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Towards in-medium heavy quarkonium dynamics from first principles

Alexander Rothkopf
Faculty of Science and Technology
Department of Mathematics and Physics
University of Stavanger

References:

T. Miura, Y. Akamatsu, M. Asakawa, A.R. arXiv:1908.06293
P. Petreczky, A.R., J. Weber, NPA982 (2019) 735
S. Kim, P. Petreczky, A.R. JHEP 1811 (2018) 088
B. Krouppa, A.R., M. Strickland PRD97 (2018) 016017
D. Lafferty, A.R., arXiv:1906.00035, A. Lehmann, A.R. (in preparation)

isoQuant SFB1225

C05 Theory

Impurity physics of the early universe

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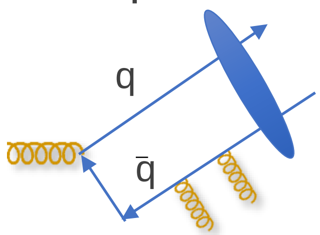
- Hard probe: susceptible to medium but distinguishable from it $Q_{probe} > T_{med}$



Quarkonium separation of scales: $M_Q > T_{med}$, $M_Q > \Lambda_{QCD}$

In vacuum: m^{Υ}=9.460GeV, Γ^{Υ} = 54(1)keV; $m^{J/\psi}$=3.096 GeV, $\Gamma^{J/\psi}$ = 93(3)keV

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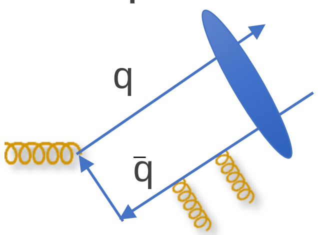

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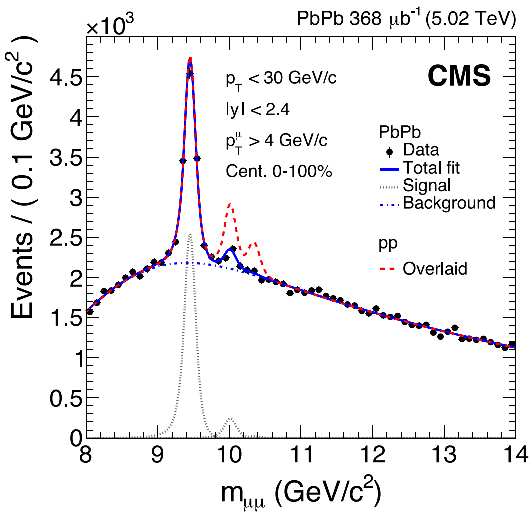

[CMS collaboration] PRL120 (2018) 142301

Bottomonium: a non-equilibrium probe of the full QGP evolution

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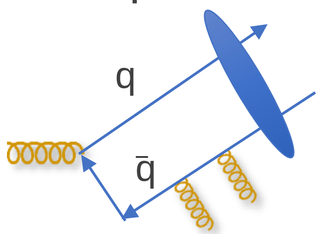

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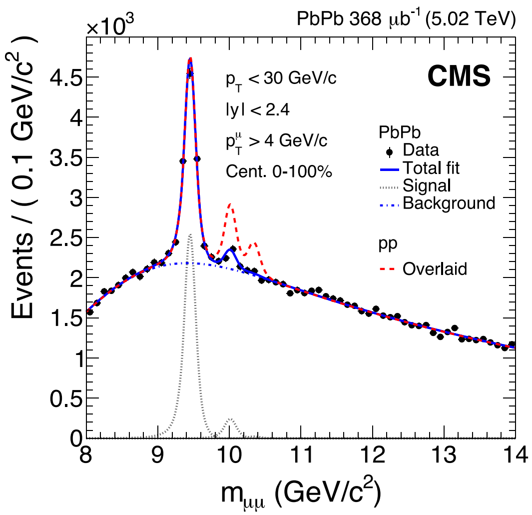

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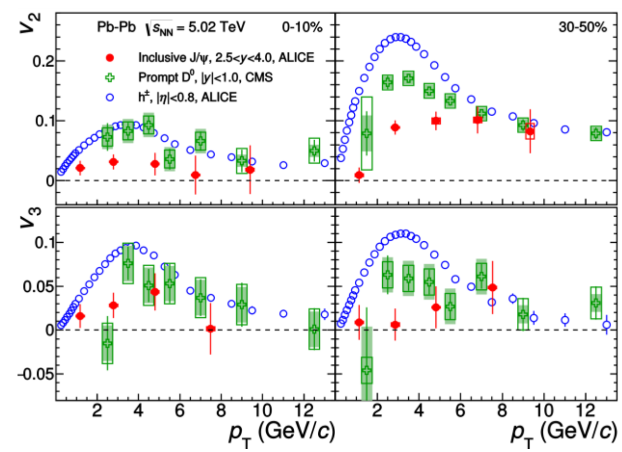

[ALICE collaboration]
JHEP 1902 (2019) 012

Charmonium: a partially equilibrated probe, sensitive to the late stages

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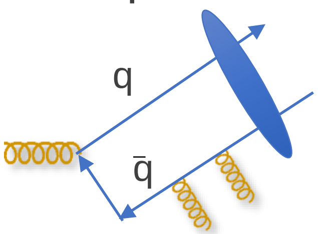

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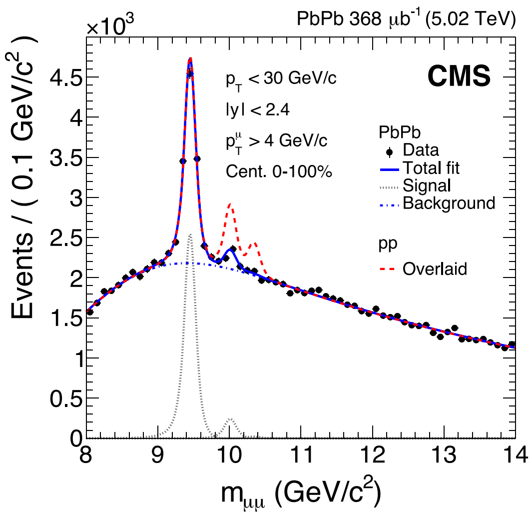

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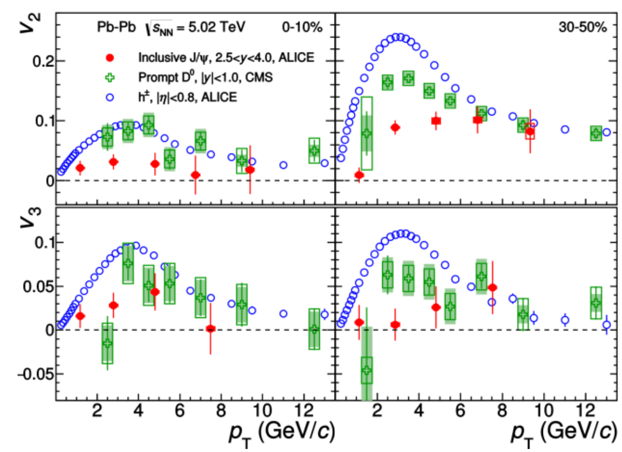

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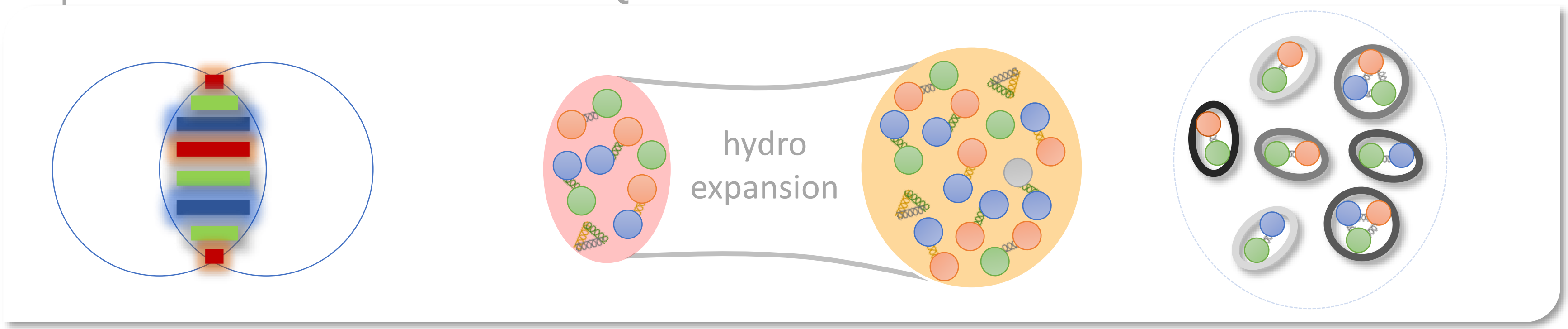
Charmonium: a partially equilibrated probe, sensitive to the late stages

- Goal: provide **first principles interpretation** to intricate phenomenology

Open theory questions

bulk: pre-thermalization Quark-Gluon-Plasma hadronization



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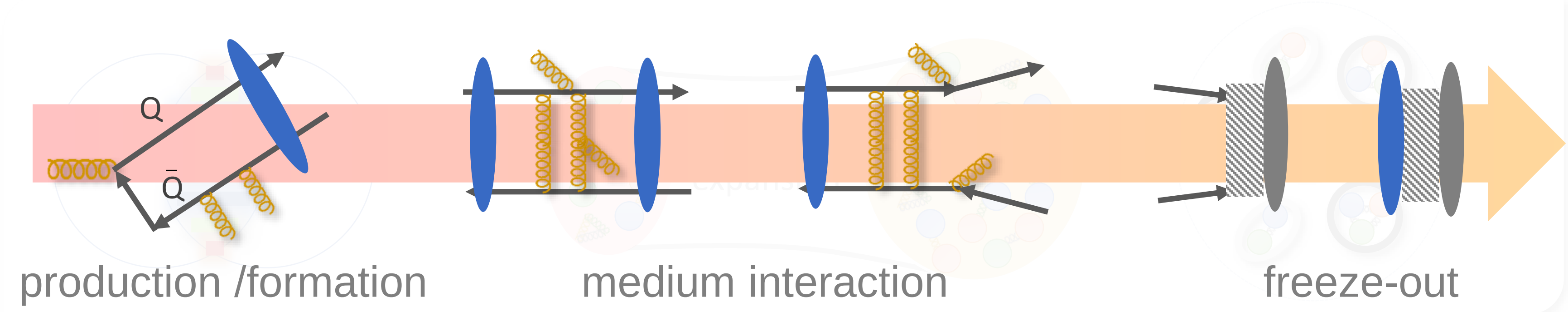

$Q\bar{Q}$: production /formation medium interaction freeze-out

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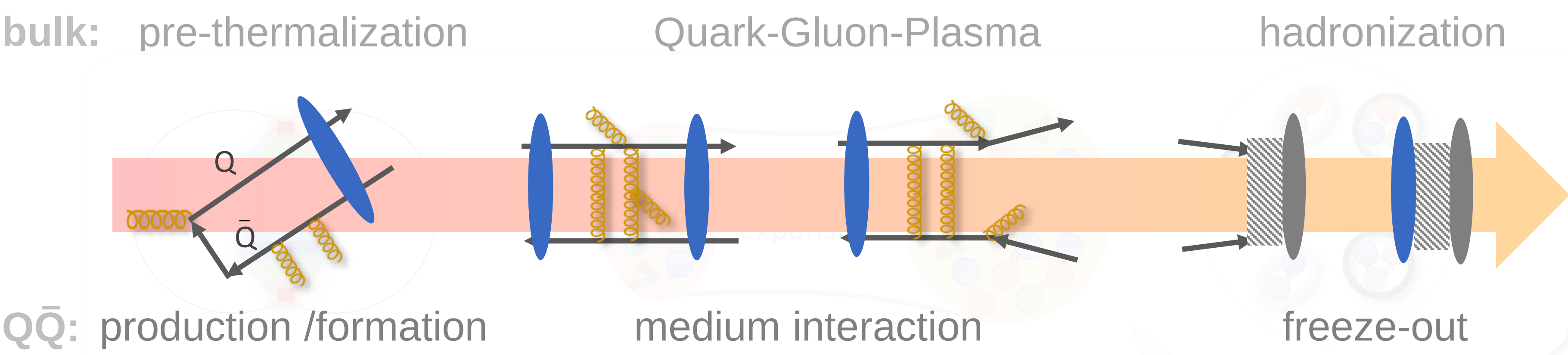

Properties of equilibrium $Q\bar{Q}$

First principles extraction of **the heavy quark potential**

Novel phenomenological definition of the **Debye mass**

Extraction of **thermal spectral properties** on the lattice

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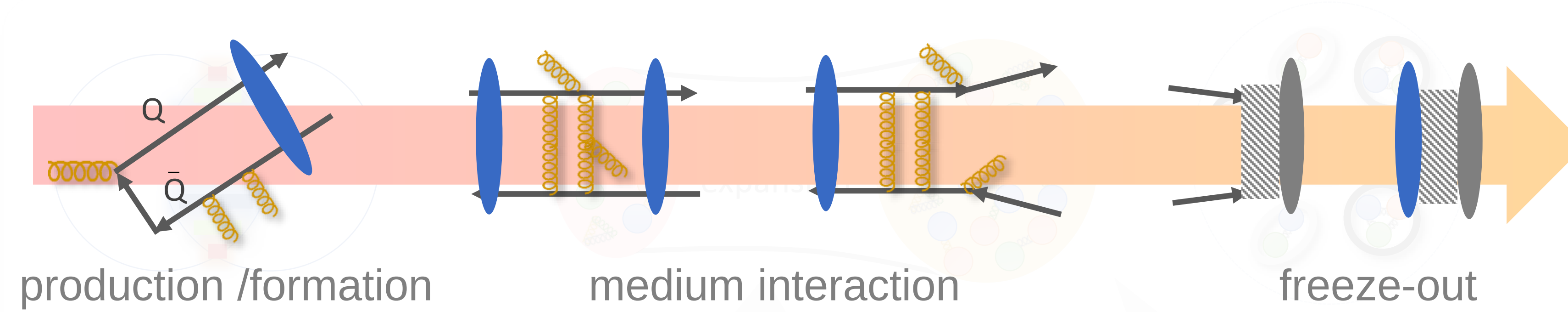

QQ̄: production /formation medium interaction freeze-out

Real-time QQ̄ evol. in local thermal equilibrium

Beyond Schrödinger:

Open-quantum-systems descr. of real-time evolution

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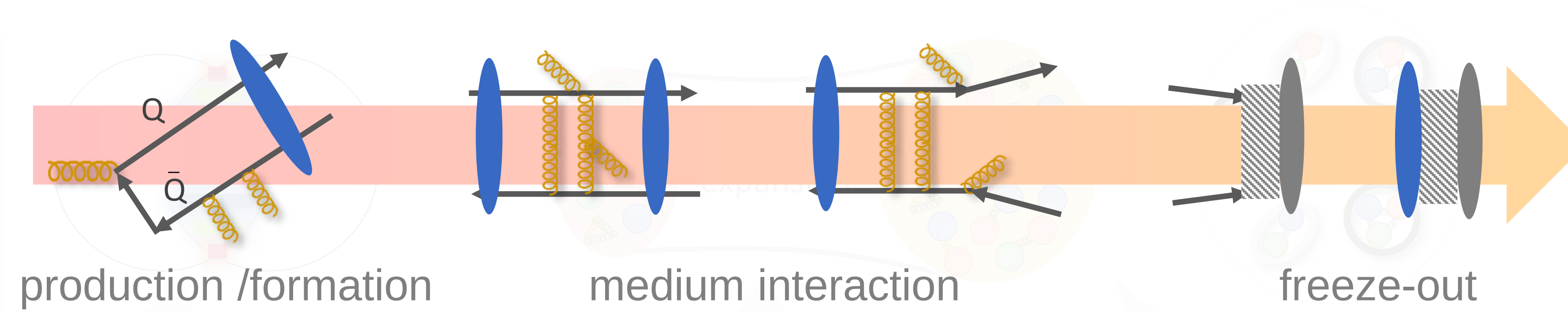

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QQ̄ realtime evol. in the initial stages

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classical statistical simulations for gluons & real-time NRQCD

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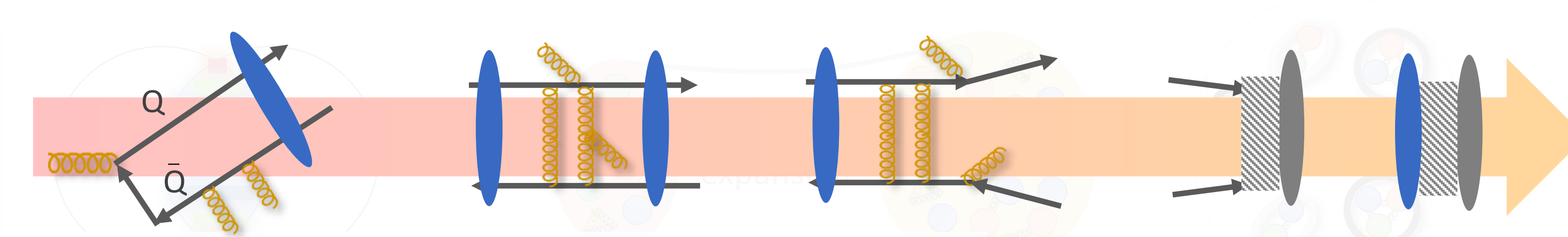

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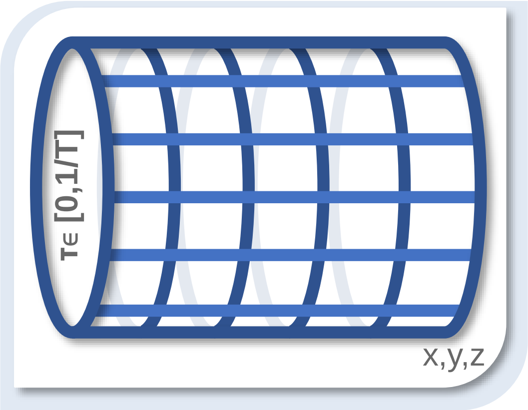
A robust tool: lattice QCD

- Non-perturbative 1st principles approach to Quantum Chromo Dynamics



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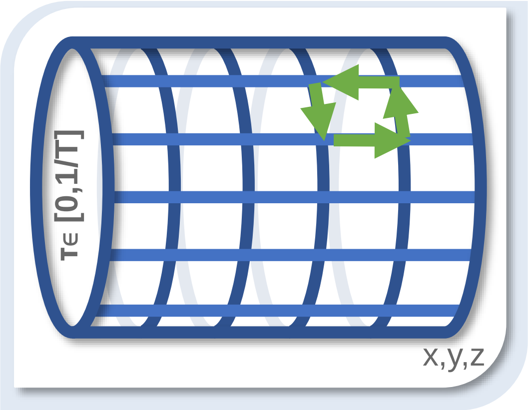

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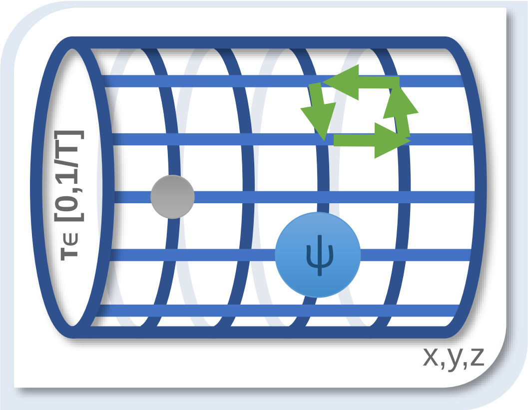

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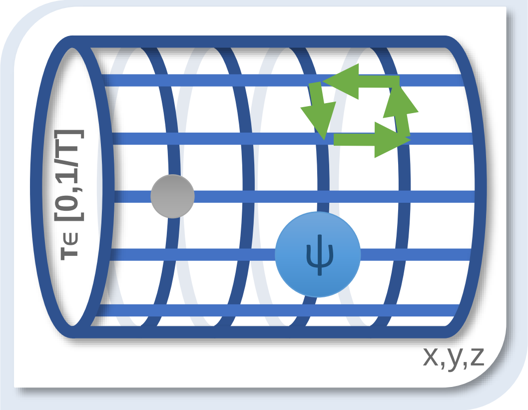

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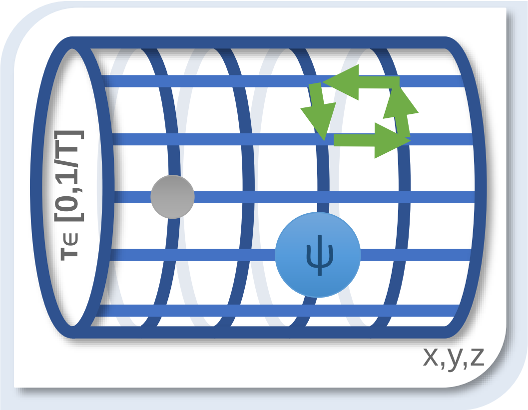

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$$\langle O(U)\rangle = \int \mathcal{D}U\, O(U)\, e^{-S_E^{QCD}[U]}$$

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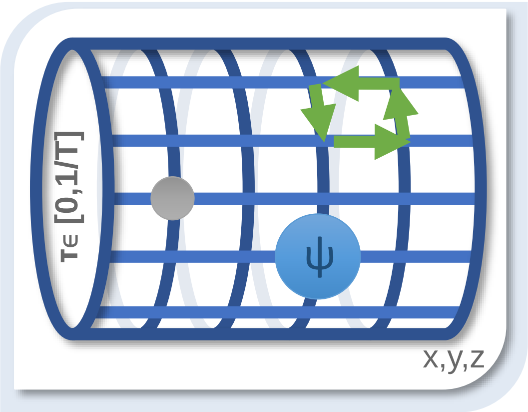

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$$\langle O \rangle = \frac{1}{N} \lim_{N\to\infty} \sum_{k=1}^{N} O(U^k) \qquad P[U] \propto e^{-S_E[U,\psi,\bar{\psi}]}$$

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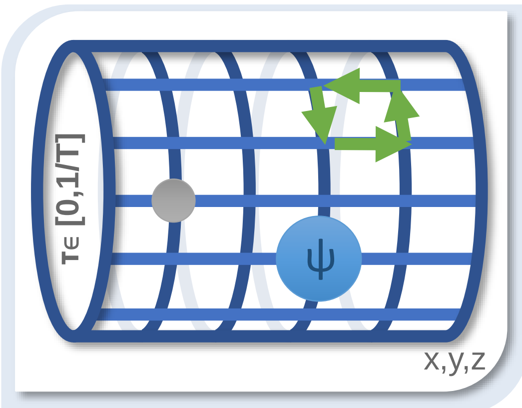

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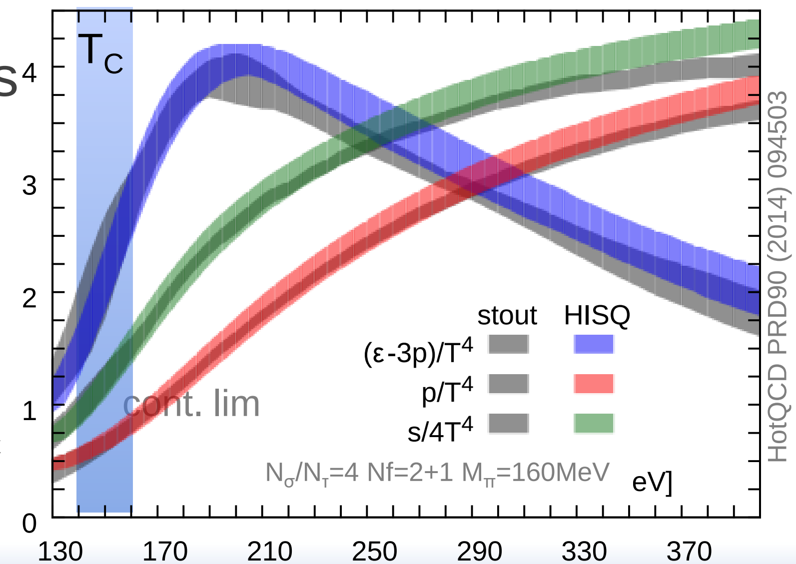
- Successful at T>0: **static** QCD properties
  - (Pseudo)critical temperature: 154±9 MeV
    WB JHEP 1009 (2010) 073 - HotQCD PRD85 (2012) 054503
  - Equation of state as input for hydro-dynamics
  - Trace anomaly $T^{\mu\mu}$ = ε-3p strong coupling at T_C
    HotQCD PRD90 (2014) 094503 - WB PLB730 (2014) 99-104



Quarkonium in thermal equilibrium

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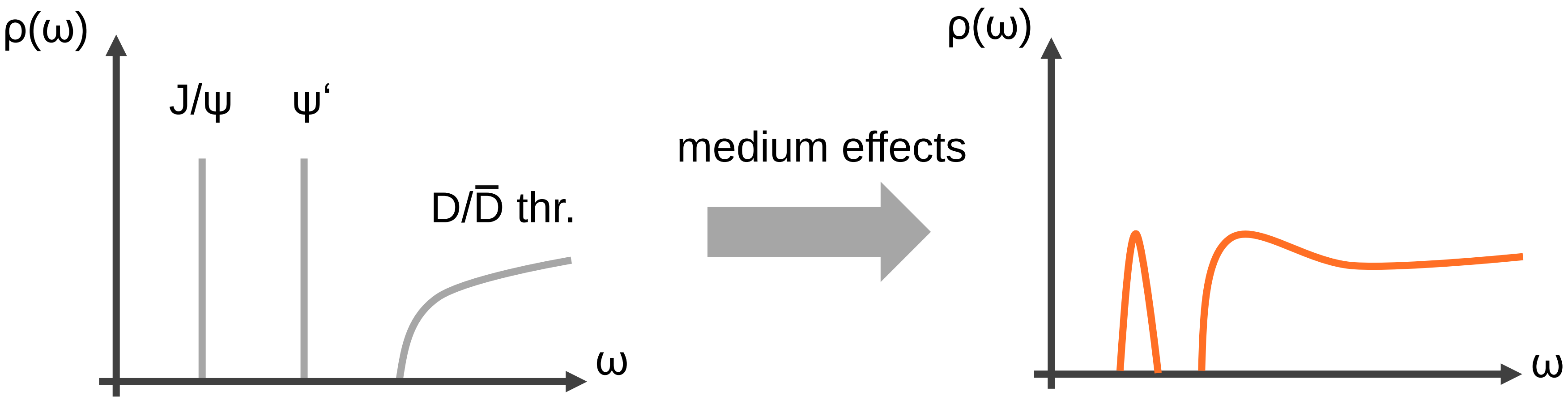
- **Intuition** on in-medium modification from **potential based** studies



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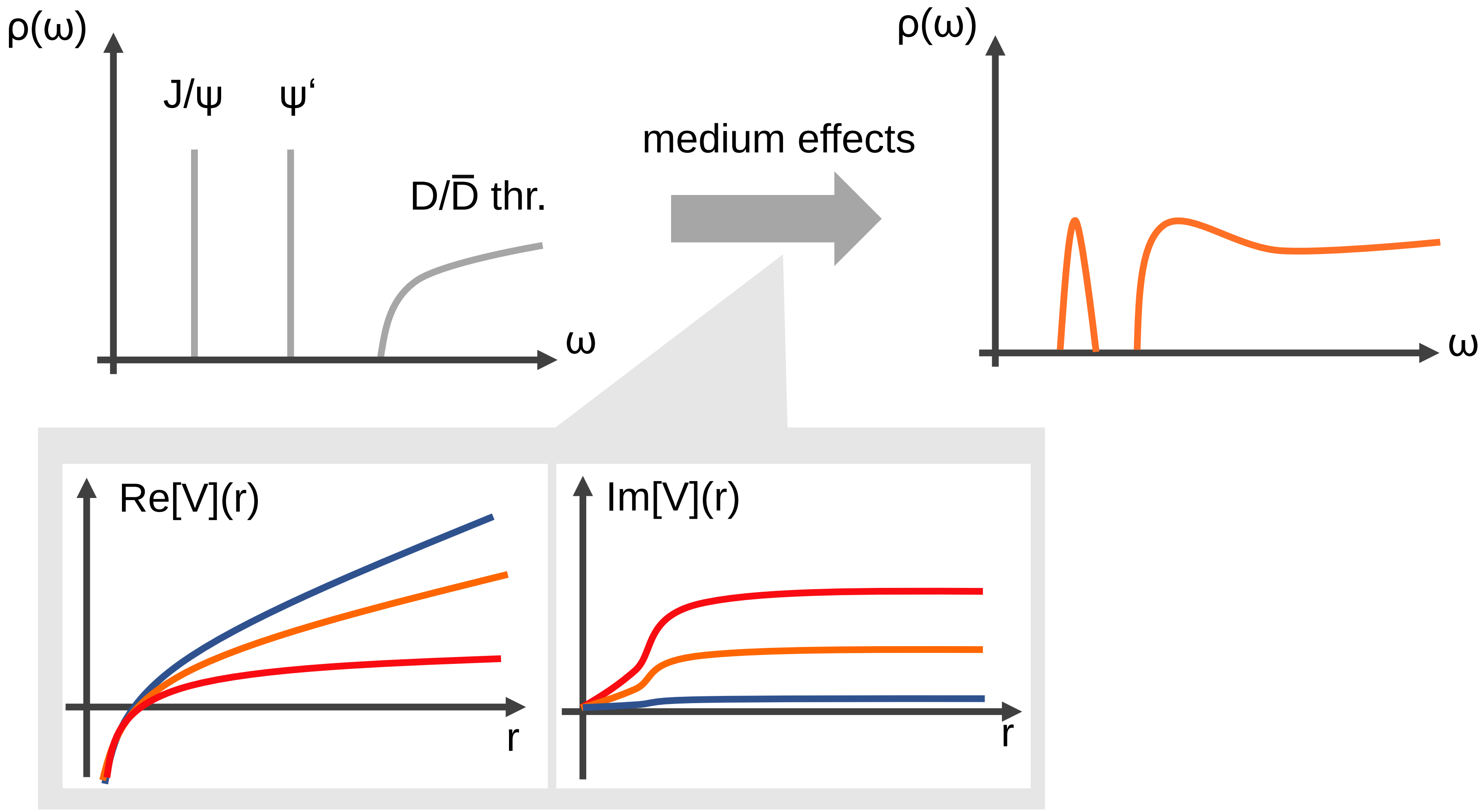

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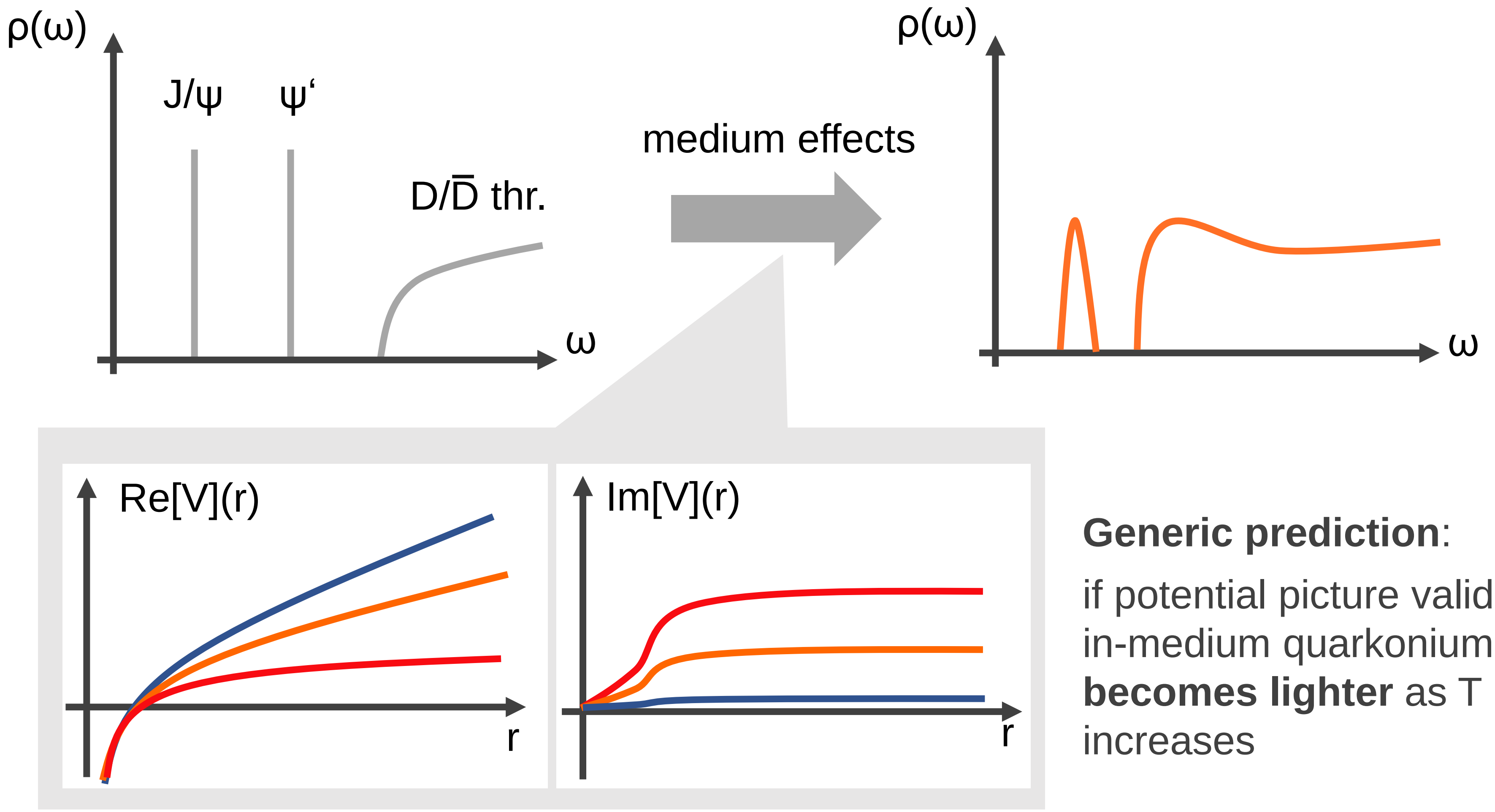

Generic prediction:

if potential picture valid, in-medium quarkonium **becomes lighter** as T increases

The QCD real-time interquark potential

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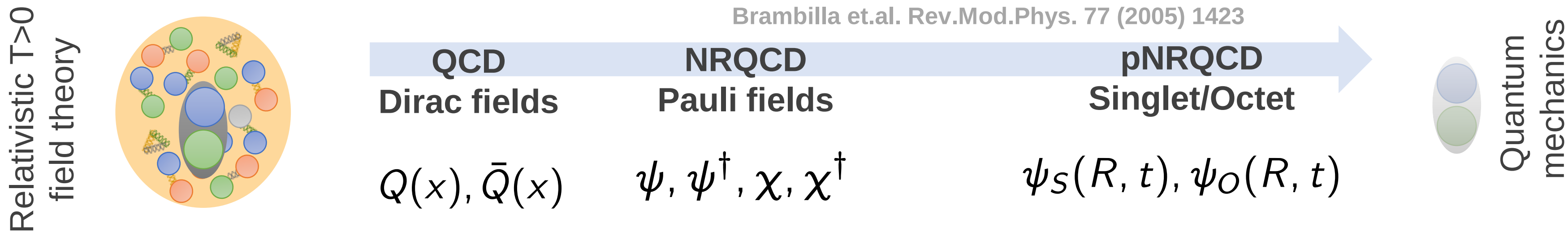
- Exploit $\frac{T}{m_Q} \ll 1, \frac{\Lambda_{QCD}}{m_Q} \ll 1$ to treat heavy quarks non-relativistically



$Q(x), \bar{Q}(x)$ $\quad \psi, \psi^\dagger, \chi, \chi^\dagger$ $\quad \psi_S(R,t), \psi_O(R,t)$

$$i\partial_t \langle \psi_s(t)\psi_s(0)\rangle = \Big(V^{QCD}(R) + \mathcal{O}(m_Q^{-1}) + \Theta(R,t)\Big)\langle \psi_s(t)\psi_s(0)\rangle$$

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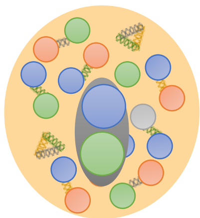

Brambilla et.al. Rev.Mod.Phys. 77 (2005) 1423

QCD	NRQCD	pNRQCD
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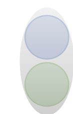

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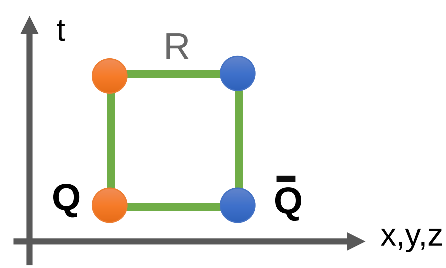
- Matching to underlying QCD in the infinite mass limit: Wilson loop

$$\langle \psi_S(R,t)\psi_S^*(R,0)\rangle_{\rm pNRQCD} \equiv W_\square(R,t) = \left\langle \mathrm{Tr}\left[\exp\left(-ig\int_\square dx^\mu A_\mu(x)\right)\right]\right\rangle_{\rm QCD}$$



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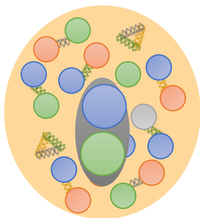

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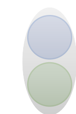

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- Wilson loop: potential emerges at late times

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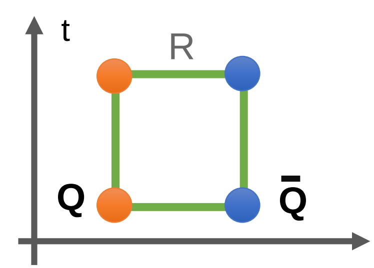

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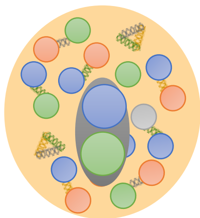

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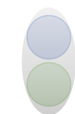

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Im[V]: Laine et al. JHEP03 (2007) 054;
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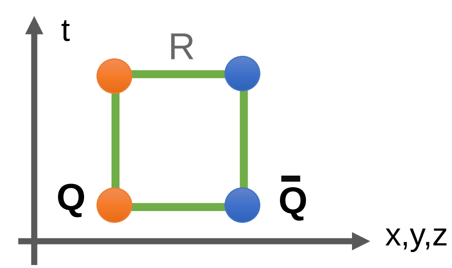

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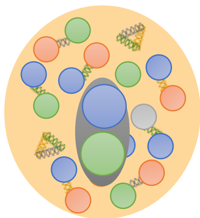

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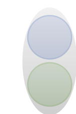
Dirac fields	Pauli fields	Singlet/Octet
$Q(x), \bar{Q}(x)$	$\psi, \psi^\dagger, \chi, \chi^\dagger$	$\psi_S(R,t), \psi_O(R,t)$



Quantum mechanics

$$i\partial_t \langle \psi_s(t)\psi_s(0)\rangle = \left(V^{\rm QCD}(R) + \mathcal{O}(m_Q^{-1}) + \Theta(R,t)\right)\langle \psi_s(t)\psi_s(0)\rangle$$

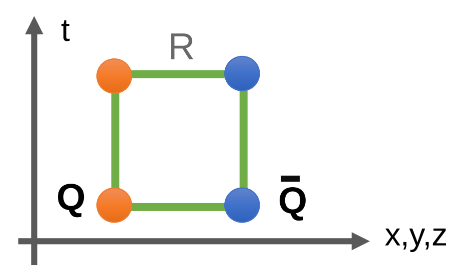
- Matching to underlying QCD in the infinite mass limit: Wilson loop

$$\langle \psi_S(R,t)\psi_S^*(R,0)\rangle_{\rm pNRQCD} \equiv W_\square(R,t) = \left\langle {\rm Tr}\left[\exp\left(-ig\int_\square dx^\mu A_\mu(x)\right)\right]\right\rangle_{\rm QCD}$$

- Wilson loop: potential emerges at late times

$$V(R) = \lim_{t\to\infty} \frac{i\partial_t W_\square(R,t)}{W_\square(R,t)} \in \mathbb{C}$$

Im[V]: Laine et al. JHEP03 (2007) 054;
Beraudo et. al. NPA 806:312,2008
Brambilla et.al. PRD 78 (2008) 014017



- In this form: Minkowski time quantities and not directly accessible on the lattice

Non-perturbative evaluation of V(R)

University of Stavanger

- How to connect to the Euclidean domain: **spectral functions**

A.R., T.Hatsuda & S.Sasaki
PRL 108 (2012) 162001

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C.Fischer, J. Pawlowski,
A.R., C. Welzbacher
PRD98 (2018) 014009

$$S_{BR}^{smooth}=\alpha\int d\omega\left(\kappa\left(\frac{\partial\rho}{\partial\omega}\right)^2+1-\frac{\rho}{m}+log\left[\frac{\rho}{m}\right]\right)$$

isoQuant SFB1225 result

Latest results on the lattice potential

University of Stavanger

- Lattices with dynamical u,d,s quarks (HISQ action, HotQCD & TUMQCD)

A. Bazavov et.al. PRD97 (2018) 014510, HotQCD PRD90 (2014) 094503

  - realistic m_π~161MeV (T=151-1451MeV)
  - fixed box (N_s=48 - **N_τ=12, N_τ=16**) & very **high statistics** 4000-9000 realizations
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P. Petreczky, A.R., J. Weber, NPA982 (2019) 735

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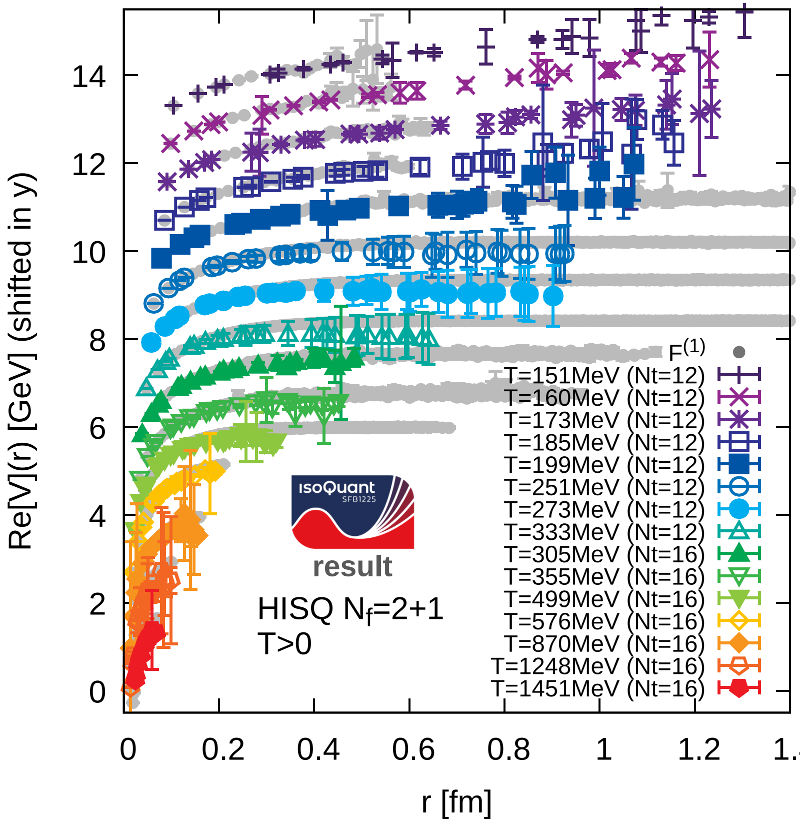

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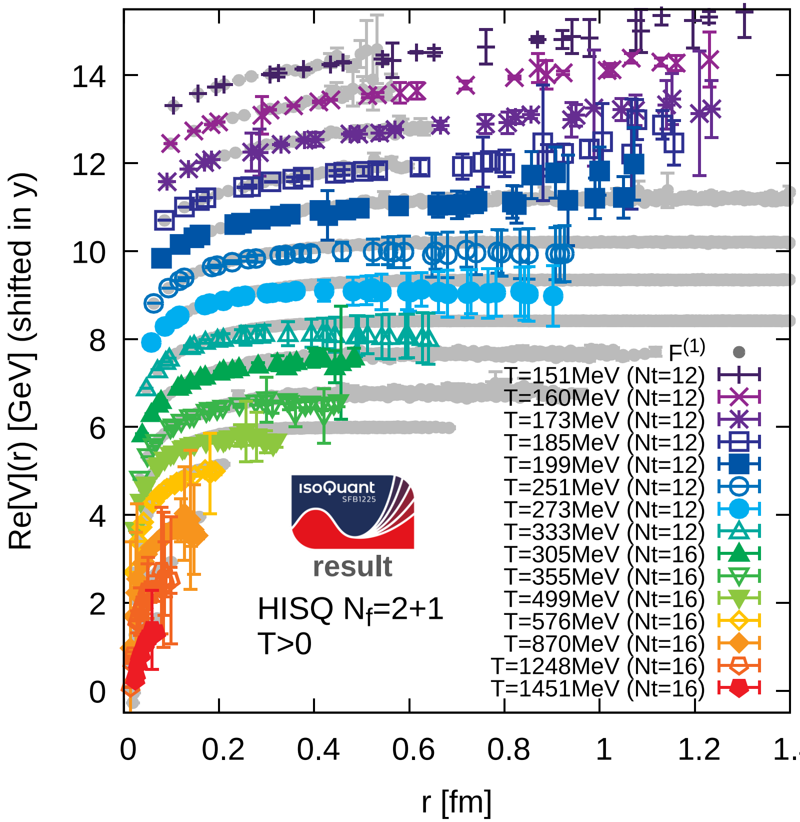

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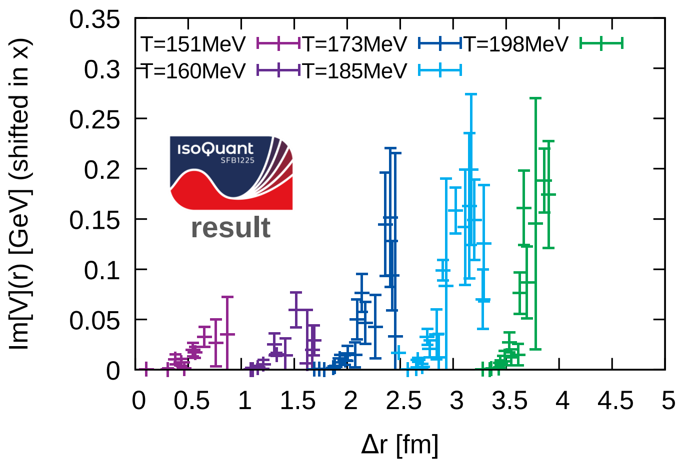
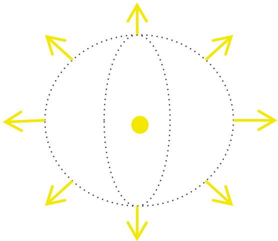


- Smooth transition from Cornell @ T=0 to Debye screened @ T>T_C
- Finite Im[V] above T_c present

An improved Gauss law approach

- For use in phenomenology applications: analytic expression for Re[V] and Im[V]



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Strategy:

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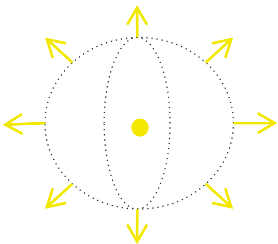

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V. V. Dixit,
Mod. Phys. Lett. A 5, 227 (1990)

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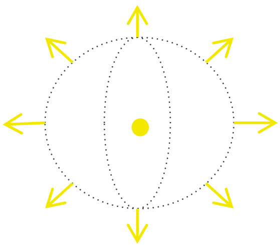

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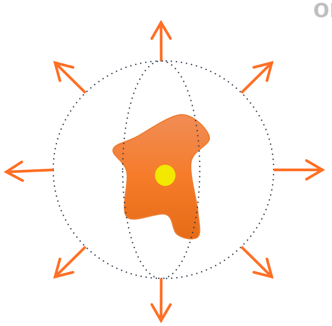
- Immerse non-perturbative charge in weak coupling HTL medium: permittivity ε

original idea: Y.Burnier, A.R. Phys.Lett. B753 (2016) 232 improved derivation D.Lafferty and A.R. arXiv:1906.00035



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University of Stavanger

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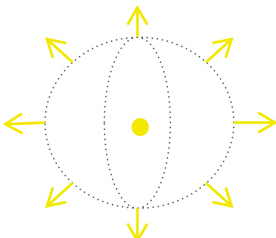

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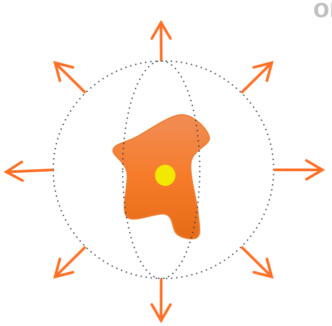

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isoQuant SFB1225 result

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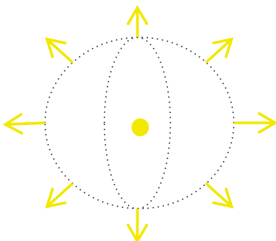

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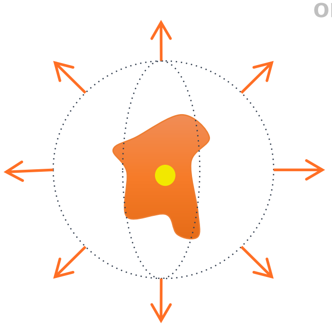

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IsoQuant SFB1225 result

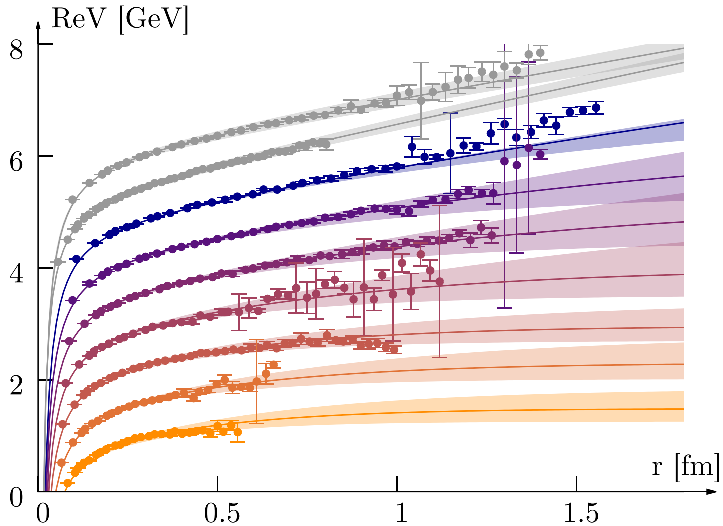
- 3 vacuum parameters and 1 temperature dependent m_D fix both Re[V] and Im[V].

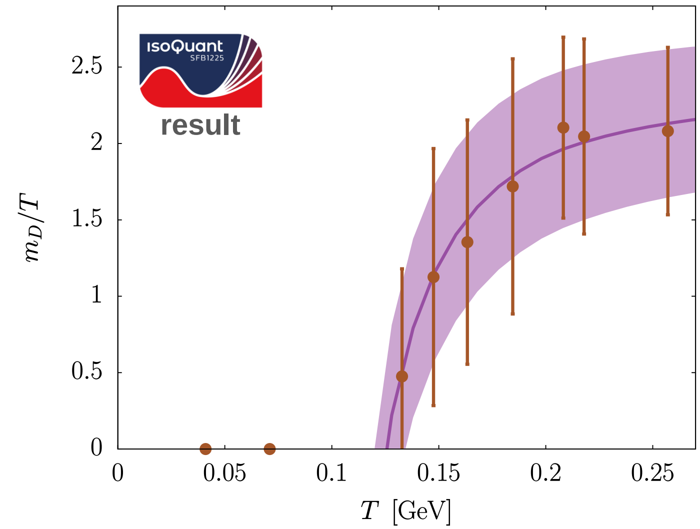
Gauss-law solution to Re[V] & Im[V]

University of Stavanger

- Gauss-Law result allows to fit Re[V] data even in the non-perturbative regime

D.Lafferty and A.R. arXiv:1906.00035





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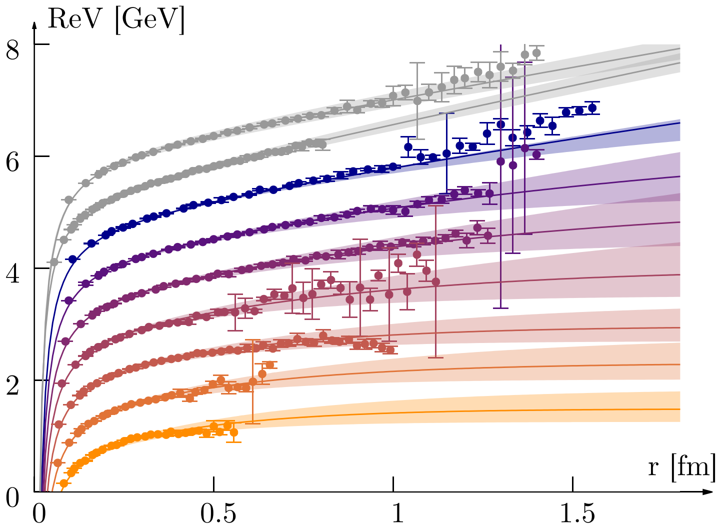

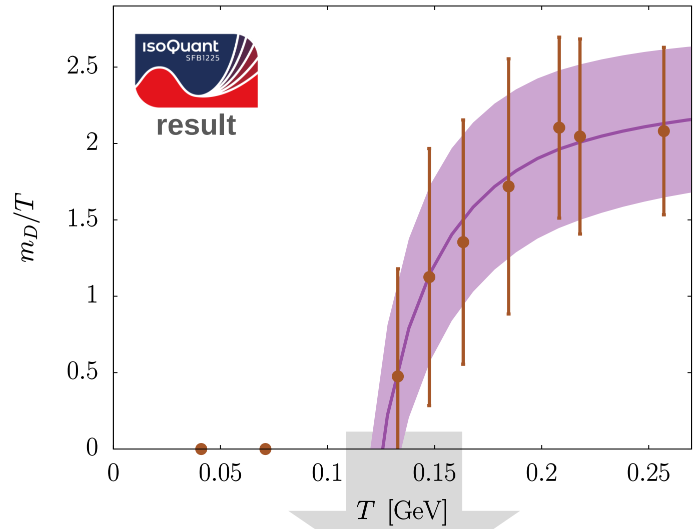

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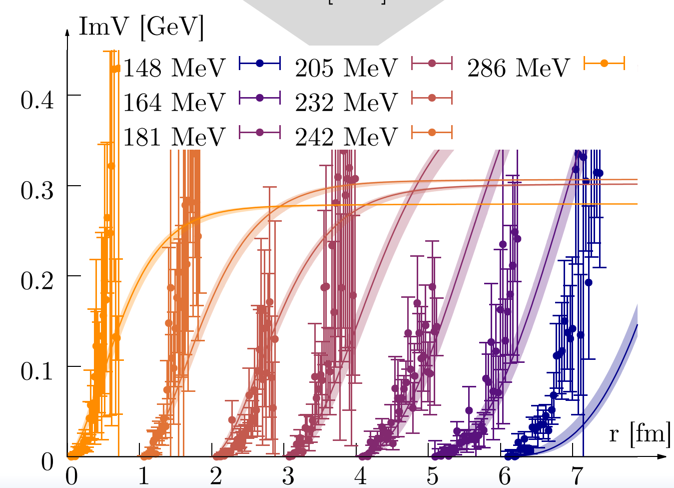

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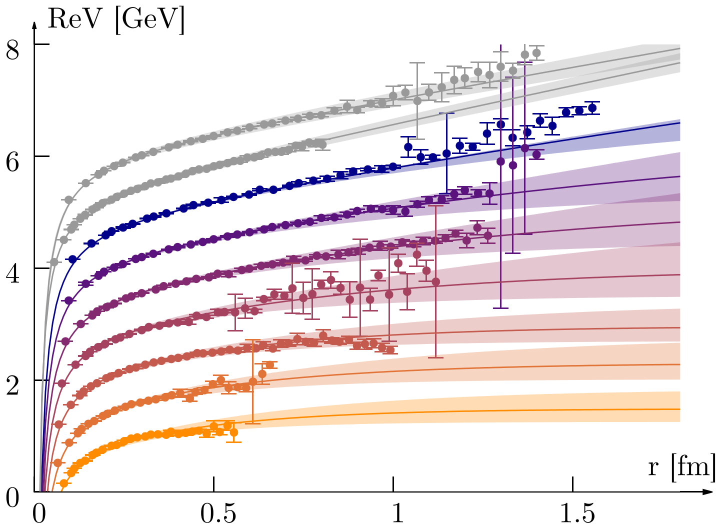

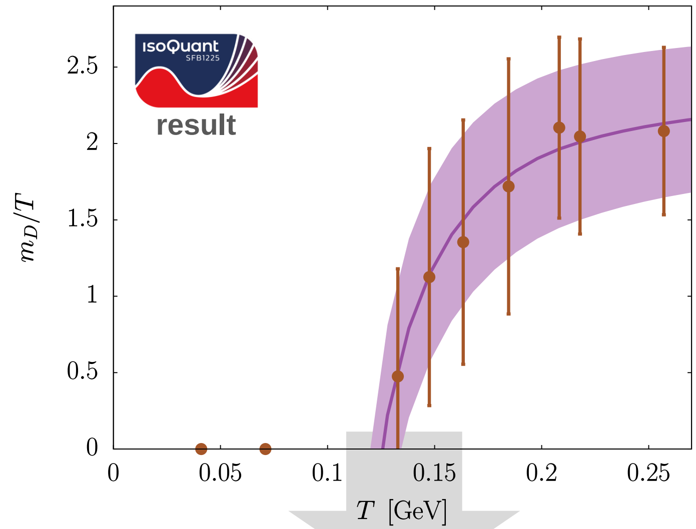

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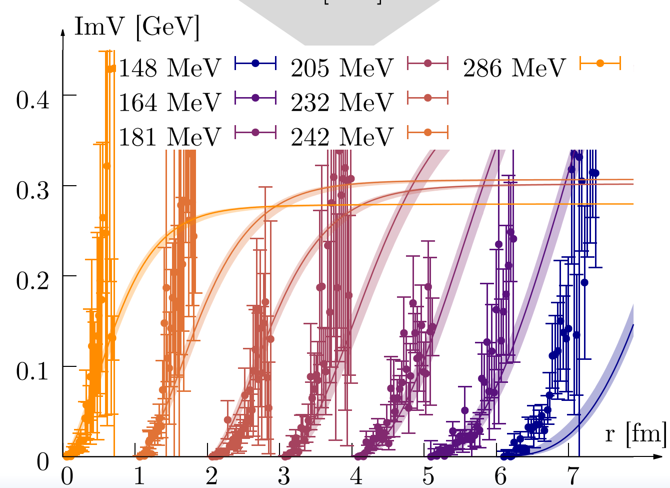
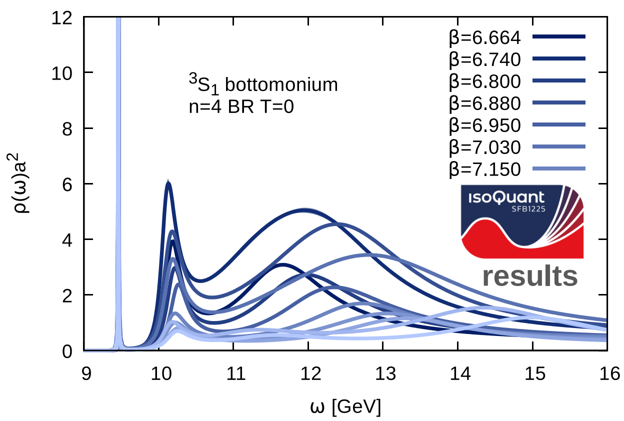


- recently extend the Gauss law to model quarkonium at finite velocity & μ_B

Equilibrium spectral properties

University of Stavanger

$b\bar{b}$ S-wave T=0 spectra



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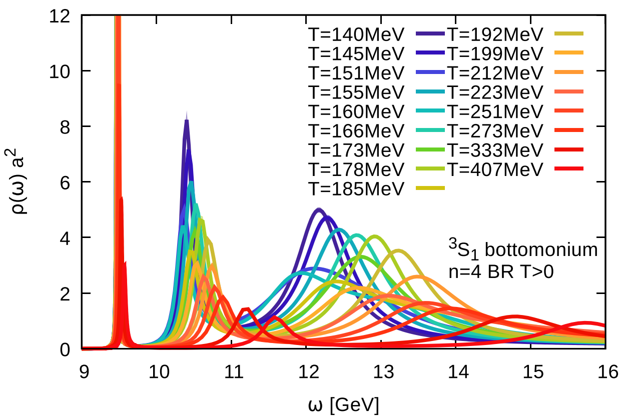

S.Kim, P. Petreczky, A.R., JHEP 1811 (2018) 088

- Crucial step: **defining correct T=0 baseline** in presence of methods artifacts

Equilibrium spectral properties

University of Stavanger

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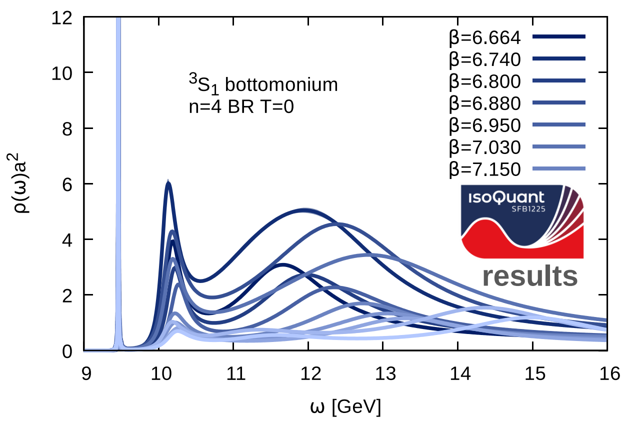

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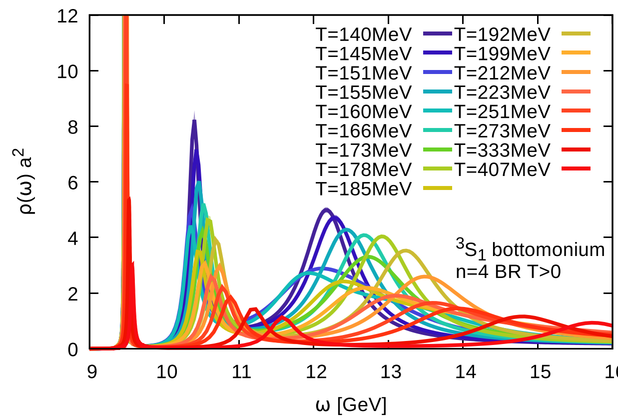

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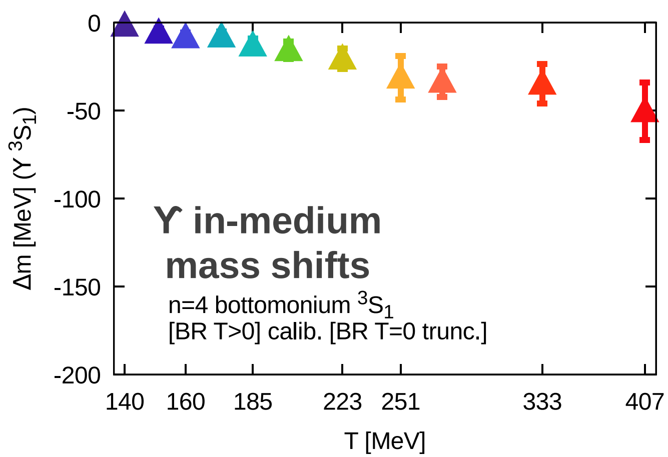
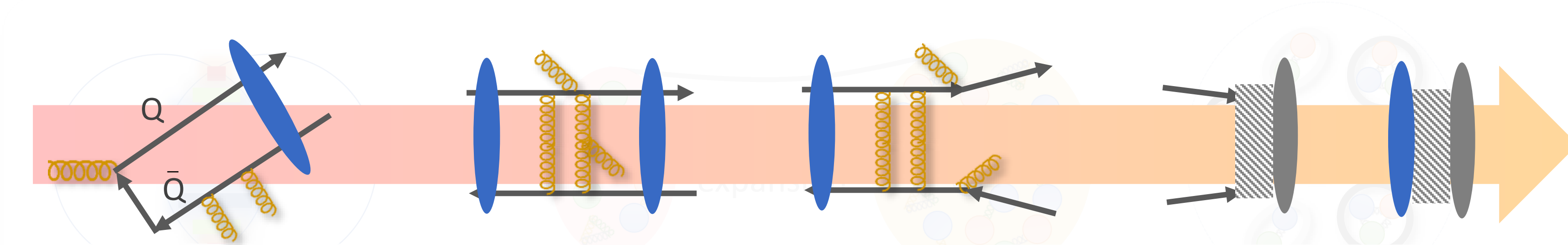


- Crucial step: **defining correct T=0 baseline** in presence of methods artifacts
- For the first time **consistent negative in medium mass shifts** – ordered by E_{bind}

Open theory questions

University of Stavanger

bulk: pre-thermalization Quark-Gluon-Plasma hadronization



$Q\bar{Q}$: production /formation medium interaction freeze-out

$Q\bar{Q}$ realtime evol. in the initial stages

First exploratory steps in the glasma

classical statistical simulations for gluons & real-time NRQCD

(see poster by Alexander Lehmann)

Real-time $Q\bar{Q}$ evol. in local thermal equilibrium

Beyond Schrödinger:

Open-quantum-systems descr. of real-time evolution

Connecting OQS to EFT language of potential

(with T. Miura, Y. Akamatsu, M. Asakawa arXiv:1908.06293)

Properties of equilibrium $Q\bar{Q}$

First principles extraction of **the heavy quark potential**

Novel phenomenological definition of the **Debye mass**

Extraction of **thermal spectral properties** on the lattice

with P. Petreczky, J. Weber: NPA982 (2019) 735
S. Kim, P. Petreczky, A.R. JHEP 1811 (2018) 088
with D. Lafferty arXiv:1906.00035

The open quantum systems picture

- Need a general approach to describe quarkonium coupled to a thermal medium
  - Overall system is closed, hermitean Hamiltonian: von Neumann equation

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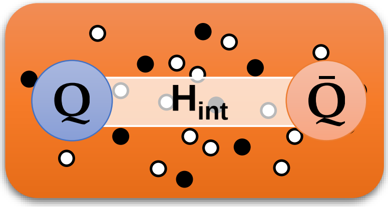

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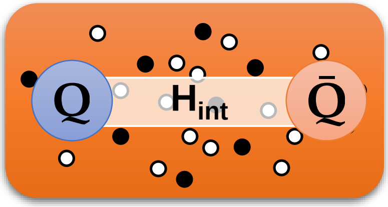

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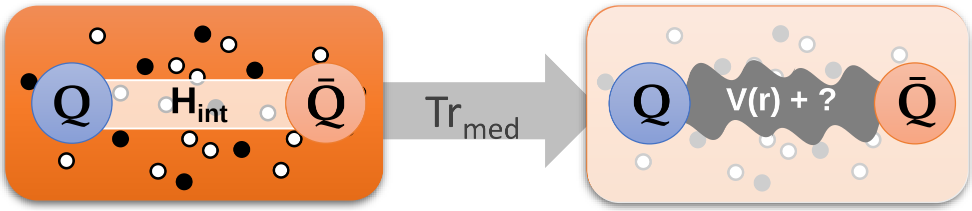

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N. Brambilla et.al. arXiv:1903.08063 & PRD97 (2018) 074009,
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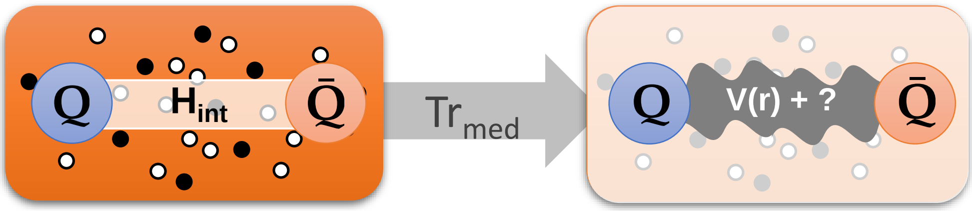

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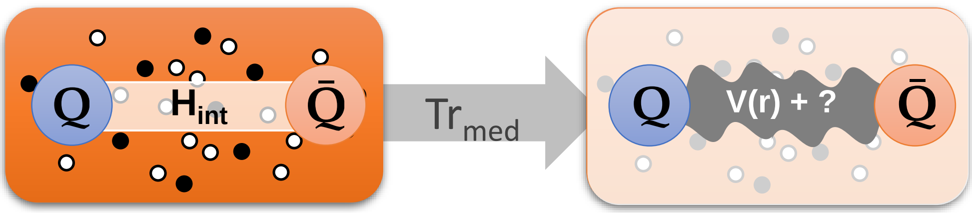

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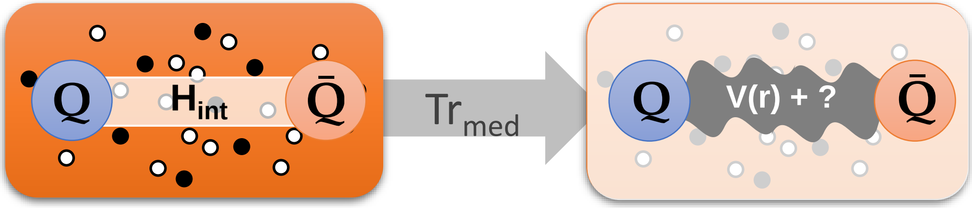

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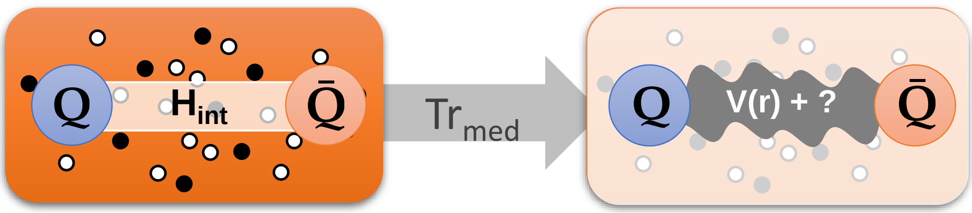

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$$\langle n|\rho_{Q\bar{Q}}|n\rangle > 0, \forall n$$

$$\rho^\dagger_{Q\bar{Q}} = \rho_{Q\bar{Q}}, \quad \mathrm{Tr}[\rho_{Q\bar{Q}}] = 1$$

Feynman Vernon influence functional

- Derivation via path integral formalism: **Feynman-Vernon influence functional**

for details see Y. Akamatsu, Phys.Rev. D87 (2013) 4, 045016 and arXiv:1403.5783

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$$\rho_{Q\bar{Q}}(t,x,y) = \int dX dY \delta(X-Y)\rho(t,x,y,X,Y)$$

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perturbative Im[V]

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medium - QQ interaction

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$$S_{FV} \approx S_{pot}\left[Re[V]\right] + S_{fluct}\left[Im[V]\right] + S_{diss}\left[Im[V]\right]$$

- In QM language: Markovian evolution via **quarkonium Lindblad equation**

$$\frac{d}{dt}\rho_{Q\bar{Q}}(t) = -i\left[H_{Q\bar{Q}},\rho_{Q\bar{Q}}\right] + \sum_{i=1}^{N_{LB}} \gamma_i \left(\hat{L}_i \rho_{Q\bar{Q}} \hat{L}_i^\dagger - \frac{1}{2}\hat{L}_i\hat{L}_i^\dagger \rho_{Q\bar{Q}} - \frac{1}{2}\rho_{Q\bar{Q}}\hat{L}_i\hat{L}_i^\dagger\right)$$

$$\tilde{D}(k) = g^2 T \frac{\pi m_D^2}{k(k^2+m_D^2)^2}$$

perturbative Im[V]

$$L_{\mathbf{k},a} = \sqrt{\frac{D(\mathbf{k})}{2}}\left[1 - \frac{\mathbf{k}}{4m_Q T}\cdot\left(\frac{1}{2}\mathbf{P}_{CM}+\mathbf{p}\right)\right] e^{i\mathbf{k}\cdot\mathbf{r}/2}(T^a\otimes 1) - \sqrt{\frac{D(\mathbf{k})}{2}}\left[1 - \frac{\mathbf{k}}{4m_Q T}\cdot\left(\frac{1}{2}\mathbf{P}_{CM}-\mathbf{p}\right)\right] e^{-i\mathbf{k}\cdot\mathbf{r}/2}(1\otimes T^a)$$

- Full Lindblad dynamics cannot be described by deterministic Schrödinger equation

The stochastic potential

- Based on scale separation & weak coupling approximation:

$$S_{FV} \approx S_{pot}\big[Re[V]\big] + S_{fluct}\big[Im[V]\big] + S_{diss}\big[Im[V]\big]$$

- In QM language corresponds to Markovian evolution by Lindblad equation

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- Include the effects of fluctuations by taking the **first order gradient expansion**

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University of Stavanger

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$$i\partial_t\psi_{Q\bar{Q}}(t) = \Big(-\frac{\nabla^2}{2m_Q} + V(r) + \Theta - \frac{i}{2}\Theta^2\Big)\psi_{Q\bar{Q}}(t)$$

- Fully unitary microscopic dynamics, no thermalization

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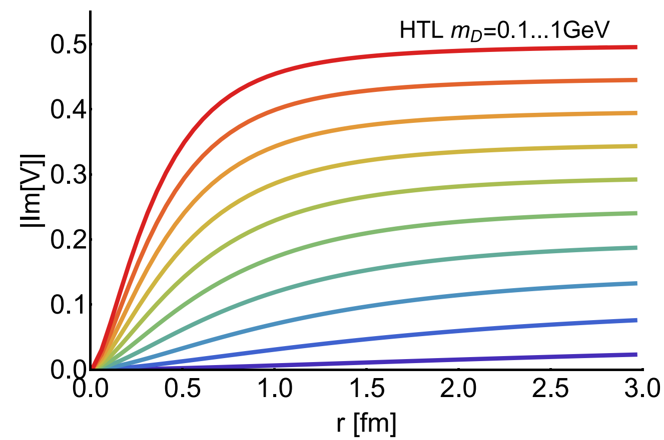

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University of Stavanger

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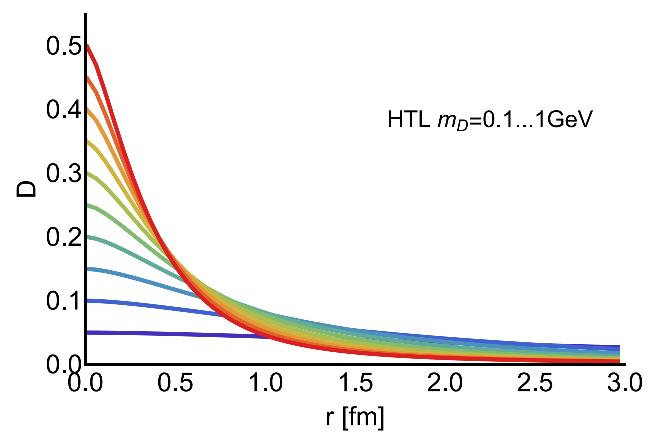

$$D(r) = Im[V](r) - Im[V](\infty)$$

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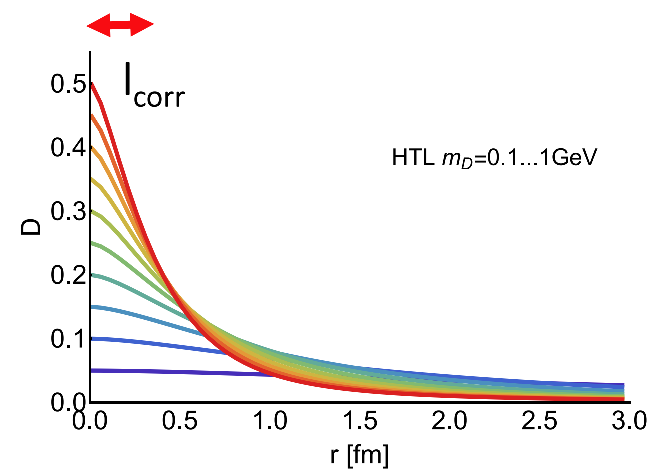

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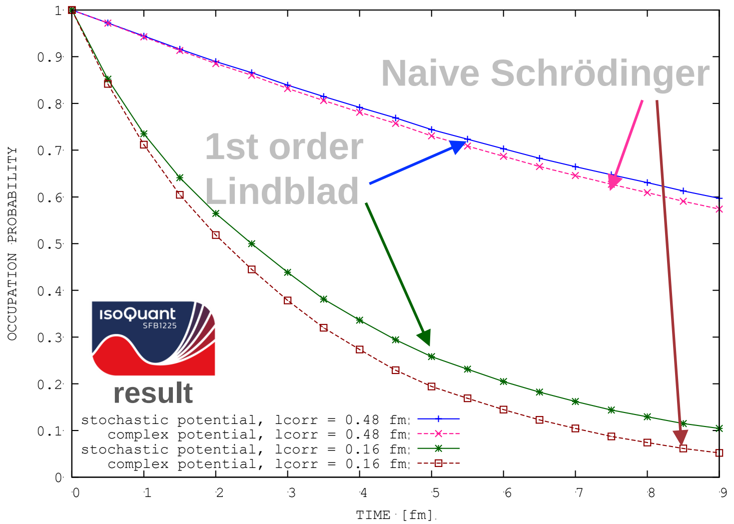

S.Kajimoto, Y.Akamatsu, M. Asakawa, A.R., PRD97 (2018) 014003

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University of Stavanger

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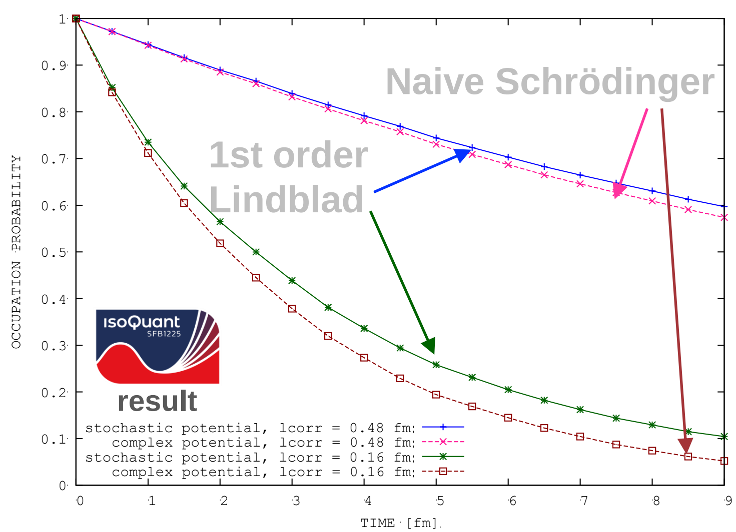

S.Kajimoto, Y.Akamatsu, M. Asakawa, A.R., PRD97 (2018) 014003

Decoherence

- A new scale appears in the dynamics: medium correlation length l_{corr}

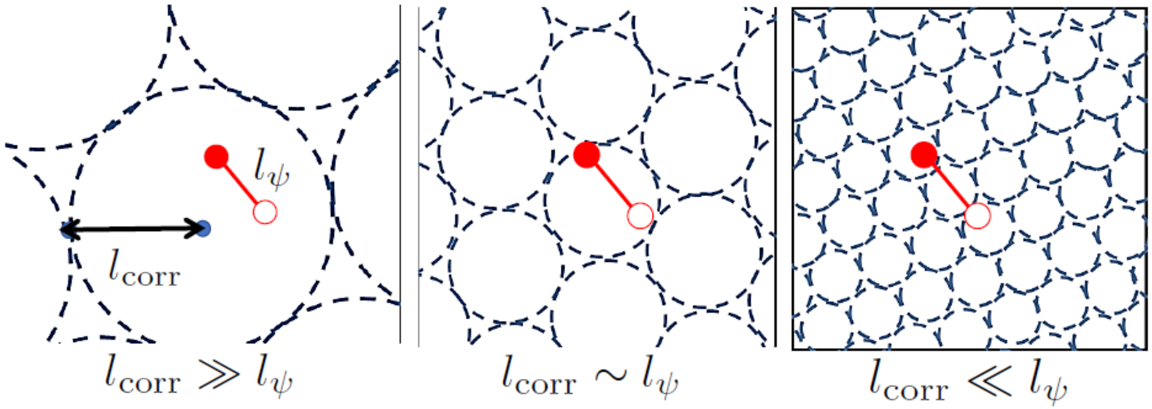

Illustration by Y. Akamatsu

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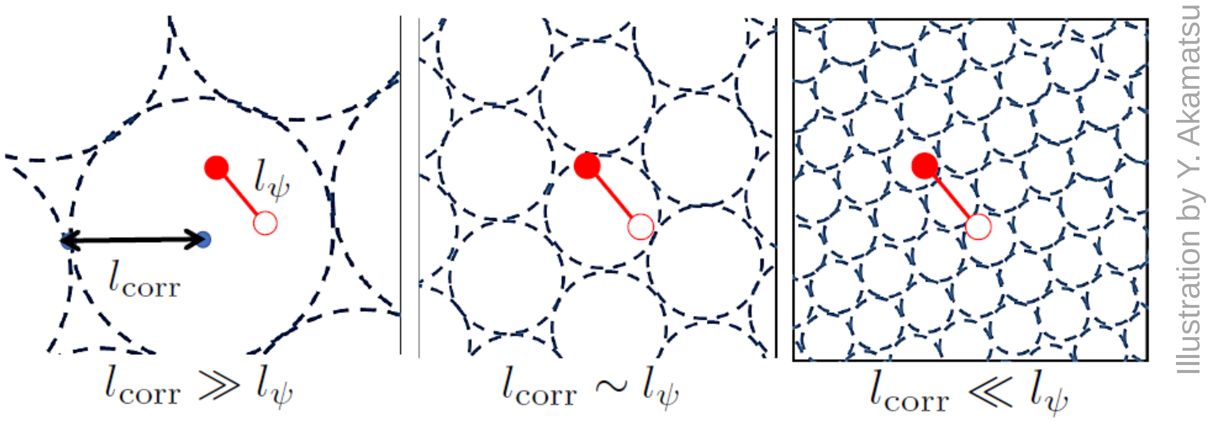
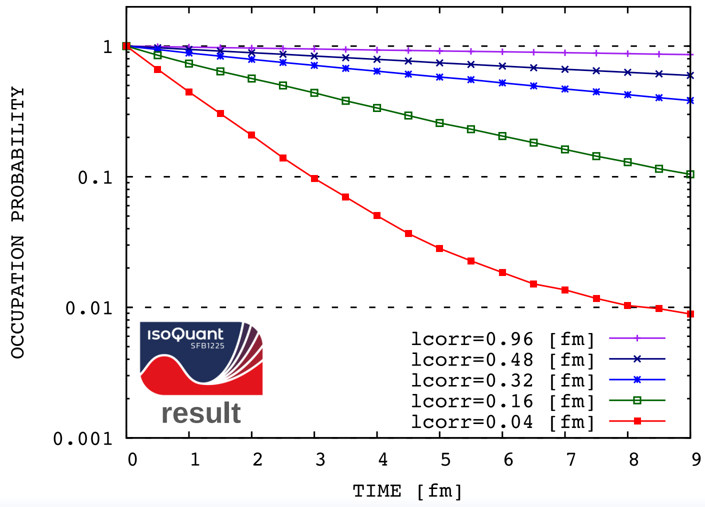


- Fluctuations induce decoherence: select preferred basis – decay of populations



stability of quarkonium decreases:
from Debye screening and fluctuations

Towards full Lindblad dynamics (I)

University of Stavanger

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- Unravel dynamics in wavefunction stochastic dynamics: **Quantum State Diffusion**

T. Miura, Y.Akamatsu, M. Asakawa, A.R., arXiv:1908.06293

$$|d\psi\rangle = |\psi(t+dt)\rangle - |\psi(t)\rangle$$

$$= -iH|\psi(t)\rangle dt + \sum_n \begin{pmatrix} 2\langle L_n^\dagger\rangle_\psi L_n - L_n^\dagger L_n \\ -\langle L_n^\dagger\rangle_\psi \langle L_n\rangle_\psi \end{pmatrix} |\psi(t)\rangle dt$$

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weak coupling parameters, V(r) Debye form

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weak coupling parameters, V(r) Debye form

- First “derivation” of phenomenological models based on nonlinear Schrödinger equation

c.f. e.g. R. Katz, P. Gossiaux Annals Phys. 368 (2016) 267

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University of Stavanger

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  c.f. e.g. R. Katz, P. Gossiaux Annals Phys. 368 (2016) 267

- First genuine Lindblad implementation: previous works could not maintain positivity of ρ

  D. De Boni, JHEP 1708 (2017) 064

Towards full Lindblad dynamics (II)

University of Stavanger

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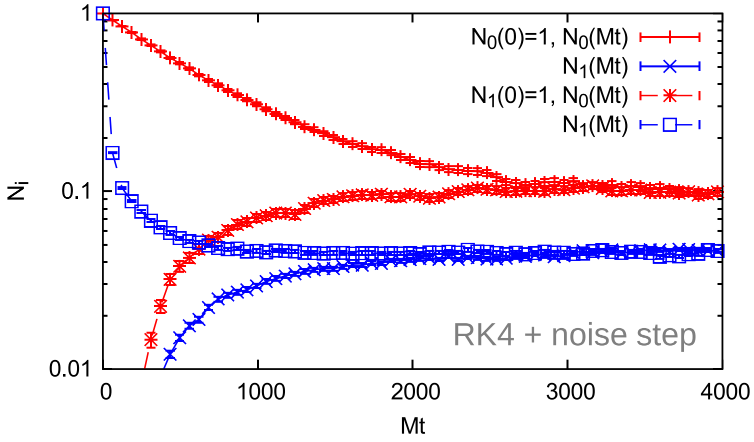
$$\frac{d}{dt}\rho_{Q\bar{Q}}(t) = -i\big[H_{Q\bar{Q}},\rho_{Q\bar{Q}}\big] + \sum_{i=1}^{N_{LB}} \gamma_i\Big(\hat{L}_i\rho_{Q\bar{Q}}\hat{L}_i^\dagger - \frac{1}{2}\hat{L}_i\hat{L}_i^\dagger\rho_{Q\bar{Q}} - \frac{1}{2}\rho_{Q\bar{Q}}\hat{L}_i\hat{L}_i^\dagger\Big)$$

- Unravel dynamics in wavefunction stochastic dynamics: **Quantum State Diffusion**

T. Miura, Y.Akamatsu, M. Asakawa A.R., arXiv:1908.06293

Proof of principle in 1d



- Encouraging: admixtures become independent of initial conditions at late times

Towards full Lindblad dynamics (II)

University of Stavanger

- Based on scale separation & weak coupling approximation:

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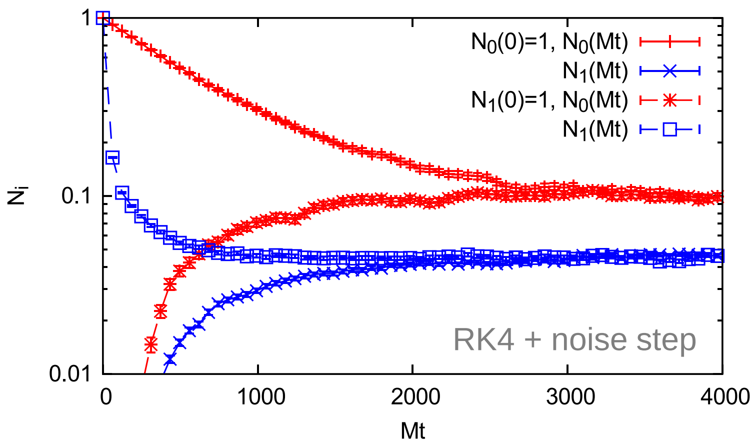

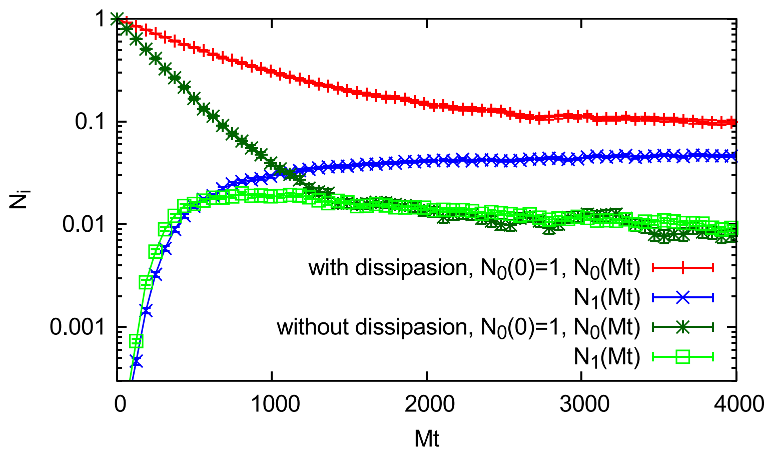

- Encouraging: admixtures become independent of initial conditions at late times
- Dissipative effects stabilize the ground state as they counteract separation of the pair

Towards full Lindblad dynamics (III)

University of Stavanger

- Based on scale separation & weak coupling approximation:

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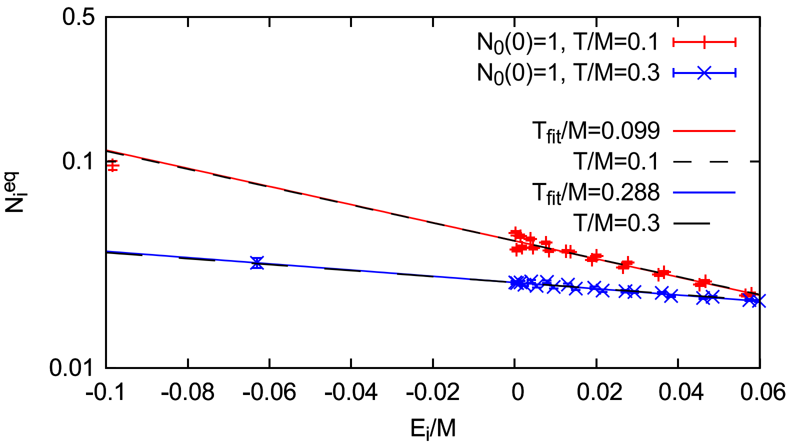

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University of Stavanger

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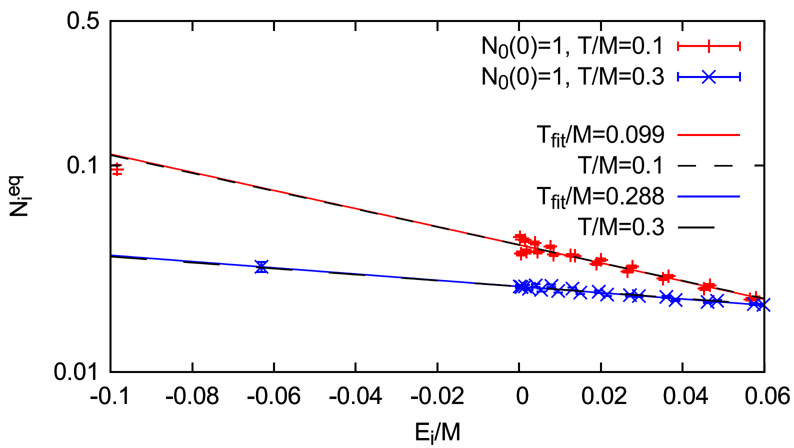

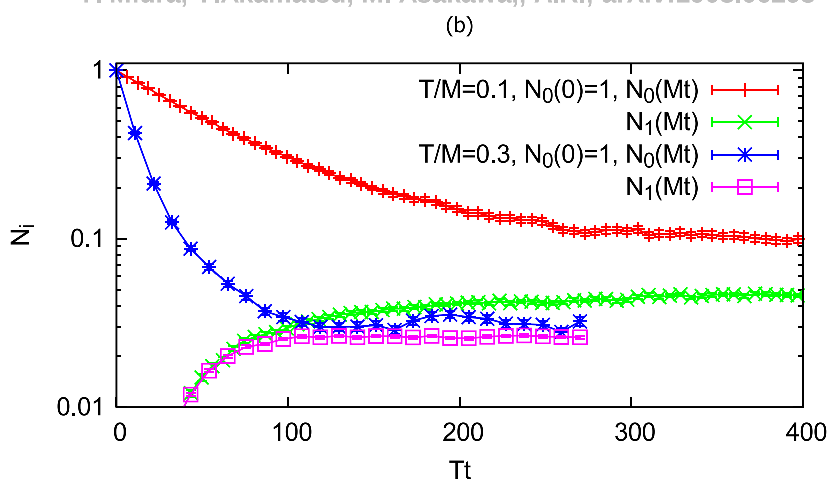

- Distribution of states at late times agrees well with Boltzmann and yields consistent T
- Smaller m_Q leads to more efficient equilibration (decoherence more effective)

Conclusion

University of Stavanger

- **Significant progress** in in-medium quarkonium theory
- Recent and ongoing studies on quarkonium dynamical properties
  - Novel extraction of the in-medium **heavy quark potential** on realistic lattices
    P. Petreczky, A.R., J. Weber, NPA982 (2019) 735
  - Improved **analytic parametrization** of V(R) using the generalized **Gauss law**
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Thank you for your attention