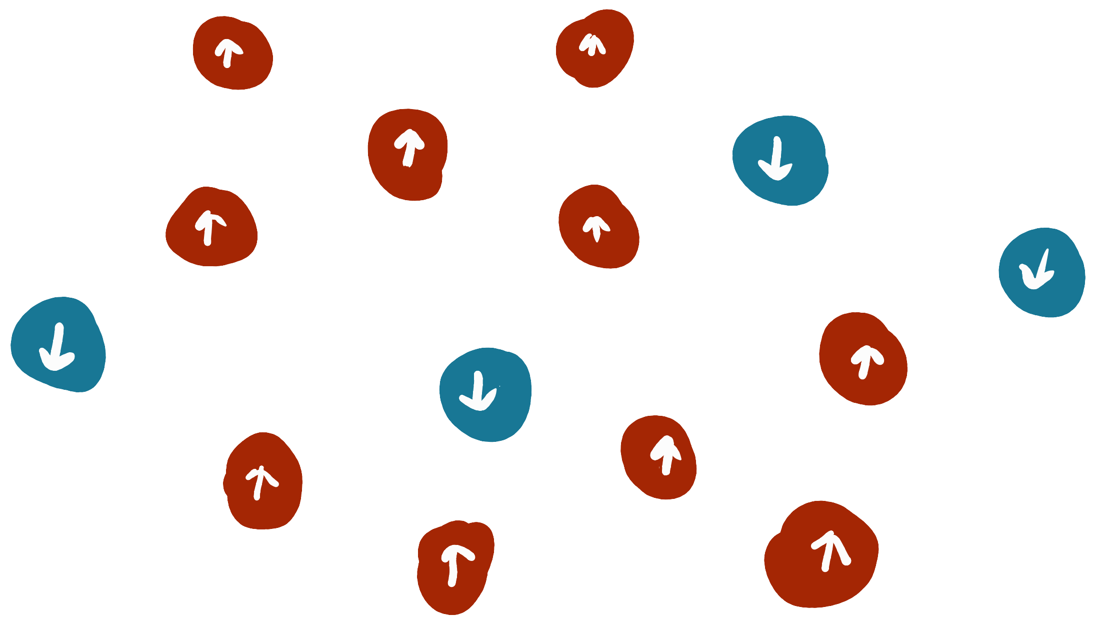
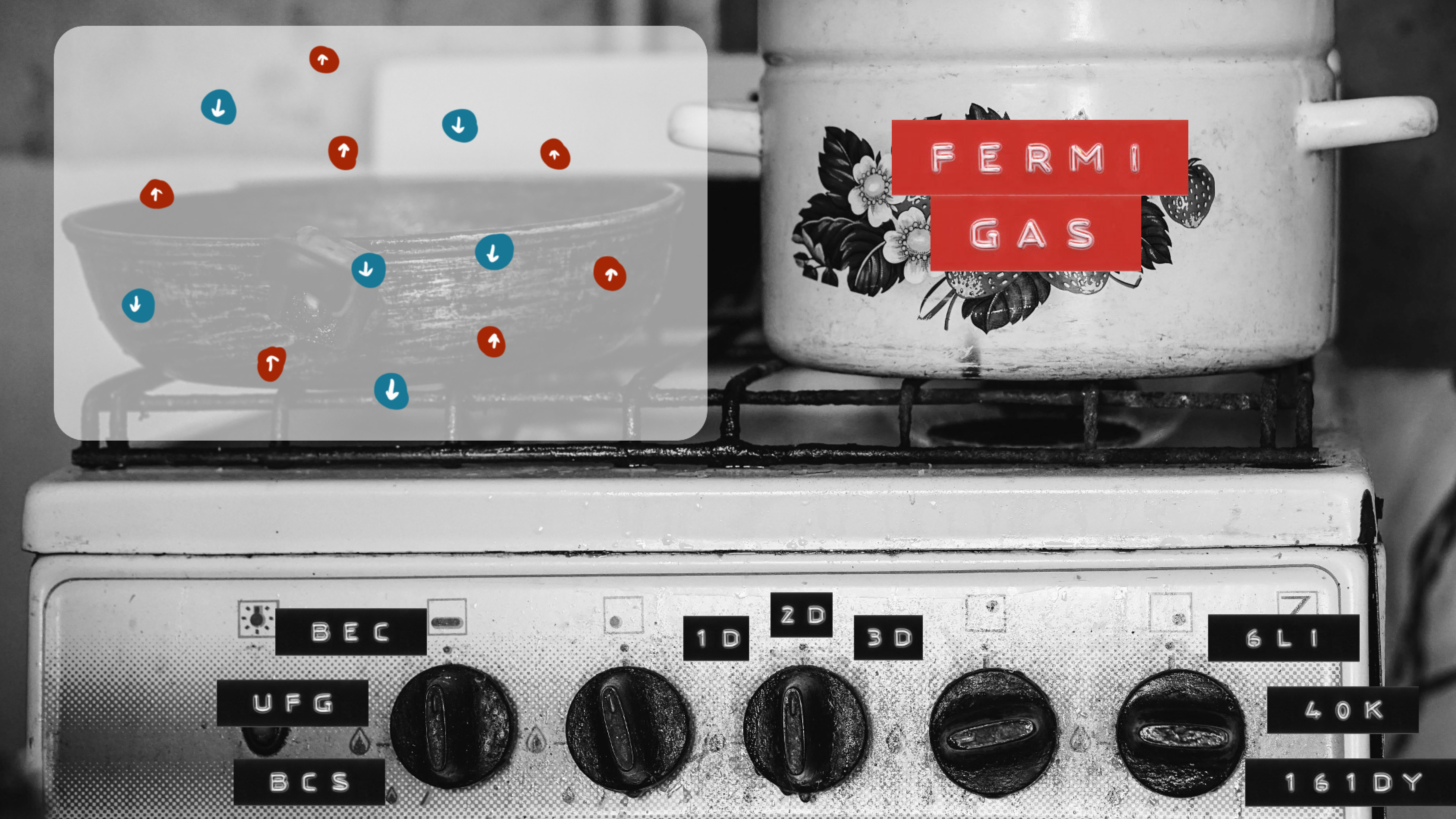
unitary fermions with finite spin-asymmetry

[Berger, Loheac, LR, Ehmann, Braun, Drut *arXiv:1907.10183*]
[LR, Loheac, Drut, Braun *Phys. Rev. Lett.* 121, 173001, 2018]



Lukas Rammelmüller, TU Darmstadt

Quantum Systems in Extreme Conditions - Heidelberg, September 24, 2019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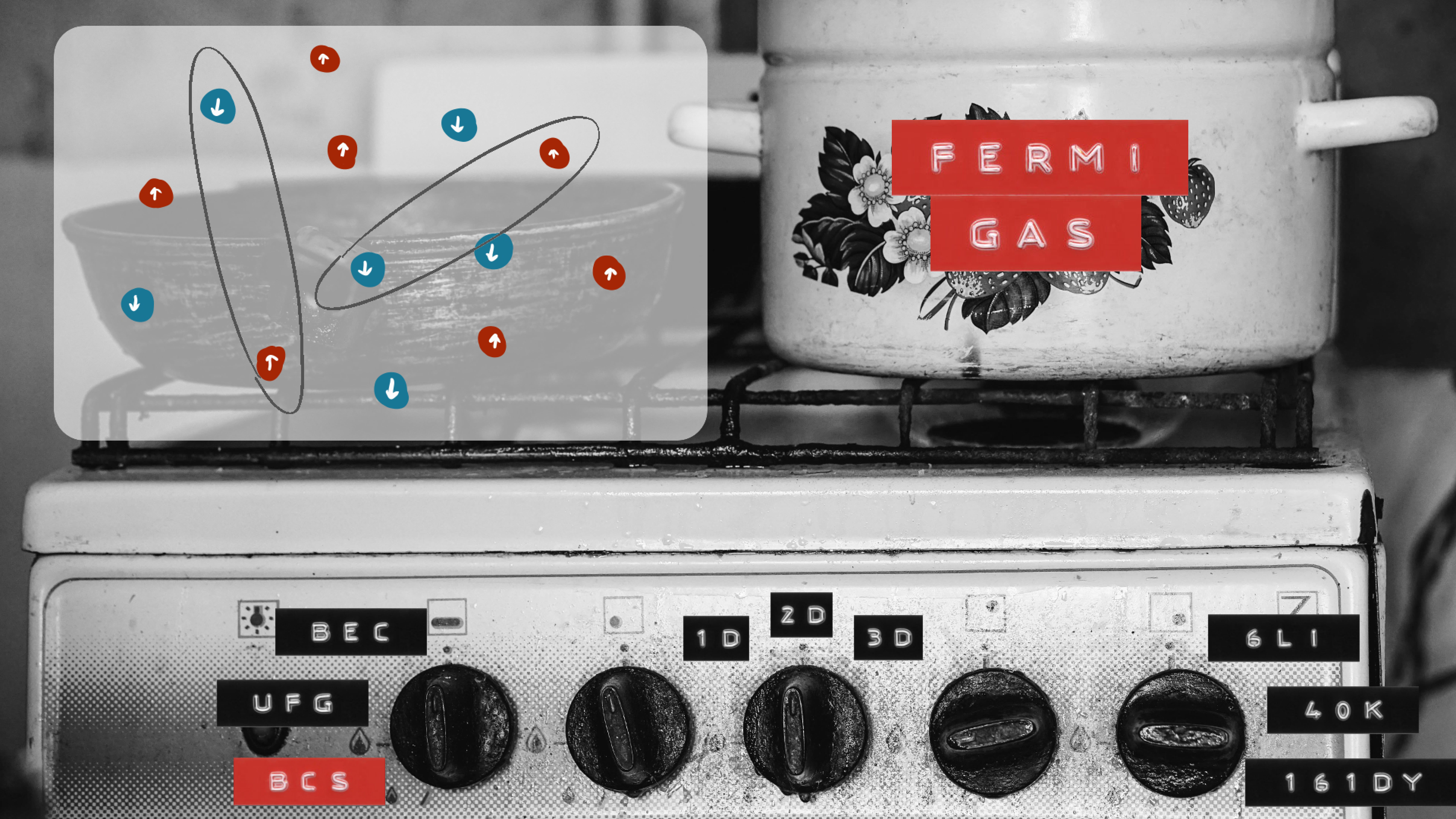
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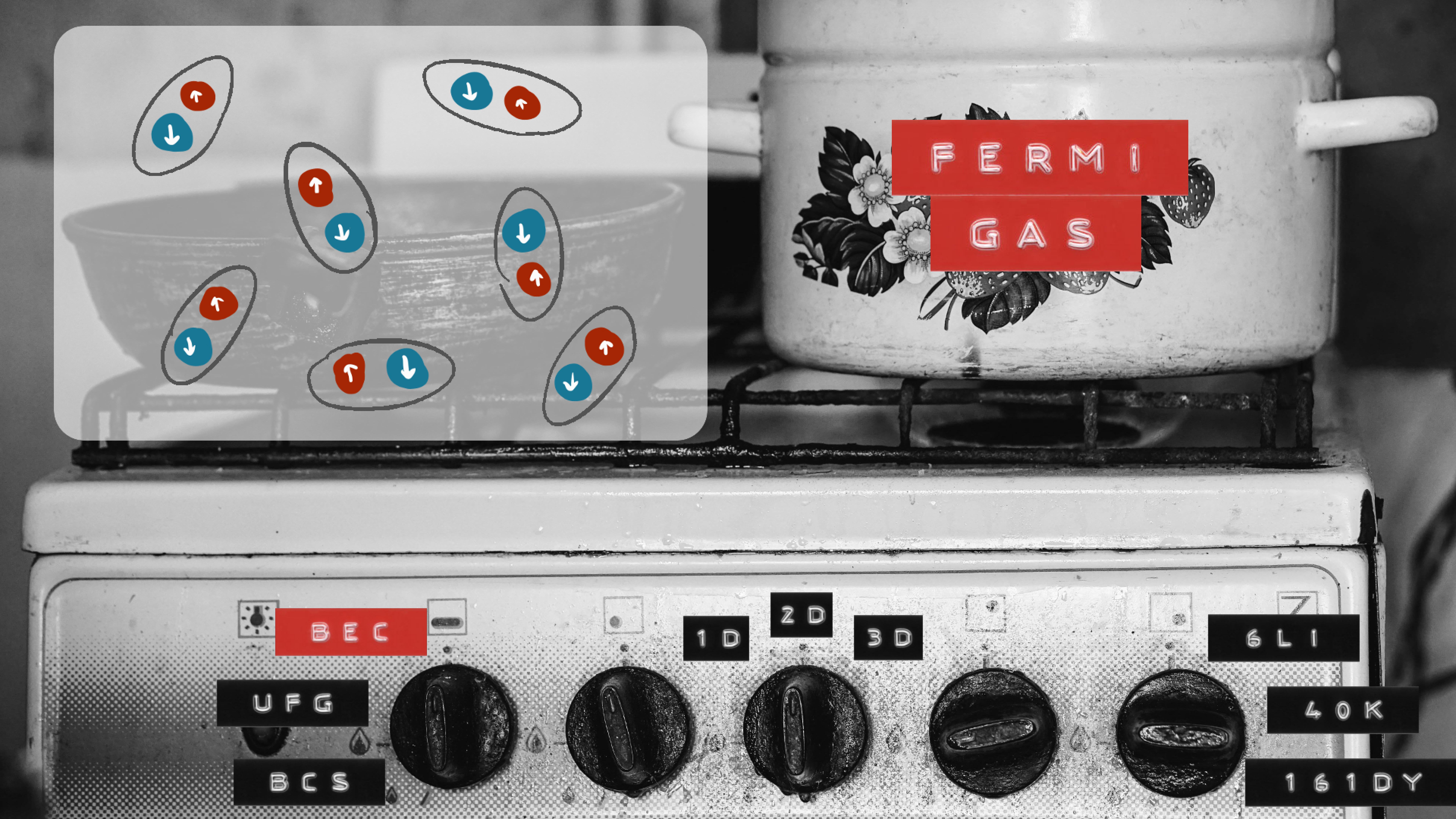
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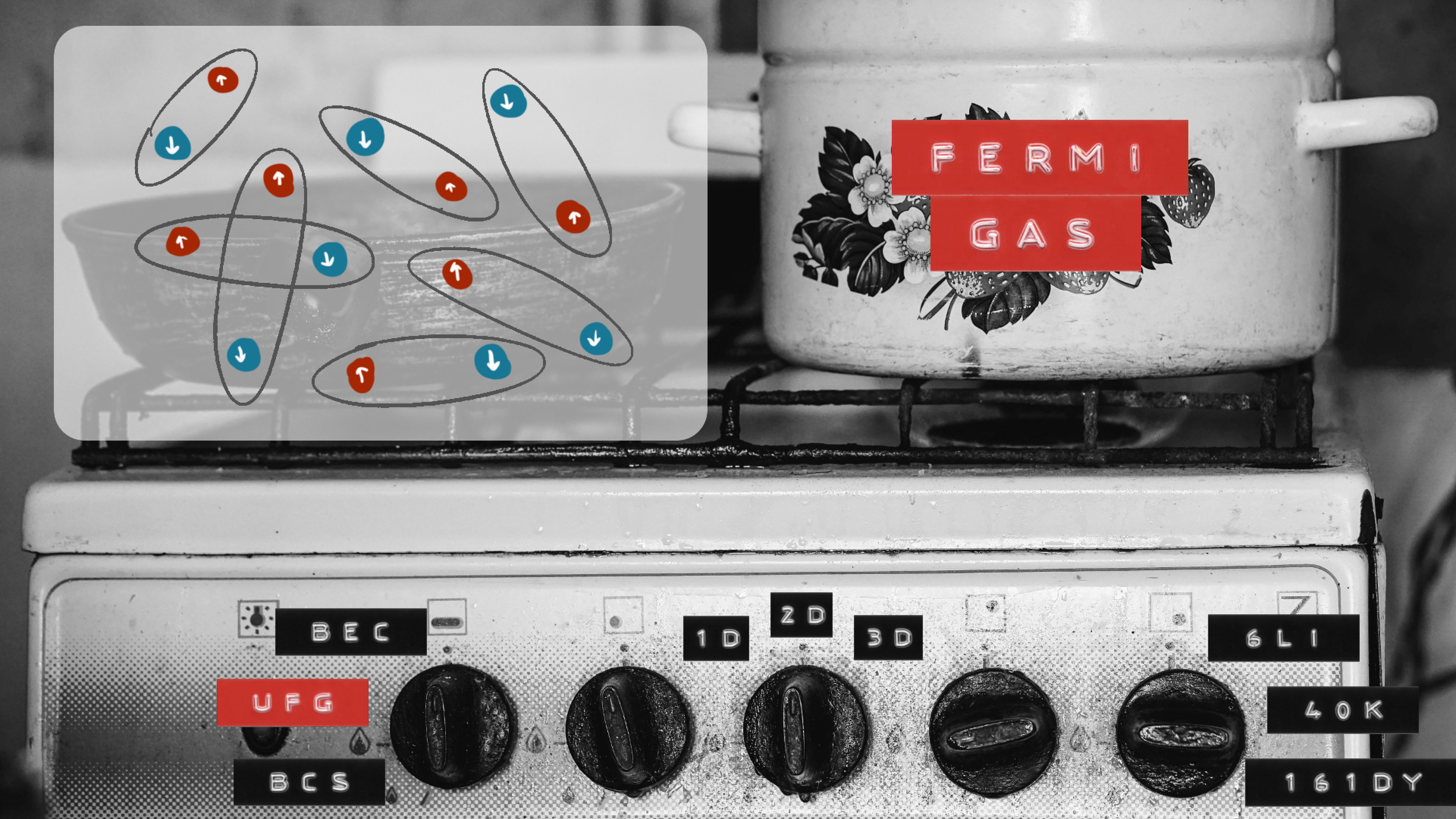
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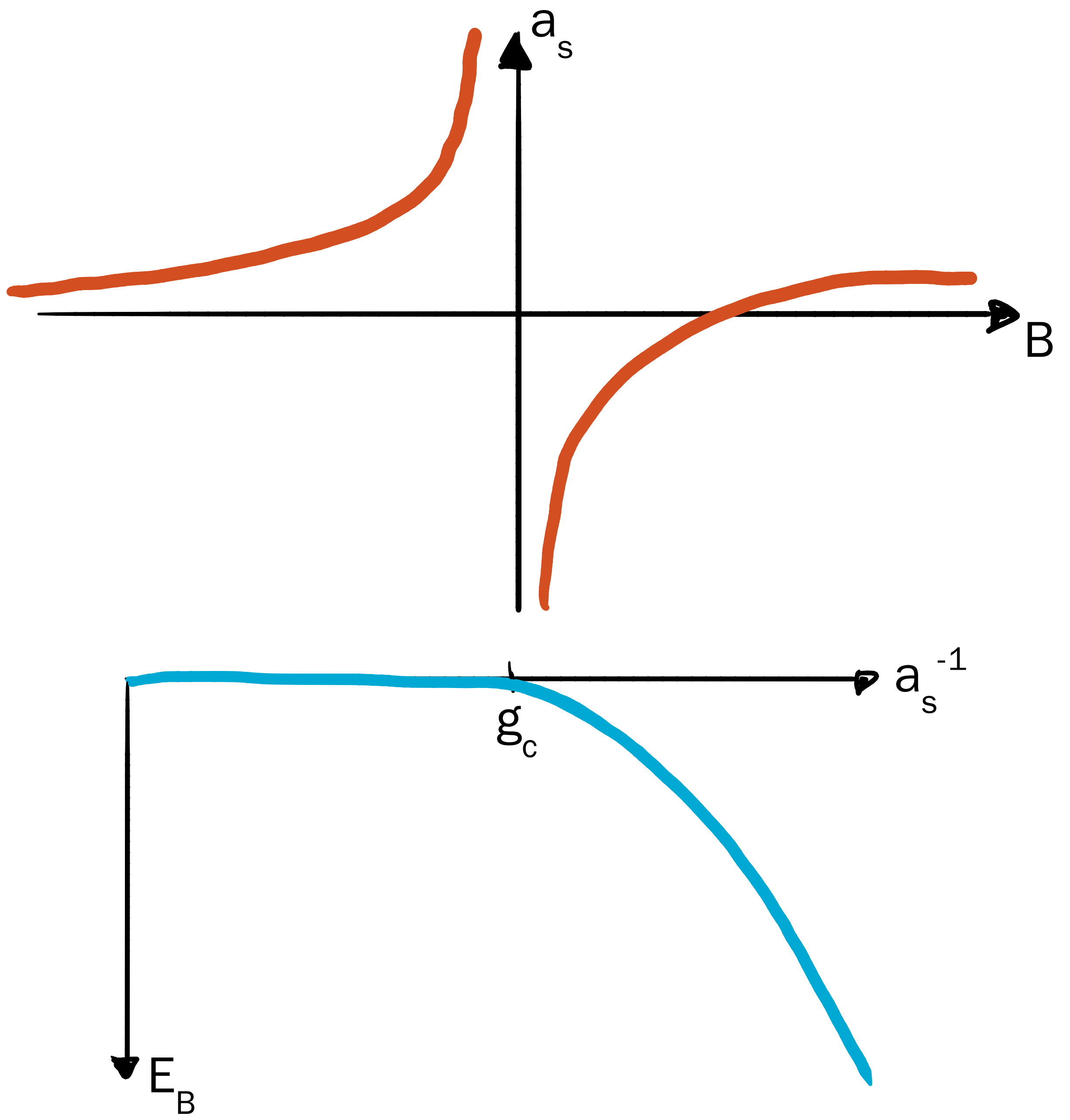
the unitary Fermi gas (UFG)

[reviews: Zwerger '12; Mukaiyama,Ueda '13]

$$a_S \gg n^{-1/3} \gg r_0$$

- density & temperature are the **only dimensionful** scales in the system
- **universal** scaling functions:

$$E = E_{FG}\ f_E(n, T)$$

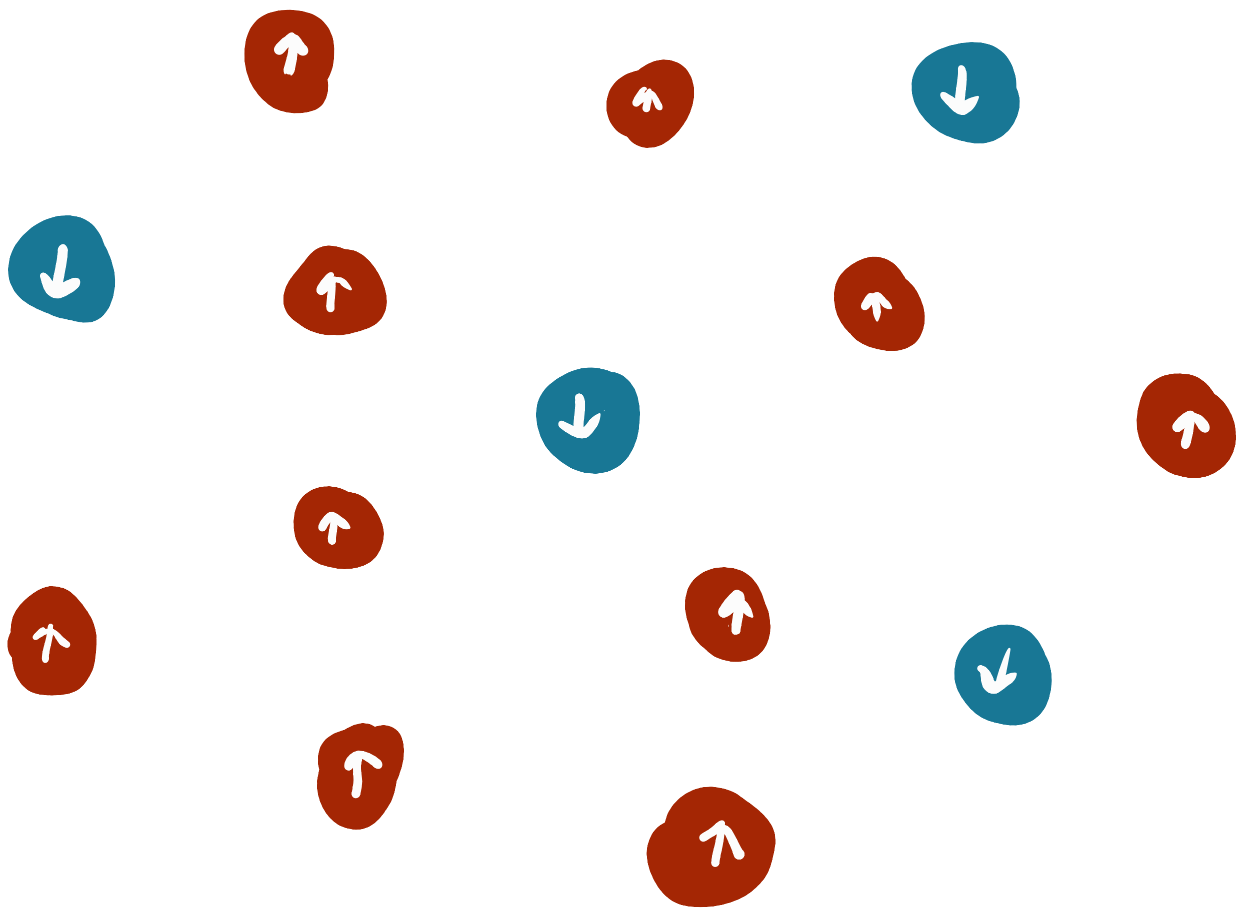


numerous experiments:

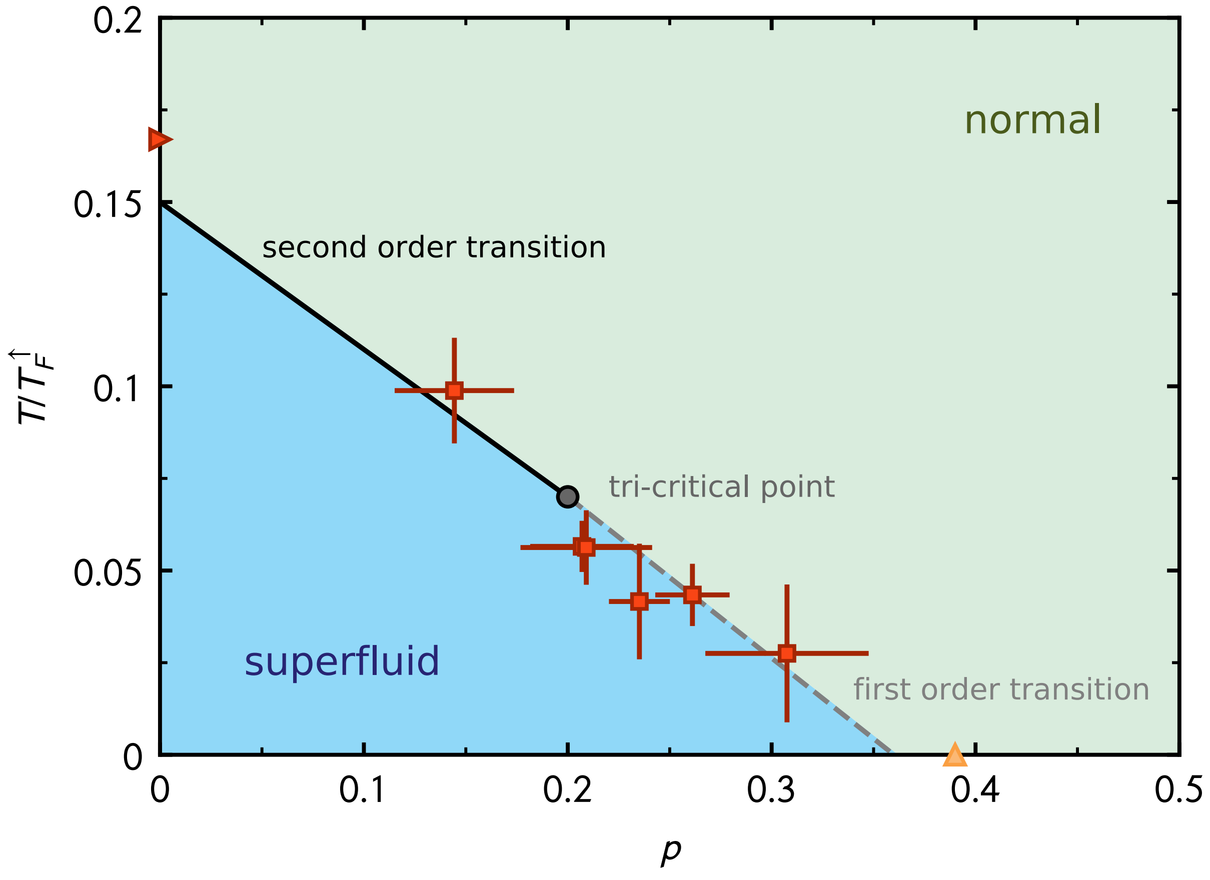
- first realizations of unitary fermions [Regal,Greiner,Jin '04; Zwierlein et al. '04; Kinast et al. '04]
- universal behavior & thermodynamics [Thomas,Kinast,Turpalov '05; Horikoshi et al. '10]
- temperature vs. polarization phase-diagram [Shin,Schunck,Schirotzek,Ketterle '08]
- measurement of equation of state [Nascimbène et al. '10; van Houcke et al. '12]
- superfluid transition [Ku,Sommer,Cheuck,Zwierlein '12]
- temperature dependence of Tan's contact [Carcy et al. '19; Mukherjee et al. '19]
- and many more...

finite spin polarization

[reviews: Chevy,Mora '10; Gubbels,Stoof '13]



[experimental PD: Shin,Schunck,Schirotzek,Ketterle '08]



Many questions remain:

critical polarization? phase diagram? pairing?

agenda

part I

quick into to **stochastic quantization & CL**
(what is it & how can it help us for fermions?)

part II

unitary fermions with finite polarization
(equations of state & thermodynamic response)

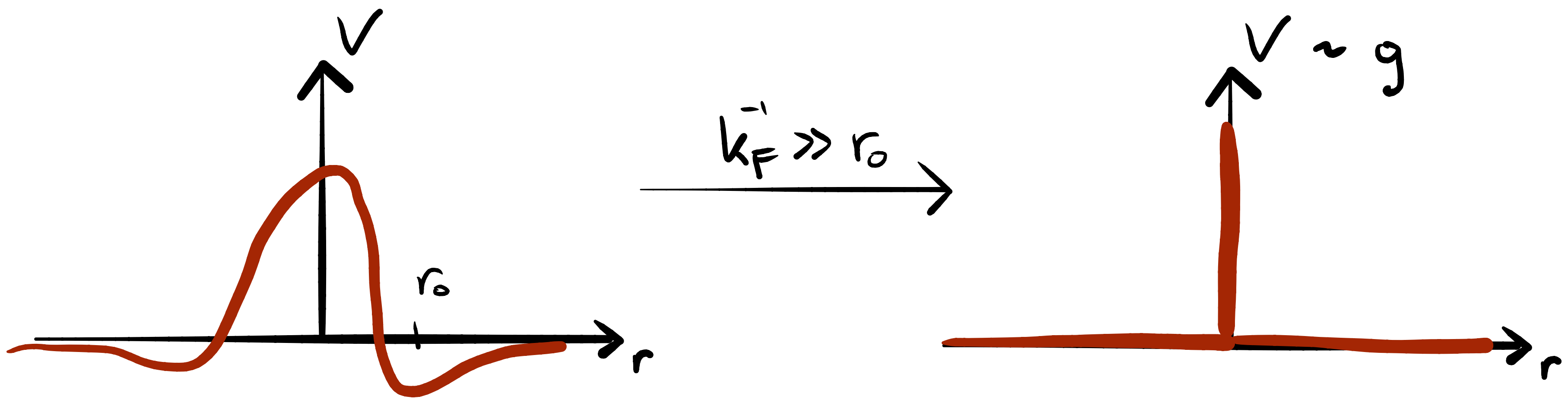
fermions with contact interaction

kinetic part

$$\hat{H} = -\sum_{s=\uparrow,\downarrow} \int d^d x \, \hat{\psi}_s^\dagger(\vec{x}) \left(\frac{\hbar^2 \vec{\nabla}^2}{2m_s} \right) \hat{\psi}_s(\vec{x})$$

$$+ \, g \int d^d x \, \hat{\psi}_\uparrow^\dagger(\vec{x}) \, \hat{\psi}_\uparrow(\vec{x}) \, \hat{\psi}_\downarrow^\dagger(\vec{x}) \, \hat{\psi}_\downarrow(\vec{x})$$

interaction part



what do we need to compute?

[lattice methods: Lee '09; Drut,Nicholson '13; Zhang '13]

$$\mathcal{Z} = \mathrm{Tr}[\mathrm{e}^{-\beta\hat{H}}] = \mathrm{Tr}[\mathrm{e}^{-\beta(\hat{T} + \hat{V})}]$$

$$\langle\mathcal{O}\rangle = \frac{1}{\mathcal{Z}}\mathrm{Tr}[\hat{\mathcal{O}}\,\mathrm{e}^{-\beta\hat{H}}]$$

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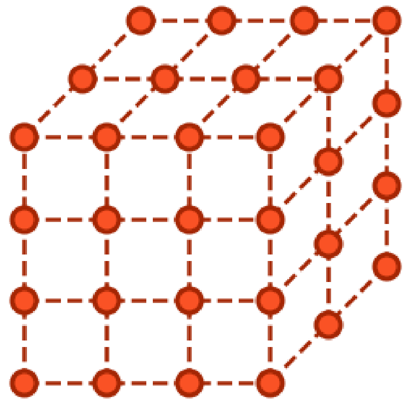

+ Trotter decomposition

+ Hubbard-Stratonovich transformation

rewrite the problem as a **path-integral**:

$$\mathcal{Z} = \int\mathcal{D}\phi\ \det M_\phi^\uparrow\ \det M_\phi^\downarrow \equiv \int\mathcal{D}\phi\ \mathrm{e}^{-S[\phi]}$$

(hard problem = $\sum$ easy problems)

stochastic quantization

[Parisi, Wu '81; Damgaard, Hüffel '87]

$$\mathcal{Z} = \int \mathcal{D}\phi \, e^{-S[\phi]}$$

(partition function)

$$\langle \hat{\mathcal{O}} \rangle = \int \mathcal{D}\phi \, \mathcal{O}[\phi] \, P[\phi]$$

(expectation values)

$$P[\phi] = \frac{1}{\mathcal{Z}} e^{-S[\phi]}$$

(probability measure)

key idea:

probability measure of a **d-dimensional Euclidean path integral**
as equilibrium distribution of a **d+1-dimensional random process**

stochastic quantization

[Parisi, Wu '81; Damgaard, Hüffel '87]

random walk governed by

Langevin equation (Brownian motion):

$$\frac{\partial \phi}{\partial t_L} = -\frac{\delta S[\phi]}{\delta \phi} + \eta$$

stochastic quantization

[Parisi, Wu '81; Damgaard, Hüffel '87]

random walk governed by

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$$\frac{\partial \phi}{\partial t_L} = -\frac{\delta S[\phi]}{\delta \phi} + \eta$$

ficticious Langevin time
(not physical)

stochastic quantization

[Parisi, Wu '81; Damgaard, Hüffel '87]

random walk governed by

Langevin equation (Brownian motion):

$$\frac{\partial \phi}{\partial t_L} = -\frac{\delta S[\phi]}{\delta \phi} + \eta$$

ficticious Langevin time (not physical)

noise term

$$\langle \eta \rangle = 0$$

$$\langle \eta_t \eta_{t'} \rangle = 2\delta(t - t')$$

the Langevin method

[Parisi, Wu '81; Damgaard, Hüffel '87]

$$\frac{\partial \phi}{\partial t_L} = -\frac{\delta S[\phi]}{\delta \phi} + \eta$$

discretization

$$\phi^{(n+1)} = \phi^{(n)} - \frac{\delta S[\phi]}{\delta \phi}\bigg|_{\phi^{(n)}} \Delta t_L + \sqrt{2\Delta t_L}\tilde{\eta}$$

(Markov chain)

the Langevin method

[Parisi, Wu '81; Damgaard, Hüffel '87]

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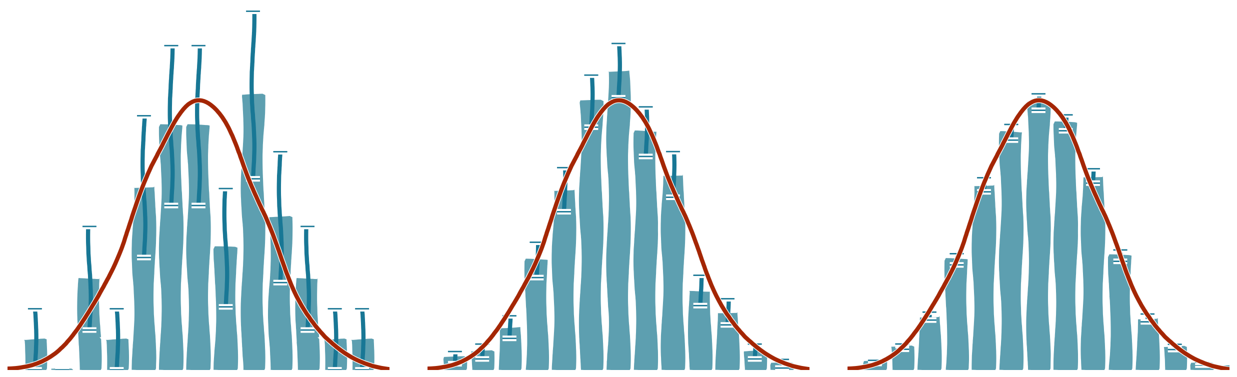
(Markov chain)

statistical evaluation

$$\langle \hat{\mathcal{O}} \rangle = \frac{1}{N} \sum_{i=1}^{N} \mathcal{O}[\phi_i]$$

$$\sigma \propto \left(\sqrt{\#\text{ of (uncorrelated) samples}}\right)^{-1}$$



how can stochastic quantization help us to study fermions?

$$\mathcal{Z} = \int \mathcal{D}\phi \, \det M_\phi^\uparrow \det M_\phi^\downarrow = \int \mathcal{D}\phi \, \mathrm{e}^{-S[\phi]}$$

how can stochastic quantization help us to study fermions?

$$\mathcal{Z} = \int \mathcal{D}\phi \, \det M_\phi^\uparrow \, \det M_\phi^\downarrow = \int \mathcal{D}\phi \, \mathrm{e}^{-S[\phi]}$$

probability measure **not positive (semi-)definite** if any of these conditions applies:

$N_\uparrow \neq N_\downarrow$
$\mu_\uparrow \neq \mu_\downarrow$
$m_\uparrow \neq m_\downarrow$
$g > 0$

the path integral & complex actions

$$\mathcal{Z} = \int \mathcal{D}\phi \;\det M_\phi^\uparrow \;\det M_\phi^\downarrow \equiv \int \mathcal{D}\phi \; \mathrm{e}^{-S[\phi]}$$

discrete Langevin equation:

$$\phi^{(n+1)} = \phi^{(n)} + \Delta\phi^{(n)}$$

$$\Delta\phi_R^{(n)} = -\mathrm{Re}\left[\frac{\delta S[\phi]}{\delta\phi}\right]_{\phi^{(n)}} \Delta t_L + \sqrt{2\Delta t_L}\,\eta$$

$$\Delta\phi_I^{(n)} = -\mathrm{Im}\left[\frac{\delta S[\phi]}{\delta\phi}\right]_{\phi^{(n)}} \Delta t_L$$

complex action $\longrightarrow$ complex Langevin equation

[Parisi '83; Klauder '84; Koonin,Adami '01; Berges,Stamatescu '05; Aarts '08; LR,Porter,Drut,Braun '17; LR,Loheac,Drut,Braun '18; Berger et al. '19]

complex probablities

$$\int \mathcal{D}\phi \; P[\phi] O[\phi] \stackrel{?}{=} \int \mathcal{D}\phi_{\mathrm{R}} \mathcal{D}\phi_{\mathrm{I}} \; P[\phi_{\mathrm{R}}, \phi_{\mathrm{I}}] O[\phi_{\mathrm{R}} + \mathrm{i}\phi_{\mathrm{I}}]$$

guaranteed convergence if PDF decays
fast enough and $S[\phi]$ is holomorphic
[Aarts,Seiler,Stamatescu '10; Aarts,James,Seiler,Stamatescu '11]

complex probablities & possible issues

$$\int \mathcal{D}\phi \; P[\phi] O[\phi] \stackrel{?}{=} \int \mathcal{D}\phi_{\mathrm{R}} \mathcal{D}\phi_{\mathrm{I}} \; P[\phi_{\mathrm{R}}, \phi_{\mathrm{I}}] O[\phi_{\mathrm{R}} + \mathrm{i}\phi_{\mathrm{I}}]$$

guaranteed convergence if PDF decays
fast enough and $S[\phi]$ is holomorphic
[Aarts,Seiler,Stamatescu '10; Aarts,James,Seiler,Stamatescu '11]

non-analyticities in the action

- zeros in measure ($\det M = 0$)
- could lead to ergodicity issues (bottlenecks)

[Aarts,Seiler,Sexty,Stamatescu '17]

non-vanishing boundary terms

- convergence to wrong limits possible
- behavior must be monitored

[Scherzer,Seiler,Sexty,Stamatescu '19]

recap:
stochastic quantization & CL

- **SQ:** interpret **Euclidean field theories**
  as equilibrium limit of a ficticious **random process**

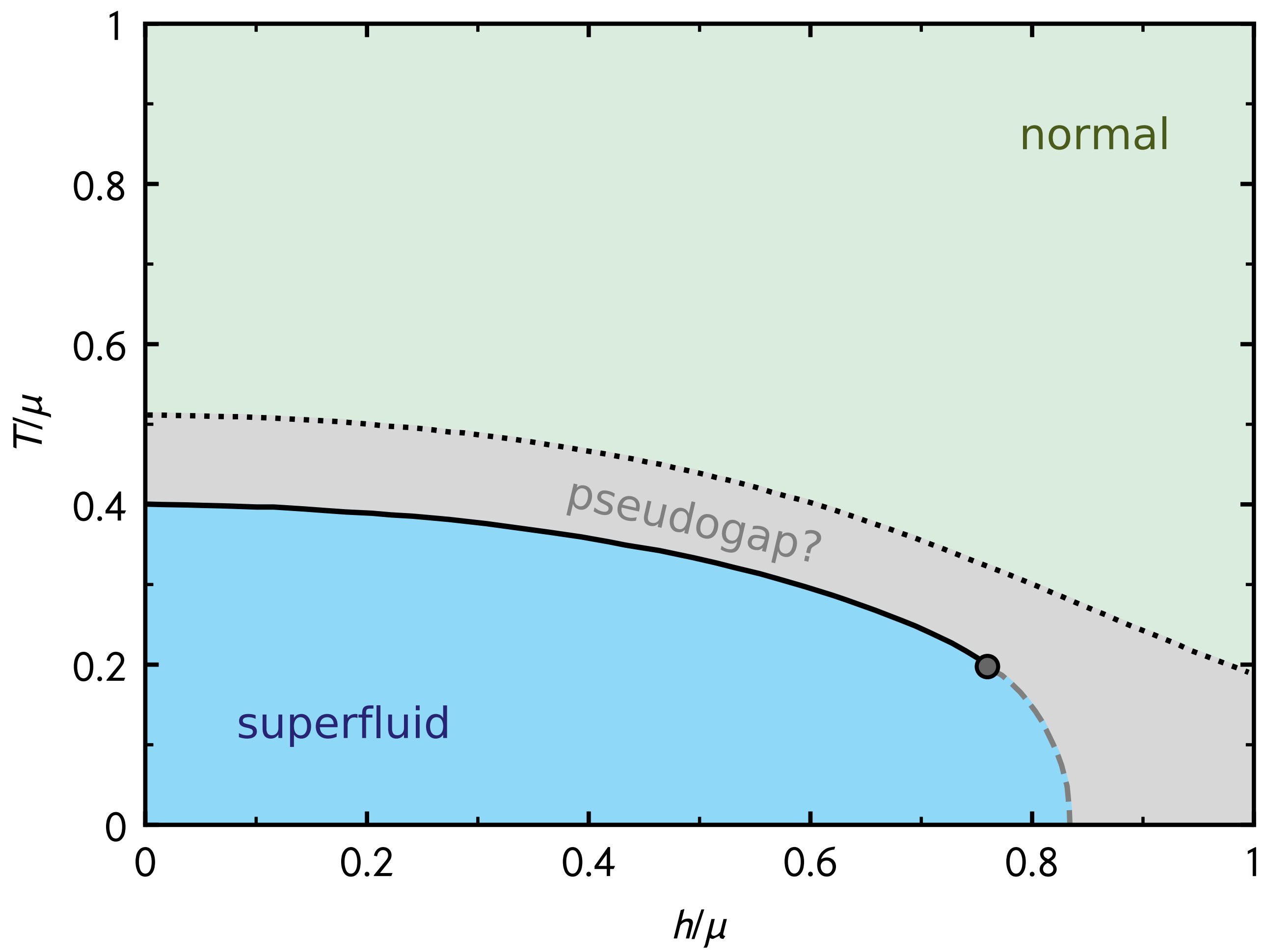
- complex Langevin provides a way
  to **evade sign problems** in some cases

- however: not guaranteed to work a-priori
  and the **behavior needs to be monitored carefully**

- recent review: [Berger et al. arXiv:1907.10183]

the spin-polarized unitary Fermi gas at finite temperature

[LR, Loheac, Drut, Braun '18]



[fRG phase diagram: Boettcher et. al '15]
[LW study of polarized UFG: Frank,Lang,Zwerger '18]

$$\mathcal{Z} = \mathrm{Tr}\left[e^{-\beta(\hat{H}-\mu_\uparrow \hat{N}_\uparrow - \mu_\downarrow \hat{N}_\downarrow)}\right]$$
$$= \mathrm{Tr}\left[e^{-\beta(\hat{H}-\mu\hat{N}-h\hat{M})}\right]$$

$$\mu = \frac{\mu_\uparrow + \mu_\downarrow}{2}$$
$$h = \frac{\mu_\uparrow - \mu_\downarrow}{2}$$

the spin-polarized unitary Fermi gas at finite temperature

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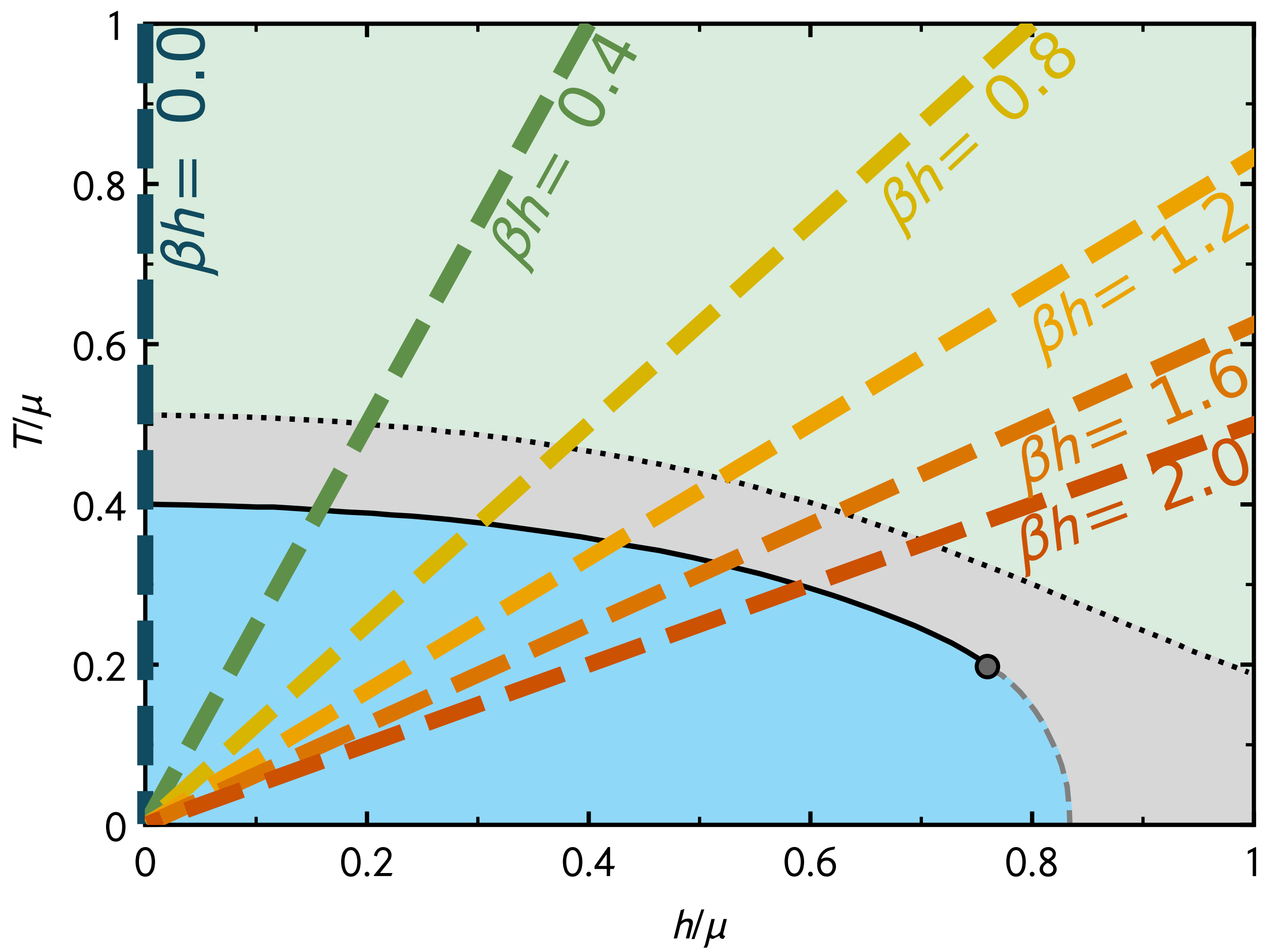

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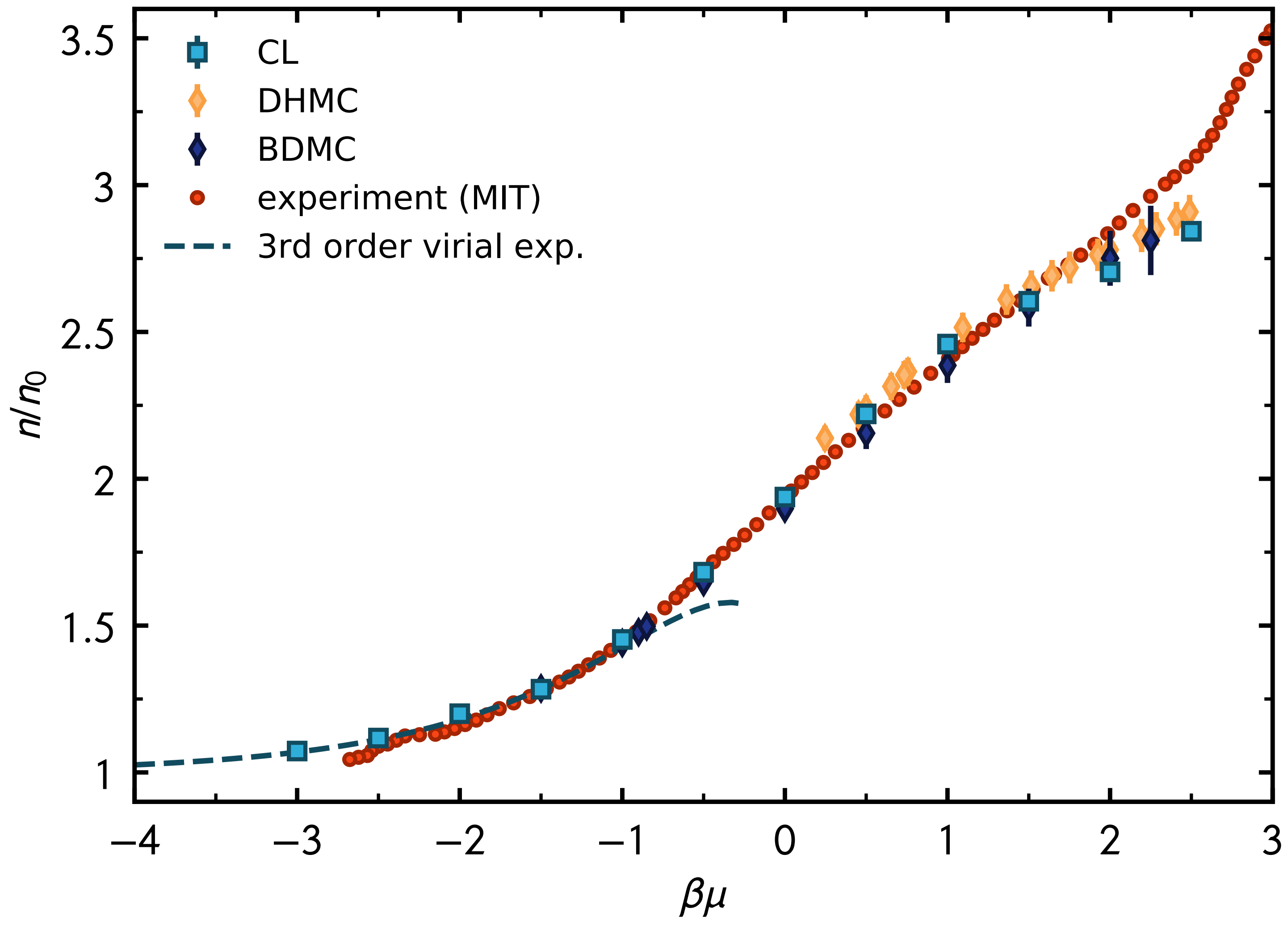
density equation of state

[LR, Loheac, Drut, Braun '18]

[experiment/BDMC: van Houcke et al. '12]
[DHMC: Drut,Lähde,Wlazlowski,Magierski '12]
[diagMC: Rossi,Ohgoe,van Houcke,Werner '18]



classical regime

$k_B T$ dominates

quantum regime

E_F dominates

density equation of state

[LR, Loheac, Drut, Braun '18]

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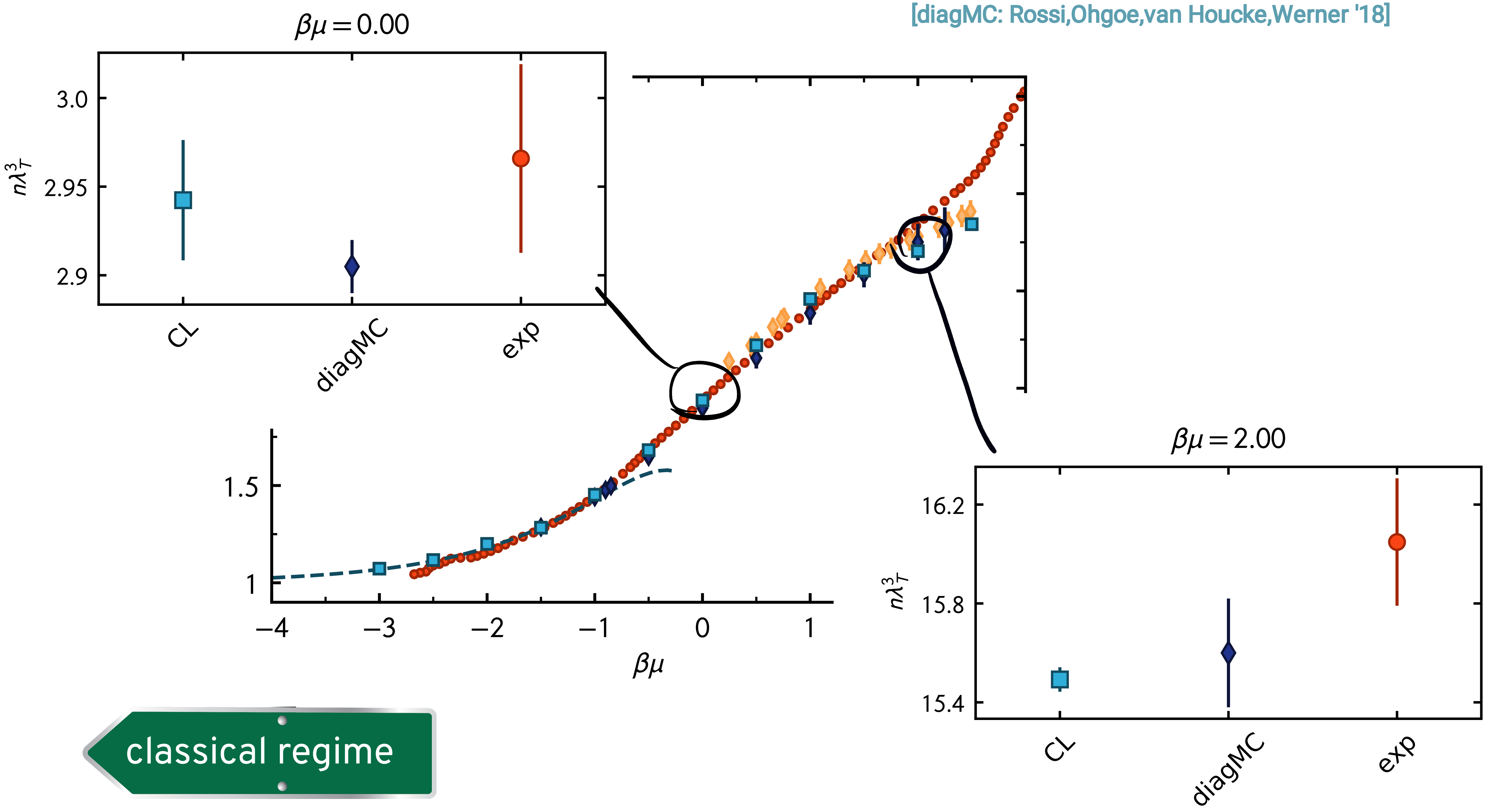

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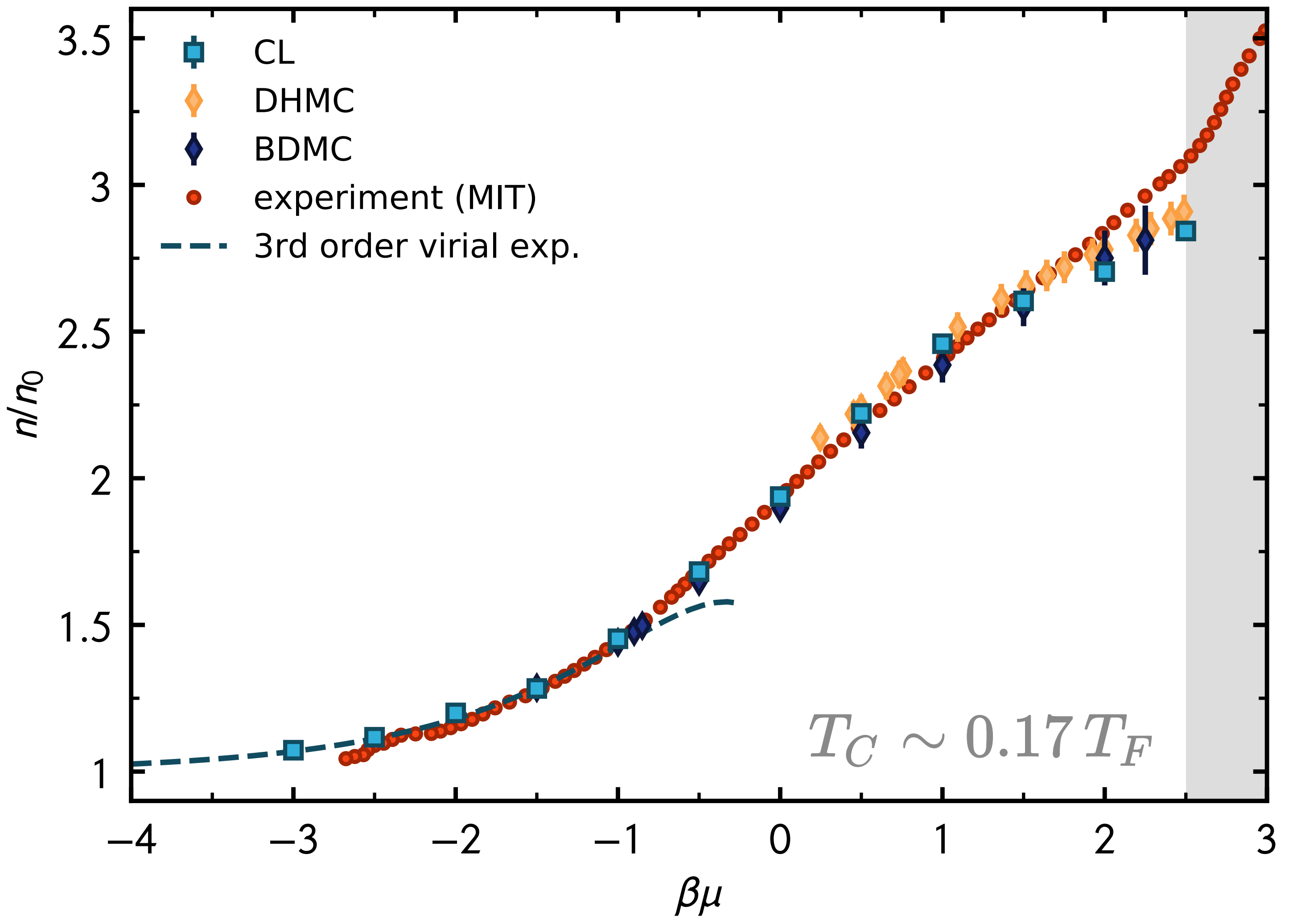
good agreement with experiment and other methods!

CL results:

finite lattice $V = 11^3$



low temperatures:
λ_T increases
($\lambda_T \ll V^{1/3}$ must be fulfilled)

classical regime

$k_B T$ dominates

quantum regime

E_F dominates

interlude: the virial expansion

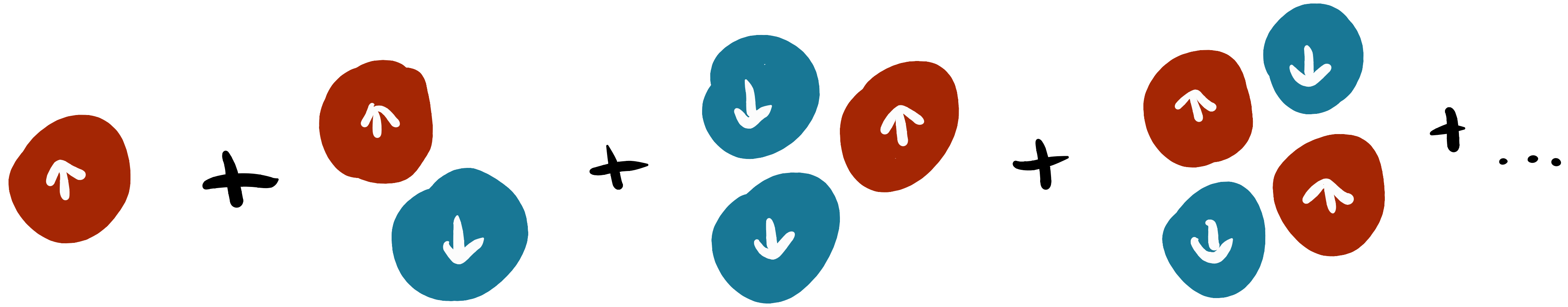
[Liu '13]

dilute gases: few-body correlations dominate

idea: describe the system as **expansion in few-body clusters**

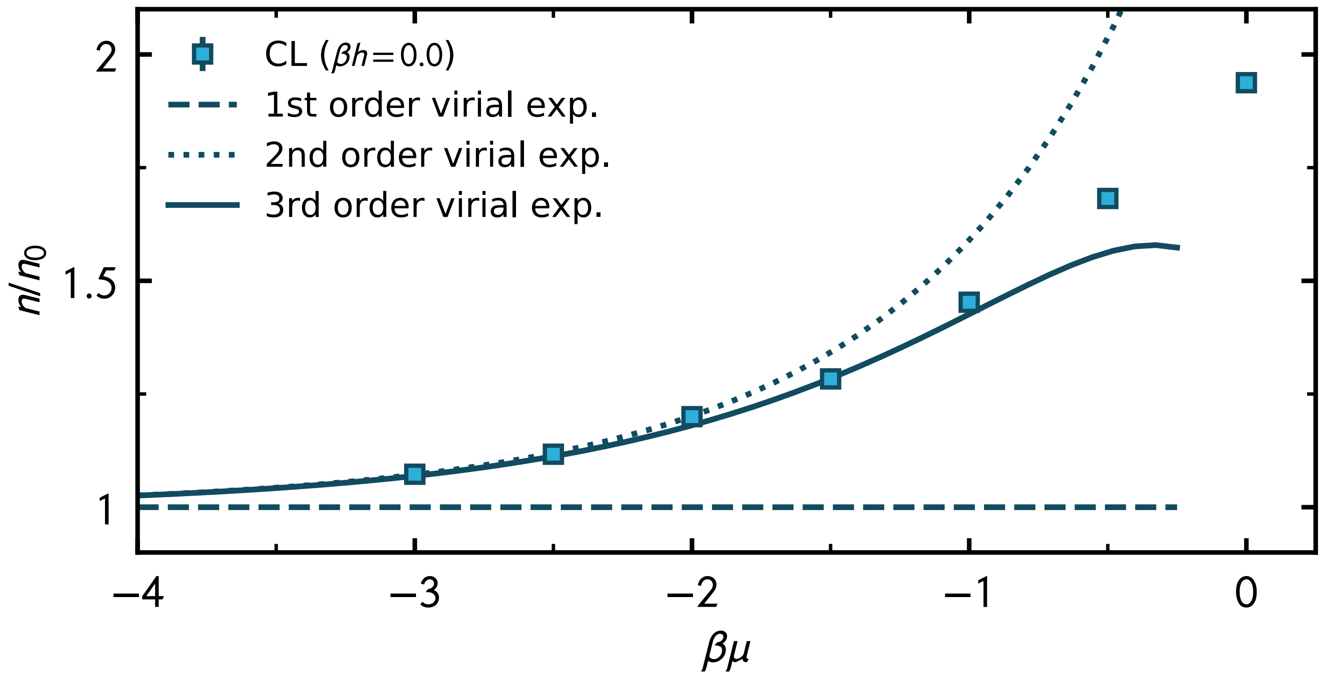
$$z = e^{\beta\mu}$$

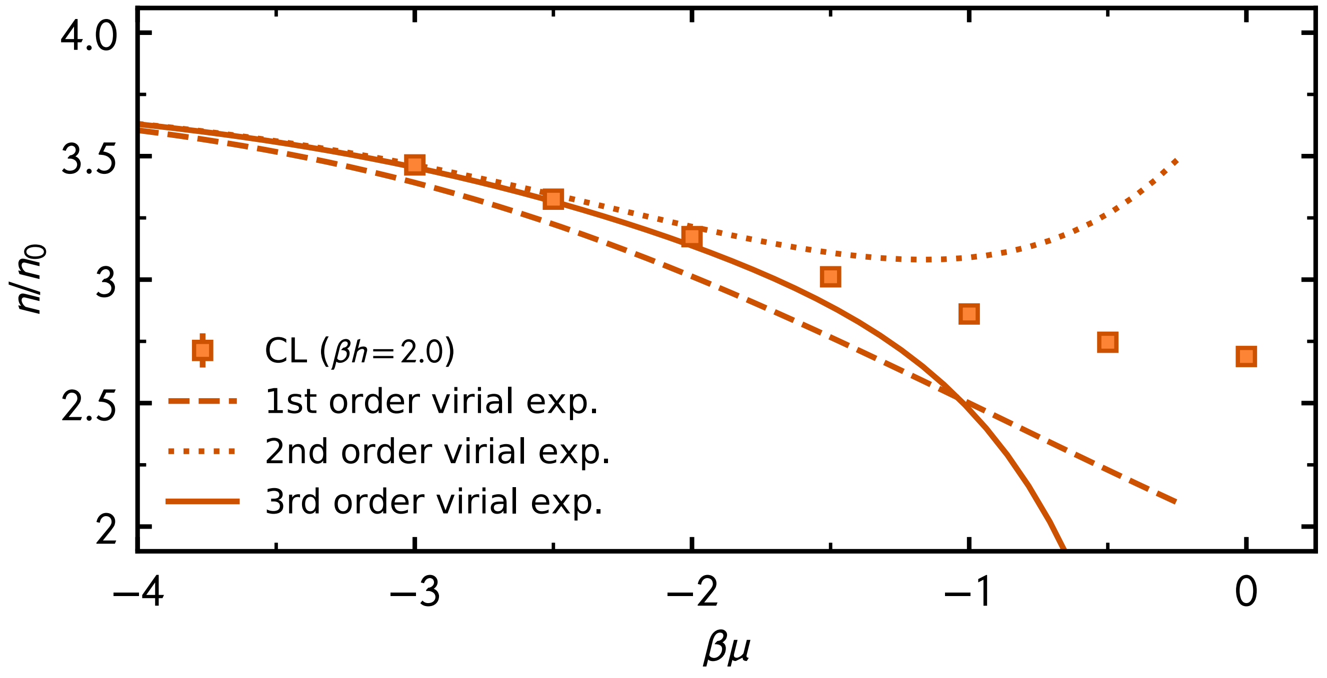
$$\ln \mathcal{Z} = \mathcal{Q}_1 \sum_n z^n b_n$$



density EOS (virial regime)

[LR, Loheac, Drut, Braun '18]

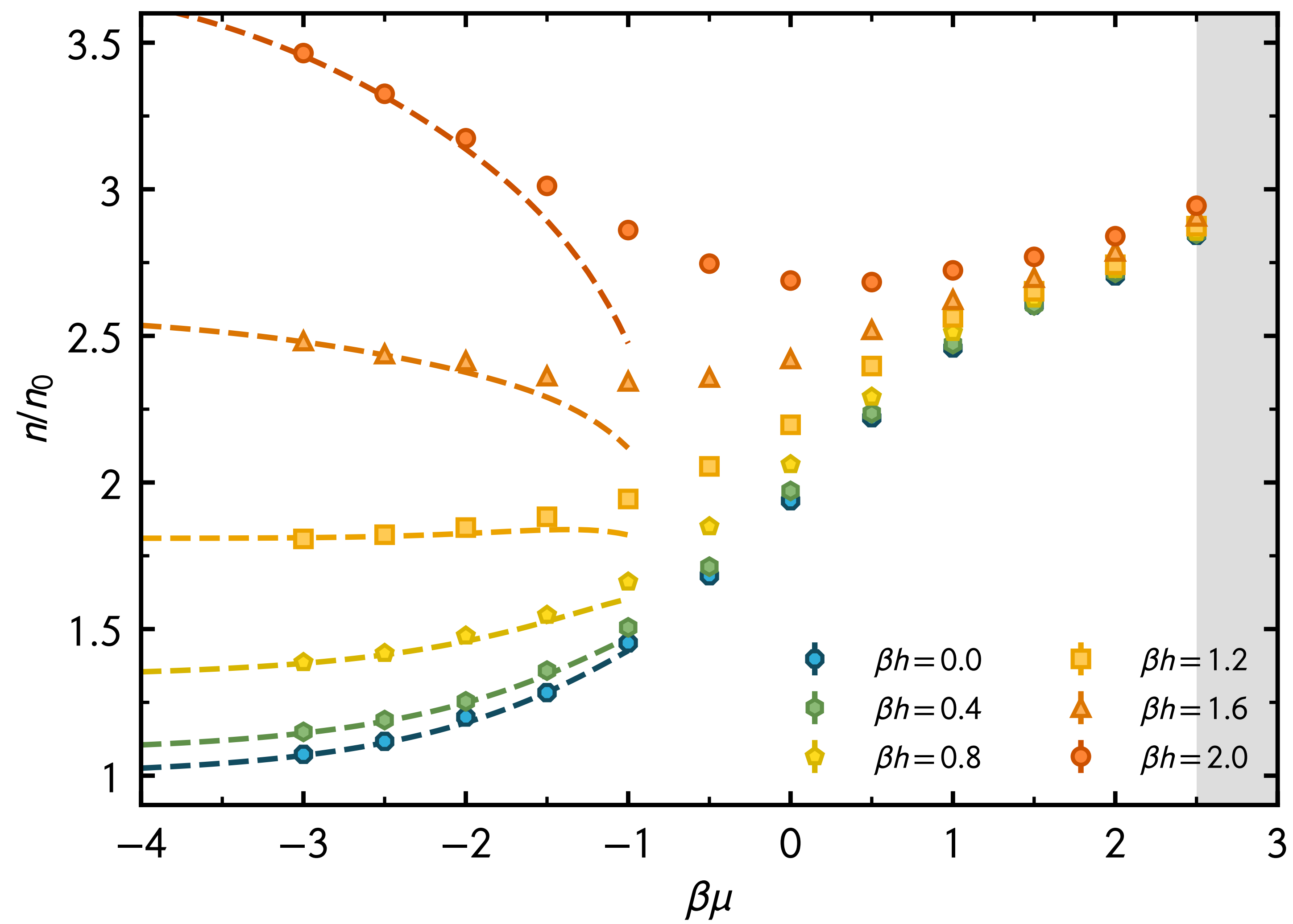




- maximum radius of convergence: $\beta\mu = 0$
- **virial expansion approaches the CL results order-by-order**
- deviation earlier for polarized systems (majority species hits convergence radius earlier)

density EOS at finite polarization

[LR, Loheac, Drut, Braun '18]



$$h = \frac{\mu_\uparrow - \mu_\downarrow}{2}$$

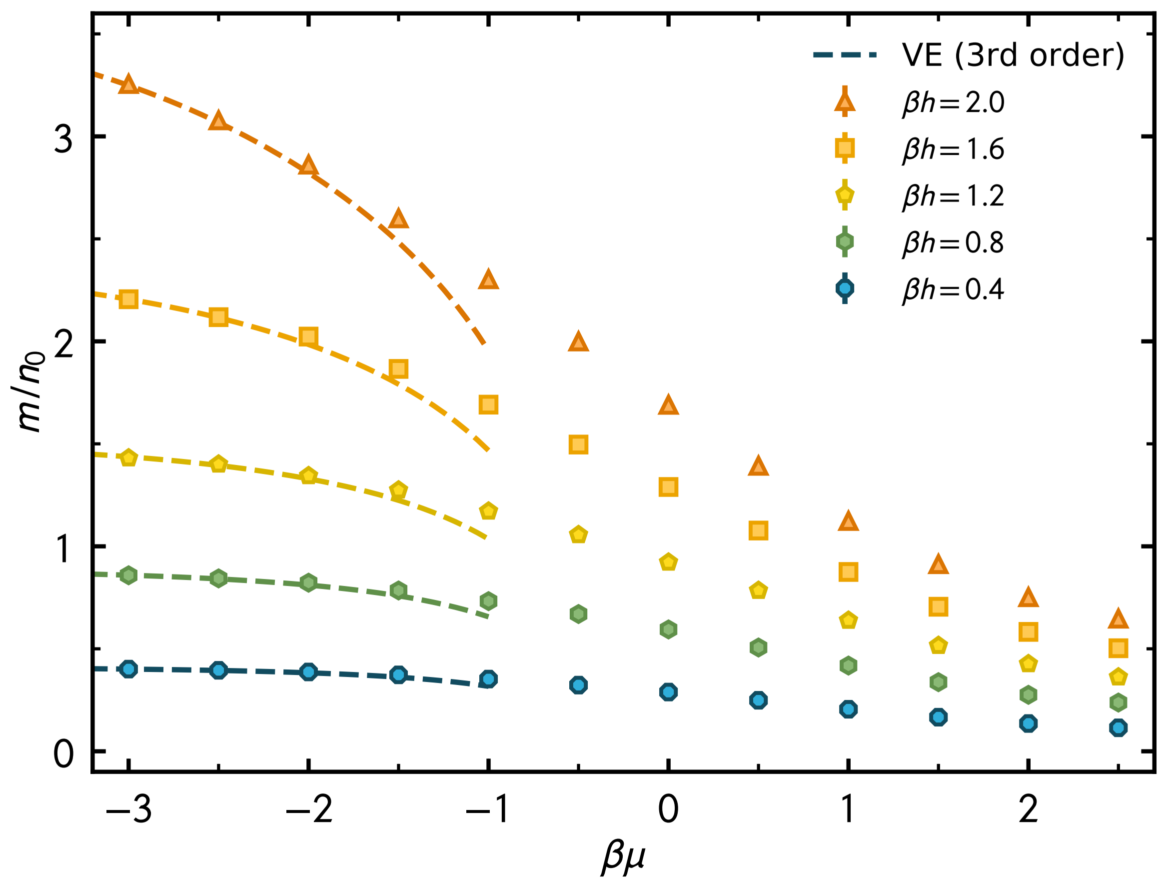
- excellent agreement with virial expansion **for all polarizations**
  - **experimentally testable** prediction

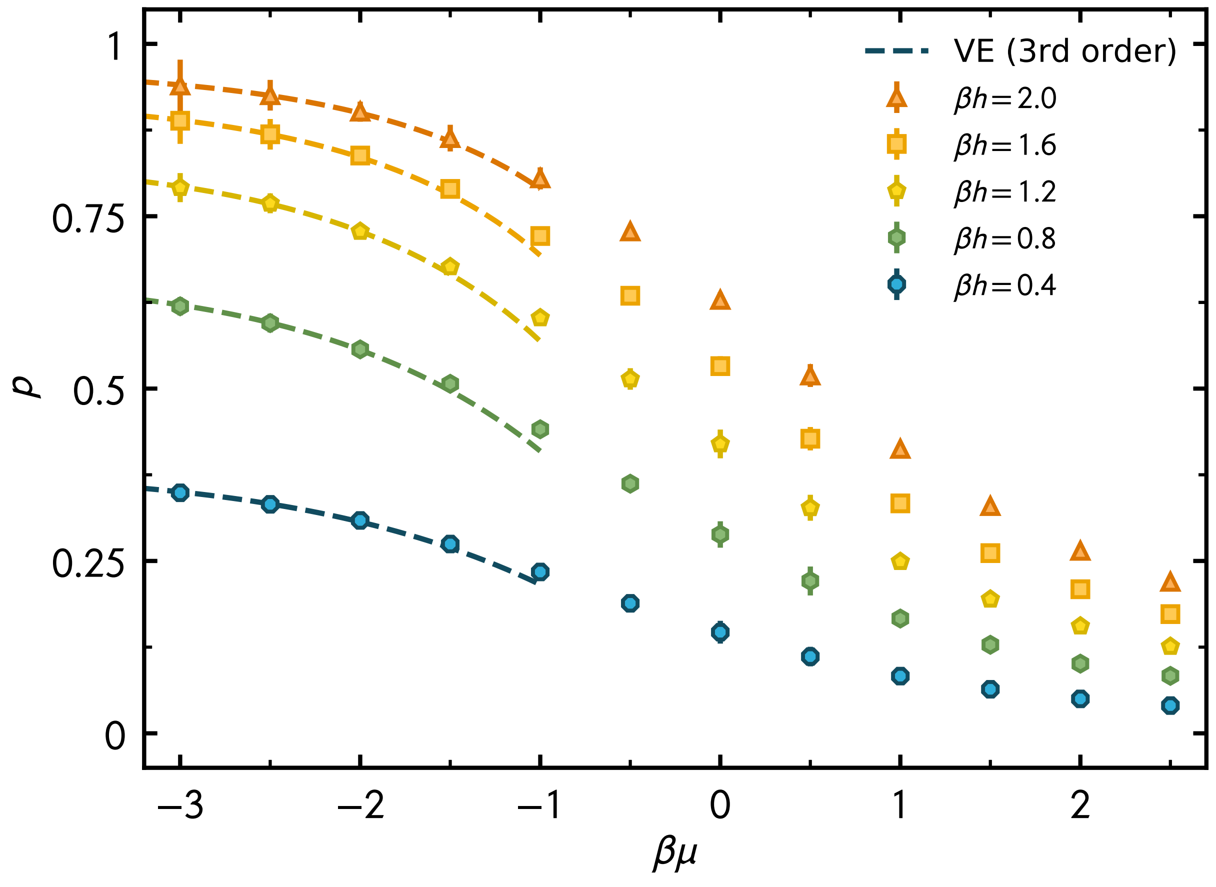
magnetization & polarization

[LR, Loheac, Drut, Braun '18]

$$m = n_\uparrow - n_\downarrow = \frac{\partial \ln \mathcal{Z}}{\partial(\beta h)}$$

$$p = \frac{n_\uparrow - n_\downarrow}{n_\uparrow + n_\downarrow} = \frac{m}{n}$$





textbook thermodynamics

$$n = \frac{\partial \ln \mathcal{Z}}{\partial(\beta\mu)}$$

$$m = \frac{\partial \ln \mathcal{Z}}{\partial(\beta h)}$$

pressure & energy

$$P(\beta\mu) = \frac{1}{\beta}\int_{-\infty}^{\beta\mu} n(x)\, dx$$

$$E = \frac{3}{2}PV$$

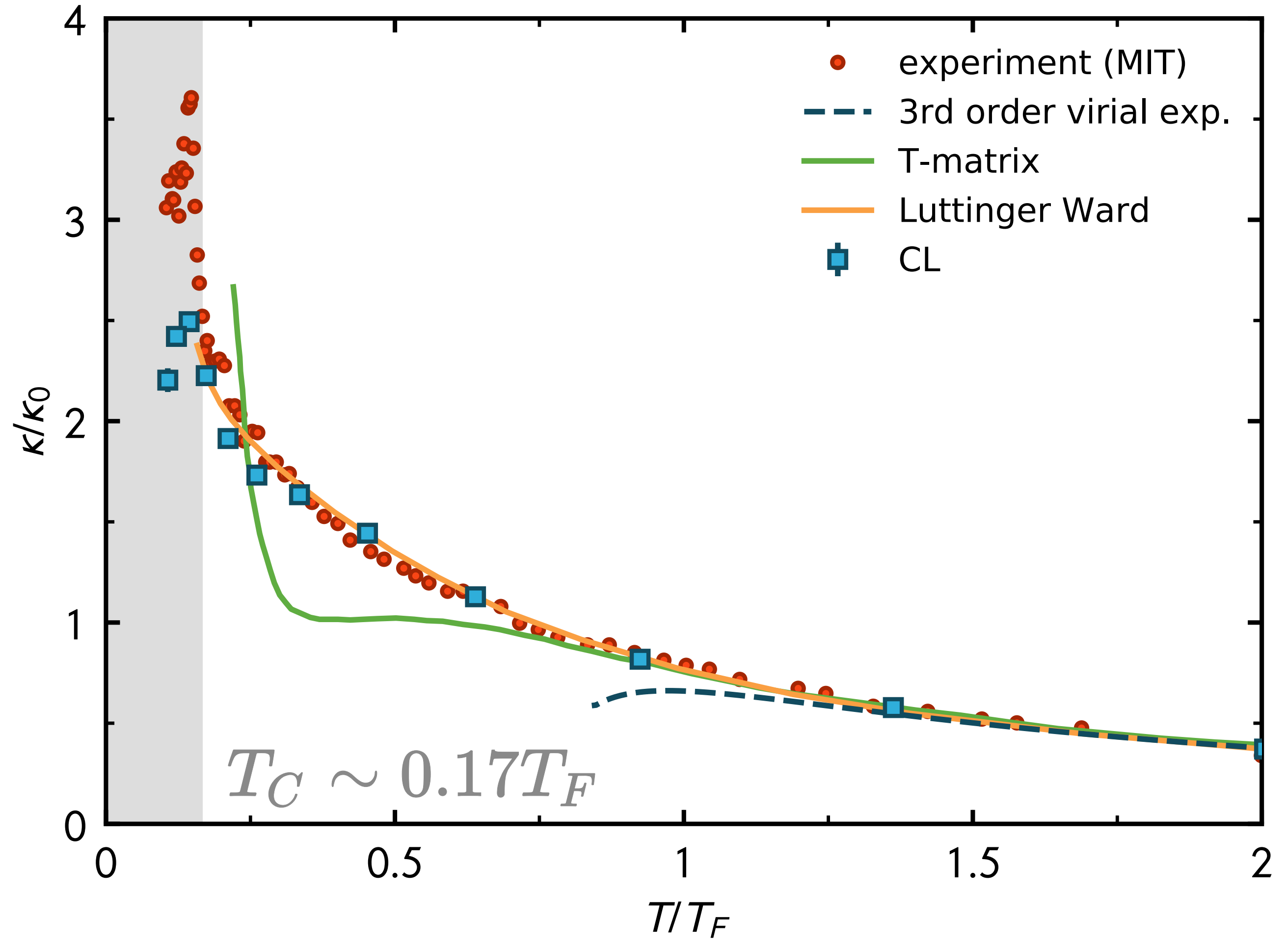
thermodynamic response

$$\kappa = \frac{1}{n}\left(\frac{\partial n}{\partial P}\right)_{T,V,h}$$

$$\chi = \left(\frac{\partial m}{\partial h}\right)_{T,V,\mu}$$

compressibility (unpolarized)

[LR, Loheac, Drut, Braun '18]



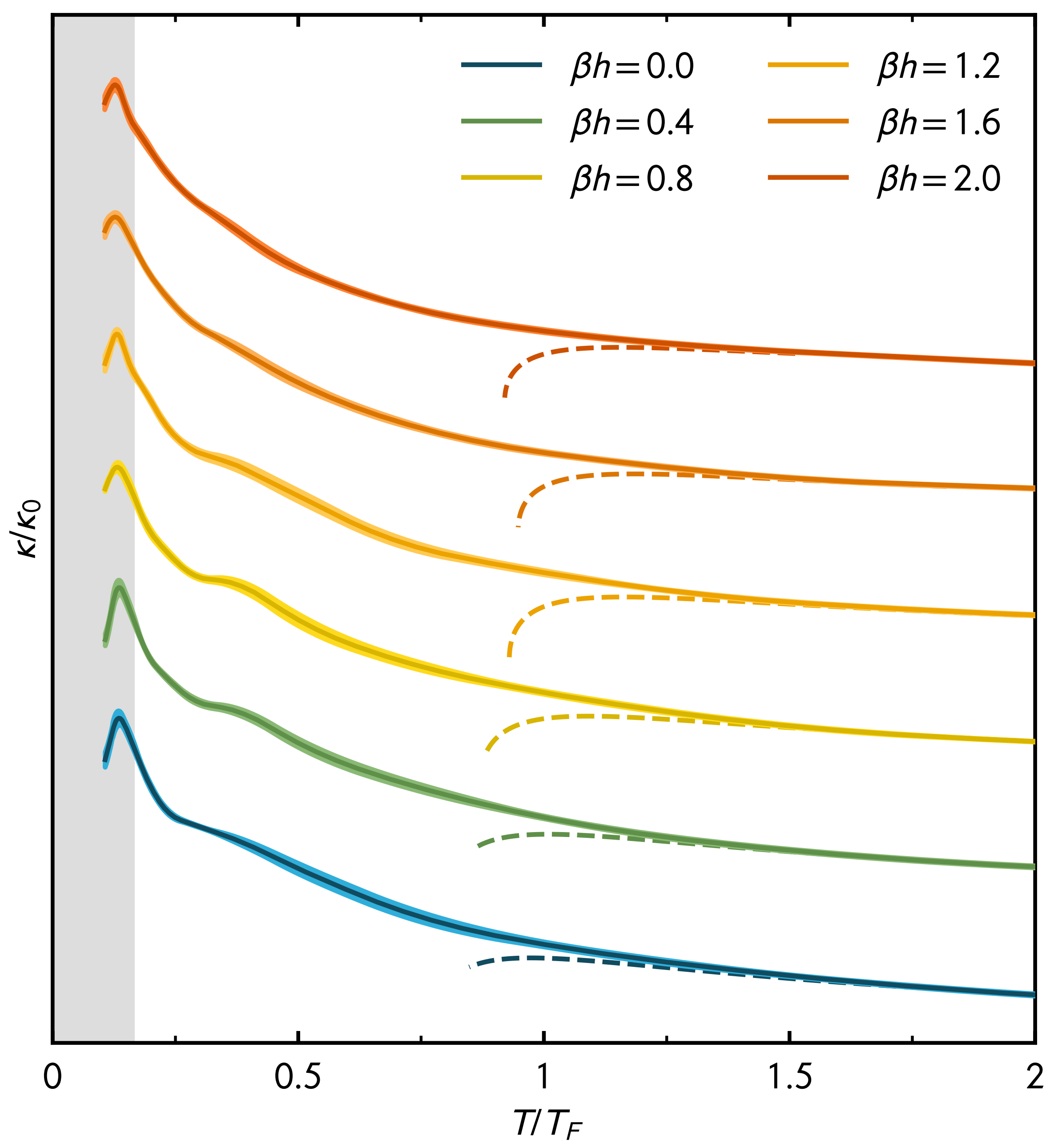
[experiment: Ku,Sommer,Cheuck,Zwierlein '12]
[Luttinger-Ward: Enns,Haussmann '12]
[T-matrix: Pantel et al. '14]

$$\kappa = \frac{1}{n}\left(\frac{\partial n}{\partial P}\right)_{T,V,h} = \frac{1}{n^2}\left(\frac{\partial n}{\partial \mu}\right)_{T,V,h}$$

- sudden increase of κ **indicates superfluid phase transition**
- features of curve recovered with CL
- quantitative disagreement at low temperatures

compressibility for polarized systems

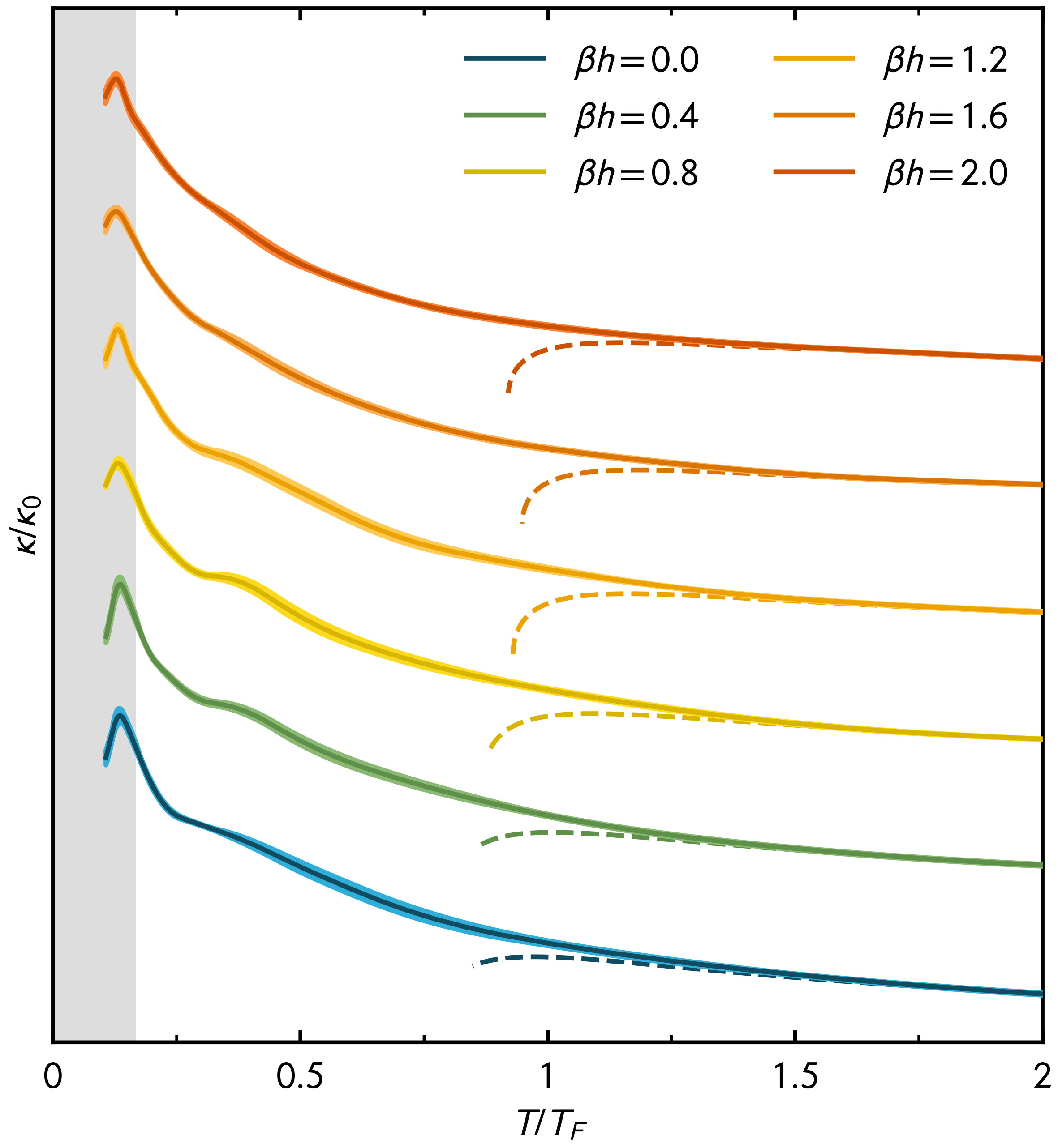
[LR, Loheac, Drut, Braun '18]



- **weak dependence of the critical temperature on polarization indicated**
- challenging to extract precise T_C

compressibility for polarized systems

[LR, Loheac, Drut, Braun '18]



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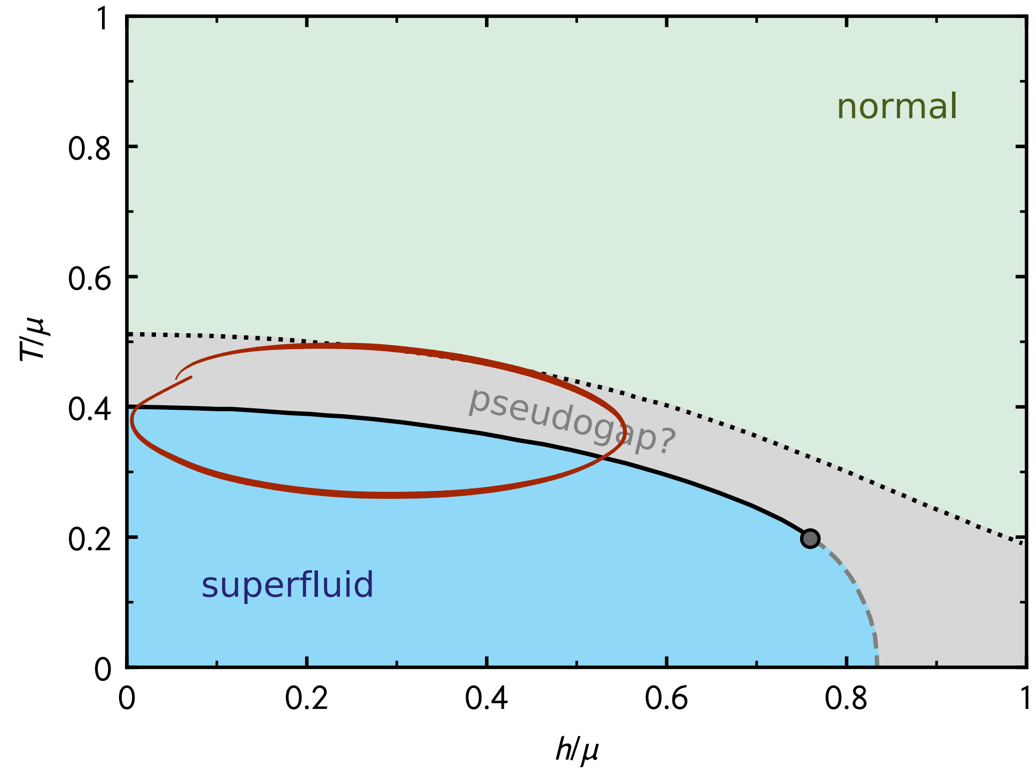
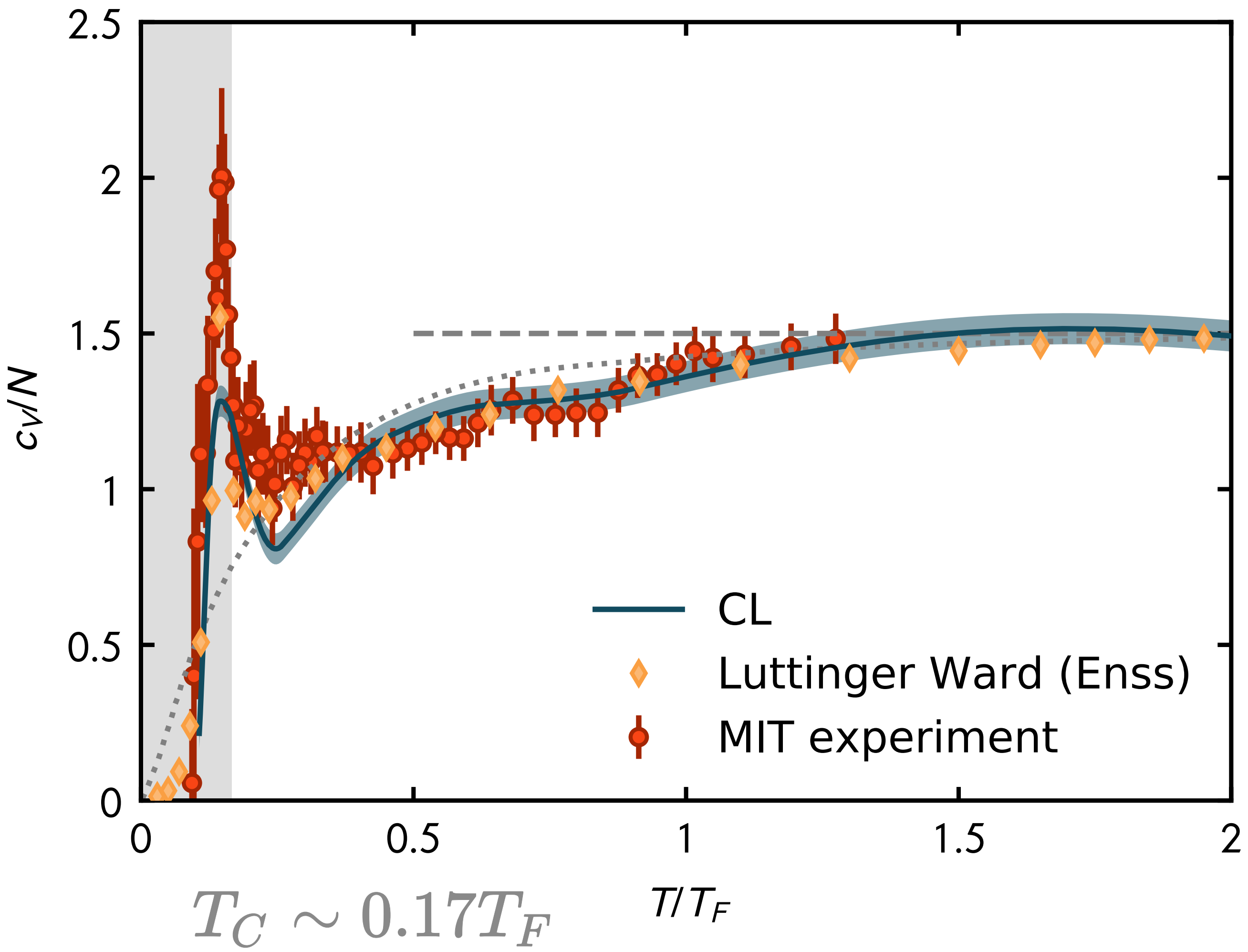


[fRG PD: Boettcher et. al '15]

specific heat (unpolarized)

[LR, Drut, Braun in preparation]



$T_C \sim 0.17 T_F$

[experiment: Ku,Sommer,Cheuck,Zwierlein '12]
[Luttinger-Ward: Enns,Haussmann '12]

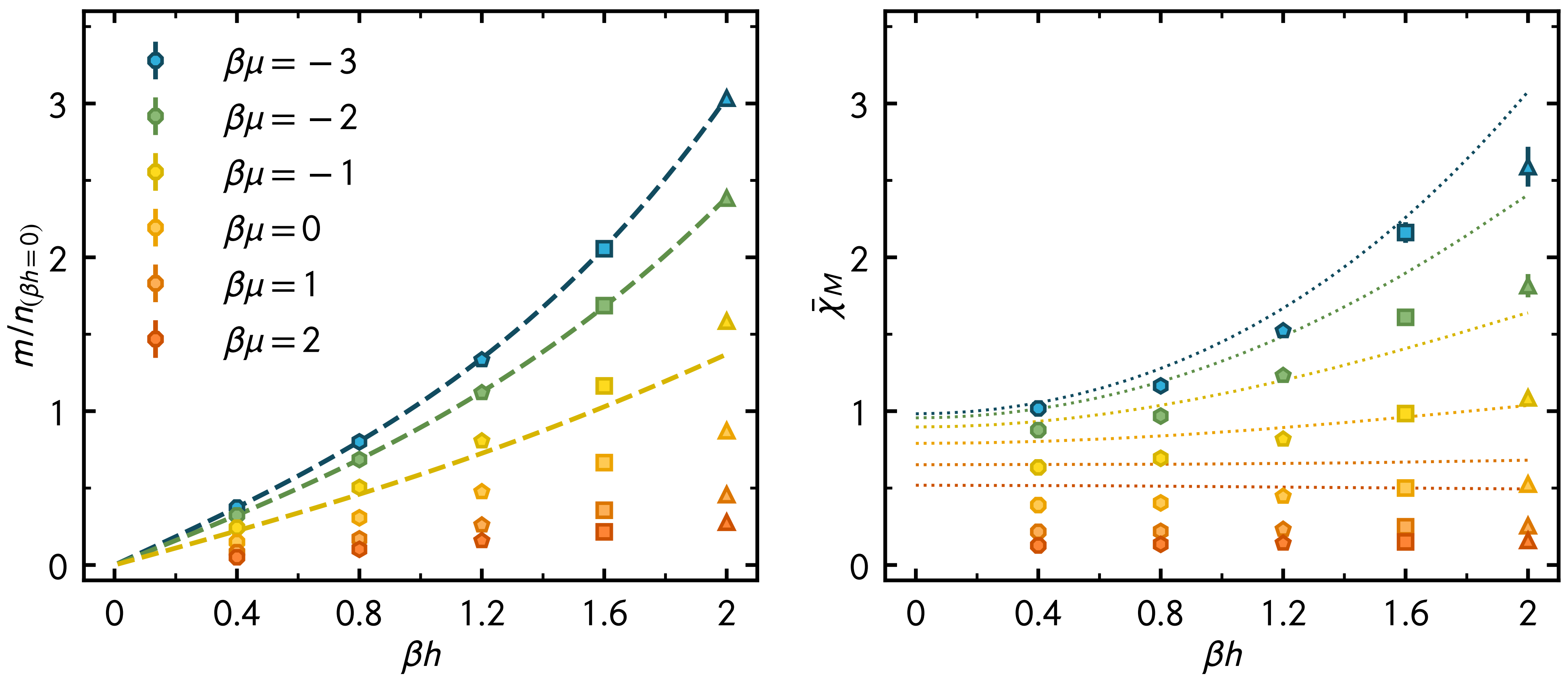
$$\frac{C_V}{N} = \frac{1}{N}\left(\frac{\partial E}{\partial T}\right)_{N,V}$$

- high temperatures: Dulong-Petit law $C_V/N = \frac{3}{2}$
- second order transition at at T_C ("lambda transition")
- **preliminary data (polarized values not yet available)**

spin susceptibility

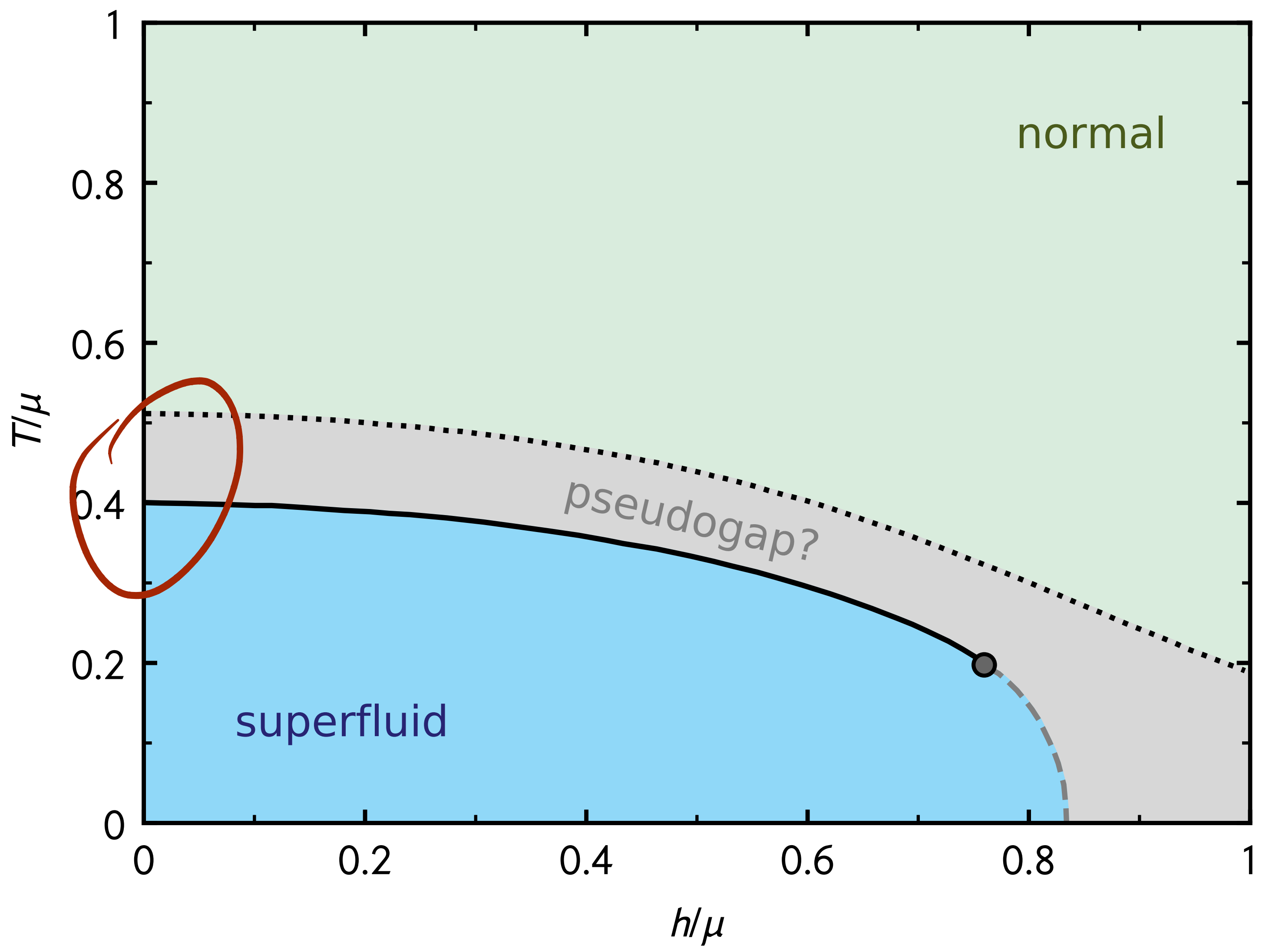
[LR, Loheac, Drut, Braun '18]

$$\chi = \left(\frac{\partial m}{\partial h}\right)_{T,V,\mu}$$



- Pauli susceptibility field independent at low field and temperature
    - **UFG: dependence on βh very similar to FG, but rescaled**

UFG phase diagram (sketch)

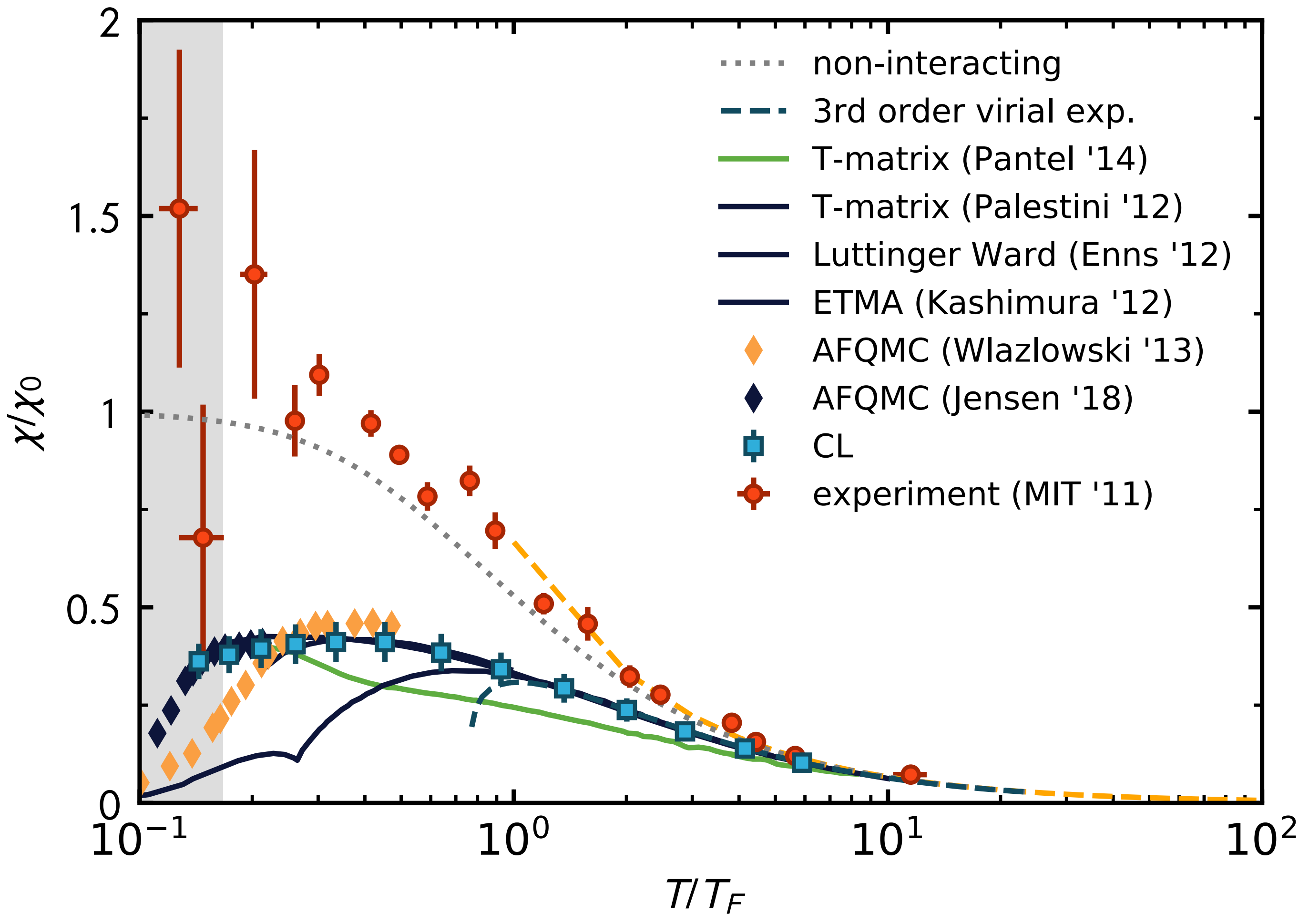


$$\mu = \frac{\mu_\uparrow + \mu_\downarrow}{2}$$
$$h = \frac{\mu_\uparrow - \mu_\downarrow}{2}$$

[fRG PD: Boettcher et. al '15]

magnetic susceptibility (unpolarized)

[LR, Loheac, Drut, Braun in preparation]



[recent review: Jensen et al. '18]

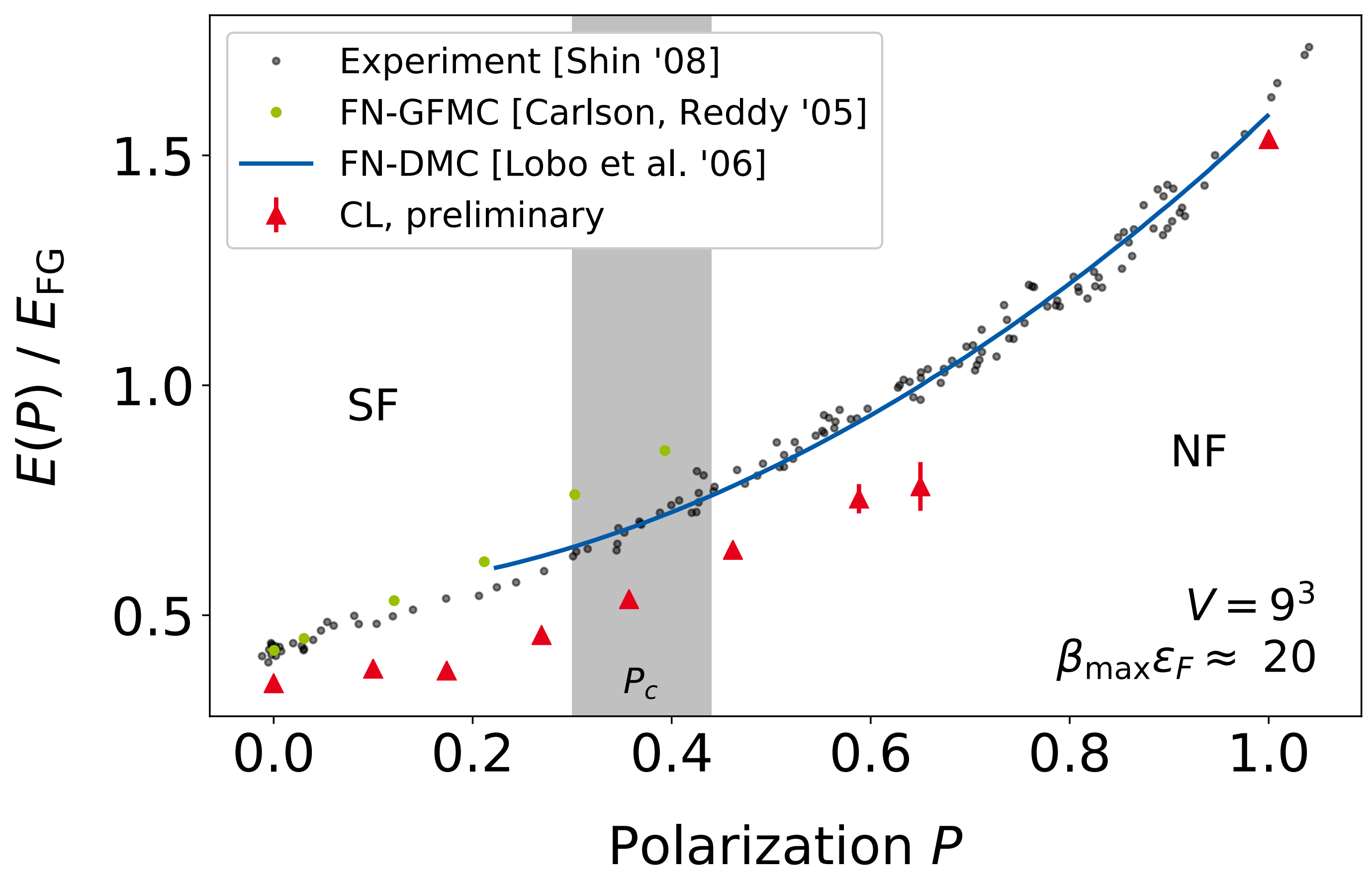
- high temperature: Curie's law $\chi \propto T^{-1}$
- theory & experiment agree at high temperatures
- Pseudogap: suppression of χ at $T > T_C$
- **low temperature: discrepancy between experiment and theory**
- **CL: pseudogap possible T^* and T_C seem to be very close**

recap: unitary fermions

- **EOS, magnetic properties & response** accessible for the unitary Fermi gas at finite temperature and polarization
- CL **matches state-of-the art results** from other methods and experiments wherever available

preliminary: UFG in the ground state

[Ehmann, LR, Drut, Braun in preparation]

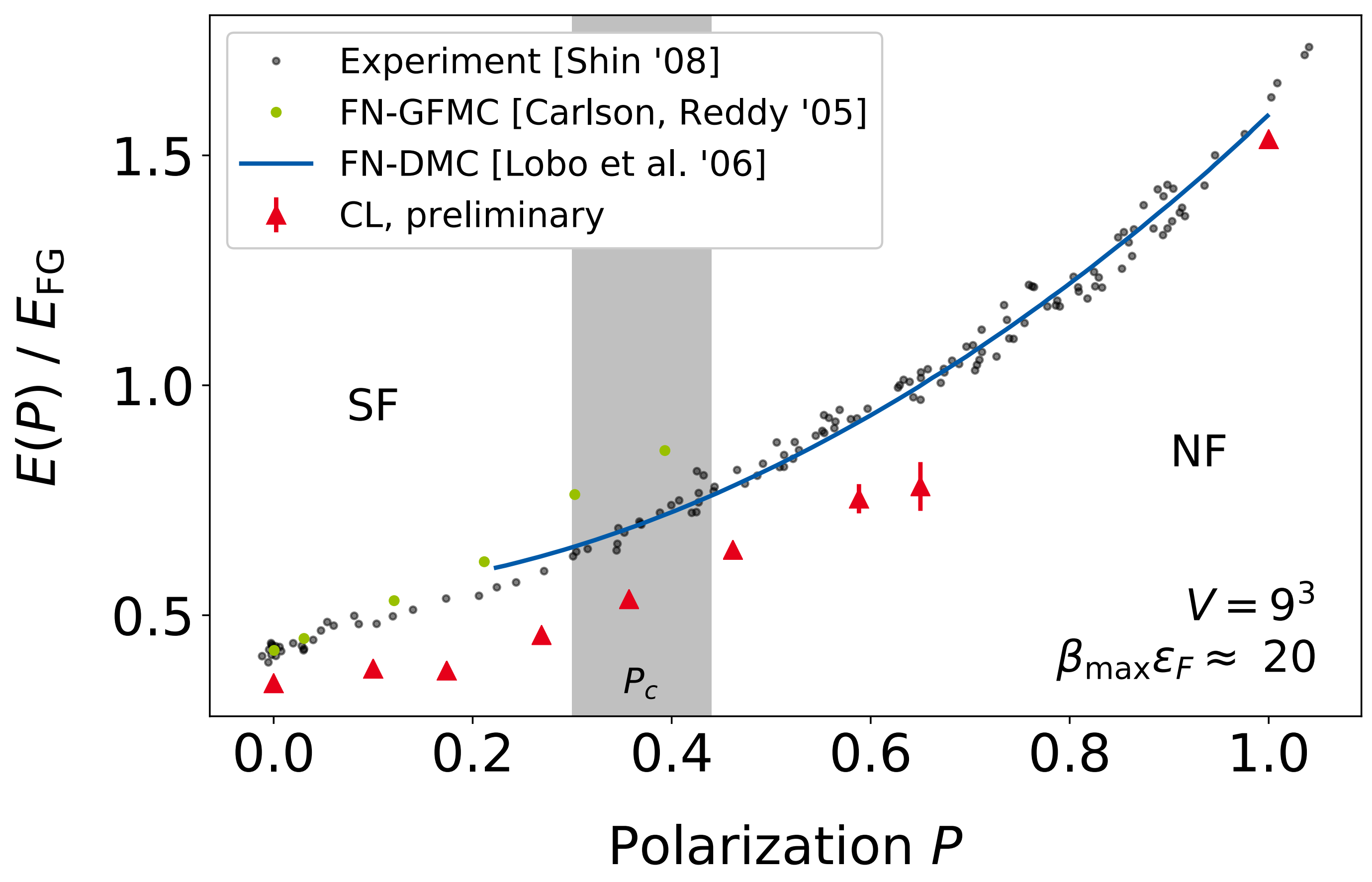


[experiment: Shin '08]
[FN_GFMC: Carlson,Reddy '05]
[FN-DMC: Lobo et al. '06]

- qualitative agreement of preliminary CL data with other MC values & experiment
- **CL does not require an ansatz**
- no continuum extrapolations as of yet
- Outlook: **FFLO ground state** near critical polarization?

preliminary: UFG in the ground state

[Ehmann, LR, Drut, Braun in preparation]



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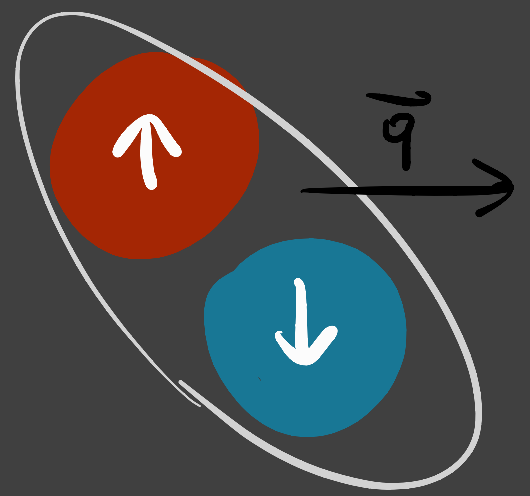
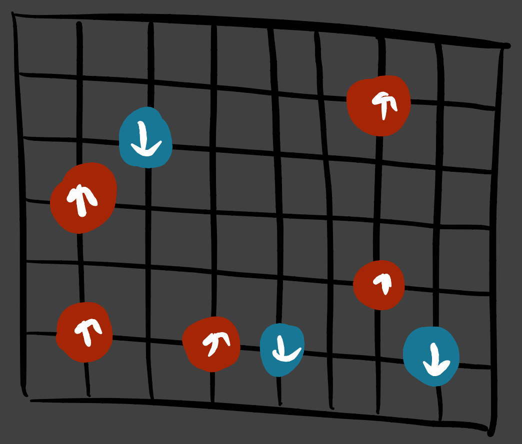
Check out the poster of Florian Ehmann for more details!

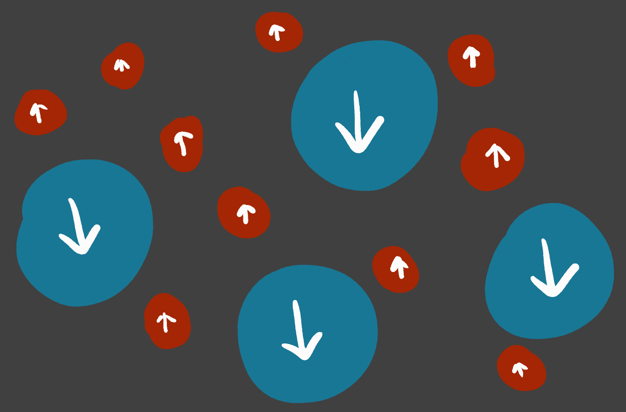
STAY TUNED!



looking for inhomogeneous phases in the UFG
[ongoing work with Florian Ehmann, Joaquin Drut & Jens Braun]



thermodynamics of 2D fermions at finite polarization
[ongoing work with Josh McKenney, Andrew Loheac, Joaquin Drut & Jens Braun]



effect of mass-imbalance on fermion pair formation

team CL

Jens Braun

Florian Ehmann

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Joaquin Drut

Andrew Loheac

Josh McKenney

Casey Berger

UNC Chapel Hill