# *Particle production from expanding QCD strings and its quantum simulation*

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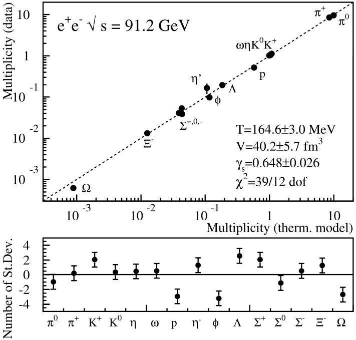
# *The thermal model puzzle*

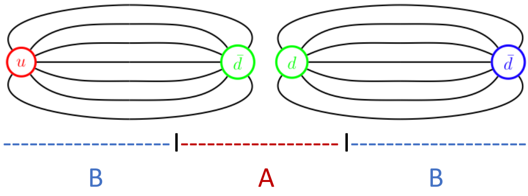
- elementary particle collision experiments such as $e^+ \, e^-$ collisions show some thermal-like features
- particle multiplicities well described by thermal model




[Becattini, Casterina, Milov & Satz, EPJC 66, 377 (2010)]

- conventional thermalization by collisions unlikely
- more thermal-like features difficult to understand in PYTHIA [Fischer, Sjöstrand (2017)]
- alternative explanations needed

## *QCD strings*



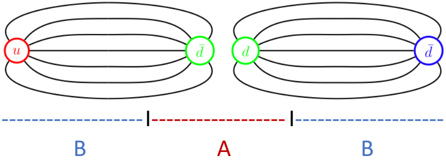

- particle production from QCD strings
- Lund string model (e. g. PYTHIA)
- different regions in a string are entangled
- subinterval $A$ is described by reduced density matrix

$$\rho_A = \mathrm{Tr}_B \rho$$

- reduced density matrix is of mixed state form
- could this lead to thermal-like effects?

# *Entropy and entanglement*

- consider a split of a quantum system into two $A + B$




- reduced density operator for system $A$

$$\rho_A = \mathsf{Tr}_B\{\rho\}$$

- entropy associated with subsystem A: **entanglement entropy**

$$S_A = -\mathsf{Tr}_A\{\rho_A \ln \rho_A\}$$

- **globally pure** state $S = 0$ can be **locally mixed** $S_A > 0$
- **coherent information** $I_{B\rangle A} = S_A - S$ can be **positive**

# *Microscopic model*

- QCD in 1+1 dimensions described by 't Hooft model

$$\mathscr{L} = -\bar{\psi}_i \gamma^\mu (\partial_\mu - ig\mathbf{A}_\mu)\psi_i - m_i \bar{\psi}_i \psi_i - \frac{1}{2}\mathrm{tr}\, \mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu}$$

- fermionic fields $\psi_i$ with sums over flavor species $i = 1, \dots, N_f$
- SU($N_c$) gauge fields $\mathbf{A}_\mu$ with field strength tensor $\mathbf{F}_{\mu\nu}$
- gluons are not dynamical in two dimensions
- gauge coupling $g$ has dimension of mass
- non-trivial, interacting theory, cannot be solved exactly
- spectrum of excitations known for $N_c \to \infty$ with $g^2 N_c$ fixed
  ['t Hooft (1974)]

# *Schwinger model*

- QED in 1+1 dimension

$$\mathscr{L} = -\bar{\psi}_i \gamma^\mu (\partial_\mu - iqA_\mu)\psi_i - m_i \bar{\psi}_i \psi_i - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

- geometric confinement
- U(1) charge related to string tension $q = \sqrt{2\sigma}$
- for single fermion one can **bosonize theory** exactly
  [Coleman, Jackiw, Susskind (1975)]

$$S = \int d^2x \sqrt{g} \Big\{ -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} M^2 \phi^2 \\ - \frac{m\, q\, e^\gamma}{2\pi^{3/2}} \cos\left(2\sqrt{\pi}\phi + \theta\right) \Big\}$$

- Schwinger bosons are dipoles $\phi \sim \bar{\psi}\psi$
- scalar mass related to U(1) charge by $M = q/\sqrt{\pi} = \sqrt{2\sigma/\pi}$
- massless Schwinger model $m = 0$ leads to free bosonic theory

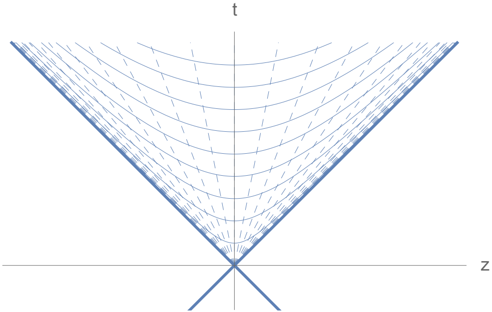
# *Transverse coordinates*

- so far dynamics strictly confined to 1+1 dimensions
- transverse coordinates may fluctuate, can be described by Nambu-Goto action ($h_{\mu\nu} = \partial_\mu X^m \partial_\nu X_m$)

$$
\begin{aligned}
S_{\mathsf{NG}} &= \int d^2x \sqrt{-\det h_{\mu\nu}}\, \{-\sigma + \ldots\} \\
&\approx \int d^2x \sqrt{g} \left\{-\sigma - \frac{\sigma}{2} g^{\mu\nu} \partial_\mu X^i \partial_\nu X^i + \ldots \right\}
\end{aligned}
$$

- two additional, massless, bosonic degrees of freedom corresponding to transverse coordinates $X^i$ with $i = 1, 2$

# *Expanding string solution 1*




- external quark-anti-quark pair on trajectories $z = \pm t$
- coordinates: Bjorken time $\tau = \sqrt{t^2 - z^2}$, rapidity $\eta = \text{arctanh}(z/t)$
- metric $ds^2 = -d\tau^2 + \tau^2 d\eta^2$
- symmetry with respect to longitudinal boosts $\eta \to \eta + \Delta\eta$

# *Expanding string solution 2*

- Schwinger boson field depends only on $\tau$

$$\bar{\phi} = \bar{\phi}(\tau)$$

- equation of motion

$$\partial_\tau^2 \bar{\phi} + \frac{1}{\tau}\partial_\tau \bar{\phi} + M^2 \bar{\phi} = 0.$$

- Gauss law: electric field $E = q\phi/\sqrt{\pi}$ must approach the U(1) charge of the external quarks $E \to q_{\mathsf{e}}$ for $\tau \to 0_+$

$$\bar{\phi}(\tau) \to \frac{\sqrt{\pi} q_{\mathsf{e}}}{q} \qquad (\tau \to 0_+)$$

- solution of equation of motion [Loshaj, Kharzeev (2011)]

$$\bar{\phi}(\tau) = \frac{\sqrt{\pi} q_{\mathsf{e}}}{q} J_0(M\tau)$$

# *Gaussian states*

- theories with quadratic action often have Gaussian density matrix
- fully characterized by field expectation values

$$\bar{\phi}(x) = \langle \phi(x) \rangle, \qquad \bar{\pi}(x) = \langle \pi(x) \rangle$$

  and connected two-point correlation functions, e. g.

$$\langle \phi(x)\phi(y) \rangle_c = \langle \phi(x)\phi(y) \rangle - \bar{\phi}(x)\bar{\phi}(y)$$

- if $\rho$ is Gaussian, also reduced density matrix $\rho_A$ is Gaussian

## *Entanglement entropy for Gaussian state*

- entanglement entropy of Gaussian state in region $A$
  [Berges, Floerchinger, Venugopalan, JHEP 1804 (2018) 145]

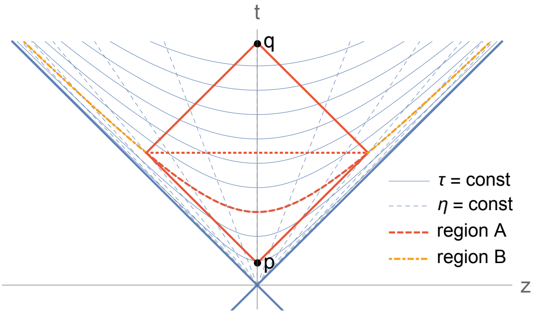
$$S_A = \frac{1}{2}\mathsf{Tr}_A \left\{ D \ln(D^2) \right\}$$

- operator trace over region $A$ only
- matrix of correlation functions

$$D(x,y) = \begin{pmatrix} -i\langle\phi(x)\pi(y)\rangle_c & i\langle\phi(x)\phi(y)\rangle_c \\ -i\langle\pi(x)\pi(y)\rangle_c & i\langle\pi(x)\phi(y)\rangle_c \end{pmatrix}$$

- involves connected correlation functions of field $\phi(x)$ and canonically conjugate momentum field $\pi(x)$
- expectation value $\bar{\phi}$ does not appear explicitly
- coherent states and vacuum have equal entanglement entropy $S_A$

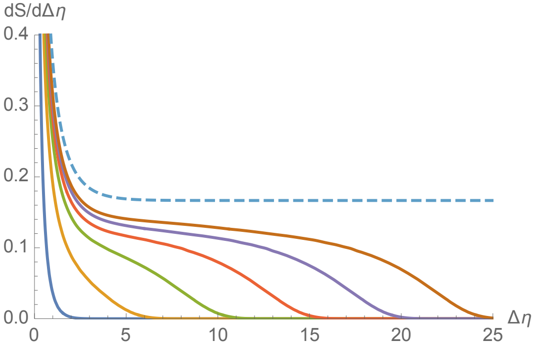
## *Rapidity interval*




- consider rapidity interval $(-\Delta\eta/2, \Delta\eta/2)$ at fixed Bjorken time $\tau$
- entanglement entropy does not change by unitary time evolution with endpoints kept fixed
- can be evaluated equivalently in interval $\Delta z = 2\tau \sinh(\Delta\eta/2)$ at fixed time $t = \tau \cosh(\Delta\eta/2)$
- need to solve eigenvalue problem with correct **boundary conditions**

# *Bosonized massless Schwinger model*

- entanglement entropy understood numerically for free massive scalars
  [Casini, Huerta (2009)]
- entanglement entropy density $dS/d\Delta\eta$ for bosonized massless Schwinger model ($M = \frac{q}{\sqrt{\pi}}$)




$M\tau = 1,\ 10^{-1},\ 10^{-2},\ 10^{-3},\ 10^{-4}$, and $10^{-5}$

## *Conformal limit*

- For $M\tau \to 0$ one has conformal field theory limit
  [Holzhey, Larsen, Wilczek (1994)]

$$S(\Delta z) = \frac{c}{3} \ln(\Delta z/\epsilon) + \text{constant}$$

  with small length $\epsilon$ acting as UV cutoff.
- Here this implies

$$S(\tau, \Delta\eta) = \frac{c}{3} \ln(2\tau \,\mathsf{sinh}(\Delta\eta/2)/\epsilon) + \text{constant}$$

- Conformal charge $c = 1$ for free massless scalars or Dirac fermions.
- Additive constant not universal but entropy density is

$$\begin{aligned} \frac{\partial}{\partial \Delta\eta} S(\tau, \Delta\eta) =& \frac{c}{6} \mathsf{coth}(\Delta\eta/2) \\ \to& \frac{c}{6} \qquad\qquad (\Delta\eta \gg 1) \end{aligned}$$

- Entropy becomes extensive in $\Delta\eta$ !

## *Universal entanglement entropy density*

- for very early times "Hubble" expansion rate dominates over masses and interactions

$$H = \frac{1}{\tau} \gg M = \frac{q}{\sqrt{\pi}}, m$$

- theory dominated by free, massless fermions
- universal entanglement entropy density

$$\frac{dS}{d\Delta\eta} = \frac{c}{6}$$

  with conformal charge $c$
- for QCD in 1+1 D (gluons not dynamical, no transverse excitations)

$$c = N_c \times N_f$$

- from fluctuating transverse coordinates (Nambu-Goto action)

$$c = N_c \times N_f + 2 \approx 9 + 2 = 11$$

# *Temperature and entanglement entropy*

- for conformal fields, entanglement entropy has also been calculated at non-zero temperature.
- for static interval of length $L$ [Korepin (2004); Calabrese, Cardy (2004)]

$$S(T,l) = \frac{c}{3}\ln\left(\frac{1}{\pi T\epsilon}\sinh(\pi L T)\right) + \text{const}$$

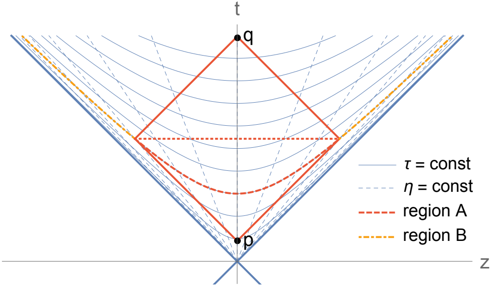
- compare this to our result in expanding geometry

$$S(\tau,\Delta\eta) = \frac{c}{3}\ln\left(\frac{2\tau}{\epsilon}\,\mathsf{sinh}(\Delta\eta/2)\right) + \text{const}$$

- expressions agree for $L = \tau\Delta\eta$ (with metric $ds^2 = -d\tau^2 + \tau^2 d\eta^2$) and time-dependent temperature

$$T = \frac{1}{2\pi\tau}$$

# *Modular or entanglement Hamiltonian 1*




- conformal field theory
- hypersurface $\Sigma$ with boundary on the intersection of two light cones
- reduced density matrix [Casini, Huerta, Myers (2011), Arias, Blanco, Casini, Huerta (2017), see also Candelas, Dowker (1979)]

$$\rho_A = \frac{1}{Z_A} e^{-K}, \qquad Z_A = \mathrm{Tr}\, e^{-K}$$

- modular or entanglement Hamiltonian $K$

# *Modular or entanglement Hamiltonian 2*

- modular or entanglement Hamiltonian is **local expression**

$$K = \int_\Sigma d\Sigma_\mu \, \xi_\nu(x) \, T^{\mu\nu}(x).$$

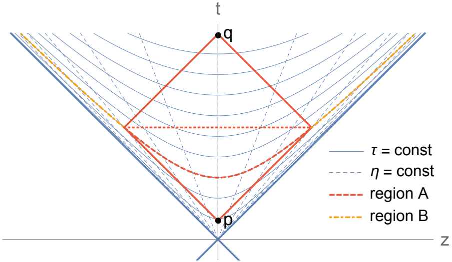
- energy-momentum tensor $T^{\mu\nu}(x)$ of excitations
- vector field

$$\xi^\mu(x) = \tfrac{2\pi}{(q-p)^2}[(q-x)^\mu(x-p)(q-p) \\ + (x-p)^\mu(q-x)(q-p) - (q-p)^\mu(x-p)(q-x)]$$

  end point of future light cone $q$, starting point of past light cone $p$
- inverse temperature and fluid velocity

$$\xi^\mu(x) = \beta^\mu(x) = \frac{u^\mu(x)}{T(x)}$$

# *Modular or entanglement Hamiltonian 3*




- for $\Delta\eta \to \infty$: fluid velocity in $\tau$-direction, $\tau$-dependent temperature

$$T(\tau) = \frac{\hbar}{2\pi\tau}$$

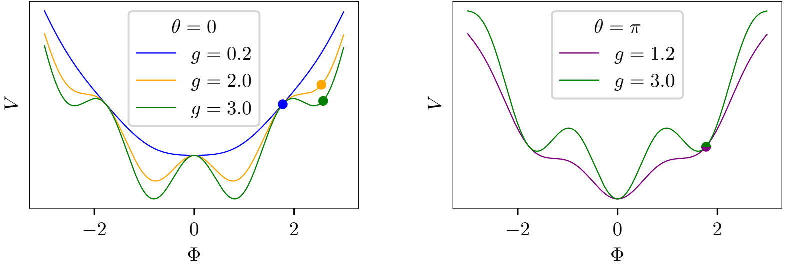
- **Entanglement between different rapidity intervals alone leads to local thermal density matrix at very early times !**
- Hawking-Unruh temperature in Rindler wedge $T(x) = \hbar c/(2\pi x)$

# *Physics picture*

- coherent state vacuum at early time contains entangled pairs of quasi-particles with opposite wave numbers
- on finite rapidity interval $(-\Delta\eta/2, \Delta\eta/2)$ in- and out-flux of quasi-particles with thermal distribution via boundaries
- technically limits $\Delta\eta \to \infty$ and $M\tau \to 0$ do not commute
  - $\Delta\eta \to \infty$ for any finite $M\tau$ gives pure state
  - $M\tau \to 0$ for any finite $\Delta\eta$ gives thermal state with $T = 1/(2\pi\tau)$

# *Bosonized massive Schwinger model*

[ongoing work with Lara Kuhn, Jürgen Berges]




- scalar theory with potential

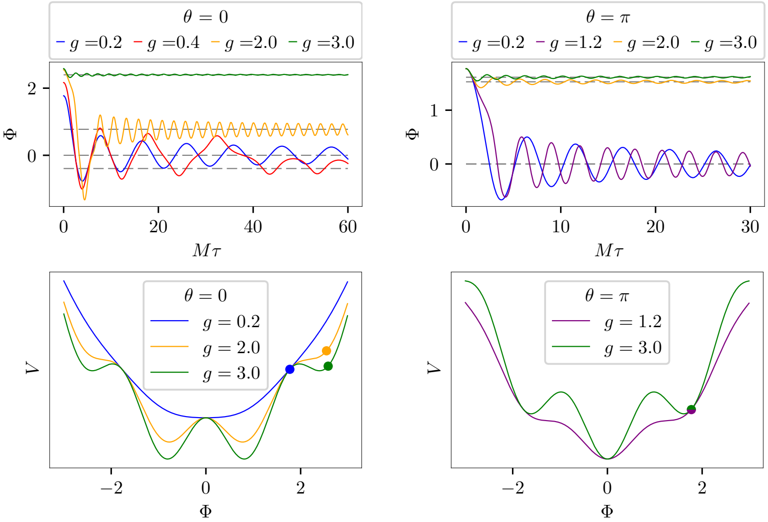
$$V(\Phi) = \frac{1}{2}M^2\Phi^2 + J\cos(2\sqrt{\pi}\phi + \theta)$$

- dimensionless coupling strength $g = 2\sqrt{\pi}J/M^2$
- initial value

$$\Phi(0) = \Phi_{\text{vac}} + \sqrt{\pi}$$

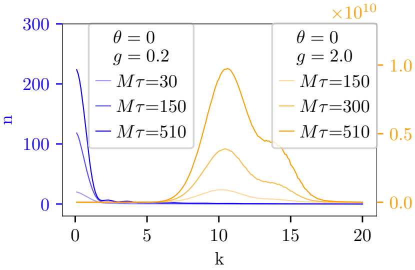
# *Time evolution of background field*

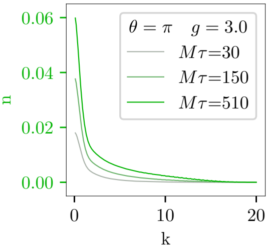
- solve equation of motion for background field in expanding geometry
- non-linear oscillations damped by expansion



# *Excitations*

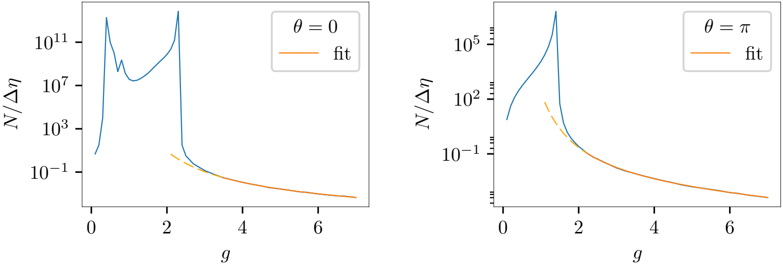
- consider now perturbations around expanding background fields
- linear problem: quantization in time dependent situation
- (quasi-) particle production
- similar: cosmology, strong electric fields
- calculate this for different parameters as function of wave number



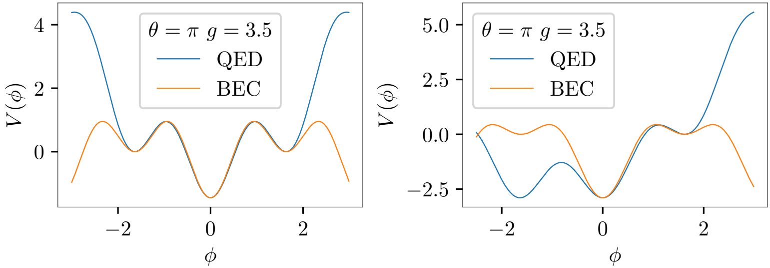
# *Asymptotic particle spectrum*

- particles per unit rapidity
- characteristic peaks due to resonance-like phenomena
- for large coupling $g = 2\sqrt{\pi}J/M^2$ exponential decay



# *Quantum simulation*

[ongoing work with Lara Kuhn]




- two one-dimensional Bose-Einstein condensates with tunnel coupling allow to realize model Lagrangian for relative phase $\phi = \varphi_1 - \varphi_2$

$$\mathscr{L} = \frac{1}{2}\dot{\phi}^2 - \frac{c_s^2}{2}(\nabla\phi)^2 - V_0\left[-\cos(\phi) + \frac{1}{2}\lambda^2\sin^2(\phi)\right]$$

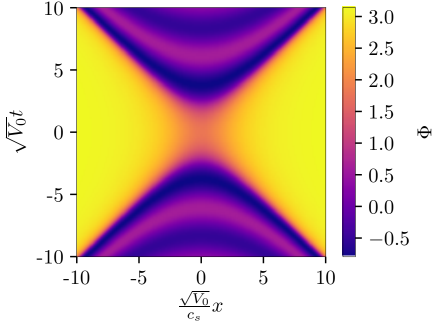
  with velocity of sound $c_s$

- $\lambda$ from periodic in time modulation of tunnel coupling [Fialko, Opanchuk, Sidorov, Drummond & Brand (2014)] (challenging to implement)

# *Tunneling and the bounce*

[ongoing work with Lara Kuhn]

- consider field in metastable vacuum
- tunneling can be described by semi-classical methods
- false vacuum decay by bounce solution [S. Coleman (1977)]
- could be induced artificially
- interior of forward sound cone resembles closely expanding string solution



## *Conclusions*

- rapidity intervals in an expanding string are entangled
- at very early times theory effectively conformal

$$\frac{1}{\tau} \gg m, q$$

- entanglement entropy extensive in rapidity $\frac{dS}{d\Delta\eta} = \frac{c}{6}$
- determined by conformal charge $c = N_c \times N_f + 2$
- reduced density matrix for conformal field theory is of locally thermal form with temperature

$$T = \frac{\hbar}{2\pi\tau}$$

- expanding QCD string dynamics could be quantum simulated through two frequency sine-Gordon model

$$V(\phi) \sim \left[ -\cos(\phi) + \frac{1}{2}\lambda^2 \sin^2(\phi) \right]$$