Hyperfine splitting in simple highly charged ions for the search of the variation of fundamental constants

Natalia S. Oreshkina, Stefano M. Cavaletto, Niklas Michel,
Zoltán Harman, Christoph H. Keitel
Max Planck Institute for Nuclear Physics, Heidelberg

Quantum Systems in Extreme Conditions
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MAX-PLANCK-GESELLSCHAFT

Introduction

Neutral atoms $\rightarrow$ Singly charged ions $\rightarrow$ Doubly charged ions ...

Highly charged ions, e.g. 3-electron (Li-like) ion:
"Small" parameters of the theory:

αZ: electron-nucleus Coulomb interaction

$1/Z$: relative strength of the electron-electron interaction

$\alpha \approx 1/137$: fine-structure constant

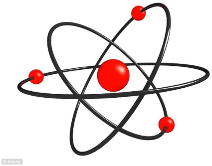
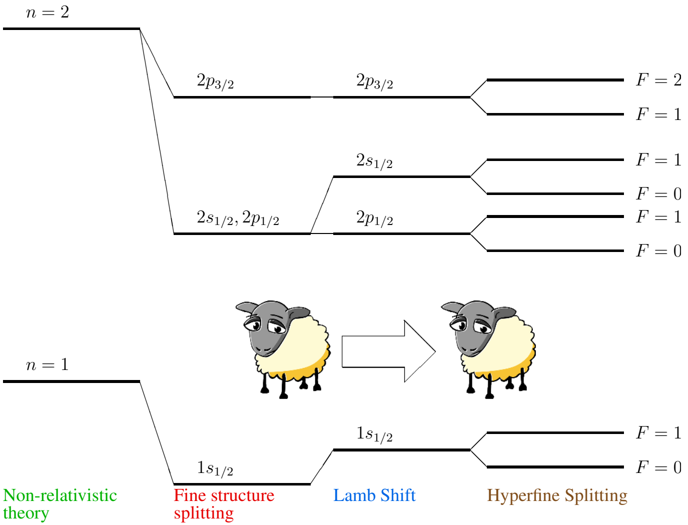


Figure source: cellcode.us

Theory: expansion of atomic properties in these parameters or all-order methods

Hydrogen-like ions



Lamb figure source: www.shutterstock.com

Schrödinger equation

Dirac equation

QED

Nuclear magnetic field

“Fundamental” “constants”

- Speed of light c
- Planck constant $\hbar$
- Electron mass m_e
- Proton mass m_p
- Electron charge e
- Quark masses: $m_u, m_d, m_s, \ldots$
- QCD scale parameter Λ_{QCD}
- Gravitational constant G
- $\ldots$

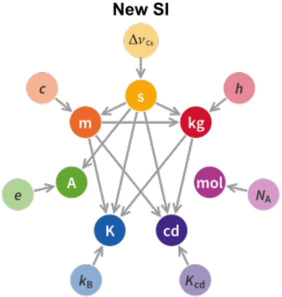


Figure source: https://en.wikipedia.org/

However: neither fundamental nor constants

Example: π

Example: π

Time variation of a fundamental dimensionless constant

Robert J. Scherrer
Department of Physics and Astronomy, Vanderbilt University, Nashville, TN 37235

We examine the time variation of a previously-uninvestigated fundamental dimensionless constant. Constraints are placed on this time variation using historical measurements. A model is presented for the time variation, and it is shown to lead to an accelerated expansion for the universe. Directions for future research are discussed.

PACS numbers: 98.80.Cq

It is well-known that only time variation of dimensionless fundamental constants has any physical meaning. Here we consider the time variation of a dimensionless constant not previously discussed in the literature: π. It is impossible to overstate the significance of this constant. Indeed, nearly every paper in astrophysics makes use of it. (For a randomly-selected collection of such papers, see Refs. [2, 3, 4, 5, 6, 7, 8, 9, 10]).

Location	Time	$\pi(t)$
Babylon	1900 BC	3.125
India	900 BC	3.139
China	263 AD	3.14
China	500 AD	3.1415926
India	1400 AD	3.14159265359

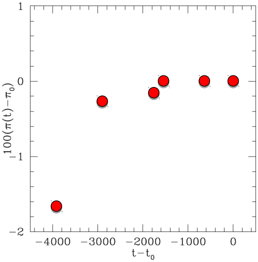


Figure courtesy: P. Scherdtfeder

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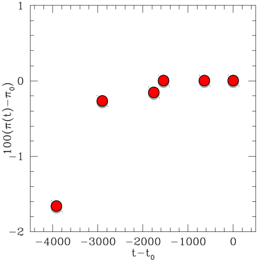


According to the Bible (Old Testament, app. 1400 BC): $\pi = 3$

Figure courtesy: P. Scherdtfeder

Fine-tuning problem

- The existence of life requires certain values for the fundamental constants
- Nuclear reactions in stars are sensitive to α

$$\alpha = \frac{e^2}{\hbar c} \approx \frac{1}{137}$$

If $\alpha \approx 1/134$ (2.5%),
or strong interaction were different by 0.4%,
or quark masses by 1-2%,

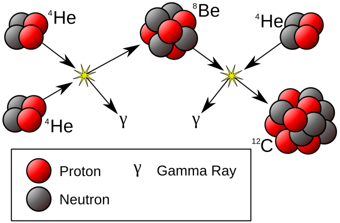


Figure source:

https://en.wikipedia.org/wiki/Triple-alpha_process

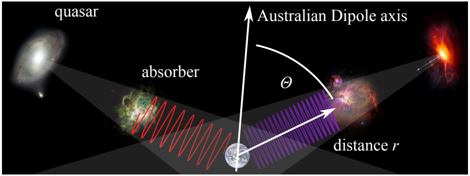
No carbon in stars, no life in the Universe.

α variation in astrophysics

From the analysis of quasar spectra

$$\Delta\alpha/\alpha = A\cos\Theta$$

$$A = (1.02 \pm 0.21)\cdot 10^{-5}$$



J. K. Webb *et al.*, PRL **107**, 191101 (2011)

...and in a laboratory

The motion of the Sun (towards a region with larger α)

$$\Delta\alpha/\alpha \approx 10^{-18} - 10^{-19}\mathrm{yr}^{-1}$$

The motion of the earth around the Sun

$$\Delta\alpha/\alpha = 1.4\cdot 10^{-20}\cos(\omega t)$$

J. C. Berengut *et al.*, Europhys. Lett. **97**, 20006 (2012)

How to measure the variation

Only variation of *dimensionless* FC can be measured:

- fine-structure constant α
- electron-proton mass ratio m_e/m_p
- quark masses on the scale of quantum chromodynamics $m_q/\Lambda_{\mathrm{QCD}}$
- ...

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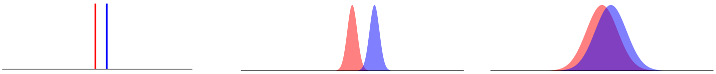
- ...

By measuring the energy two times, Today and Tomorrow, we can theoretically see



How to measure the variation

If a transition energy depends on α, (m_e/m_p), and $m_q/\Lambda_{\mathrm{QCD}}$, one can write:

$$\frac{\Delta E}{E} = K_\alpha \frac{\Delta\alpha}{\alpha} + K_{e/p} \frac{\Delta(m_e/m_p)}{m_e/m_p} + K_q \frac{\Delta(m_q/\Lambda_{\mathrm{QCD}})}{m_q/\Lambda_{\mathrm{QCD}}}$$

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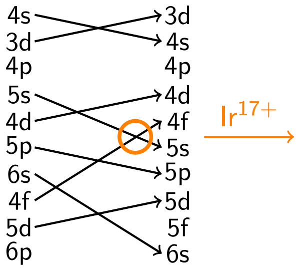
Conditions:

1. Maximized enhancement factors K
2. Minimized linewidth Γ
3. Optical transition in stable atomic system
4. Doable

Configurations-level crossing HCI ions: Ir^{17+}

Talk of Julian Berengut

$Z = 77$, 60 electrons:



J. C. Berengut *et al.*, PRL **106**, 210802 (2011)

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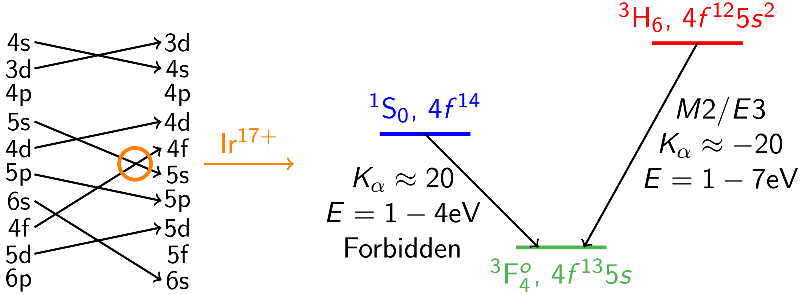

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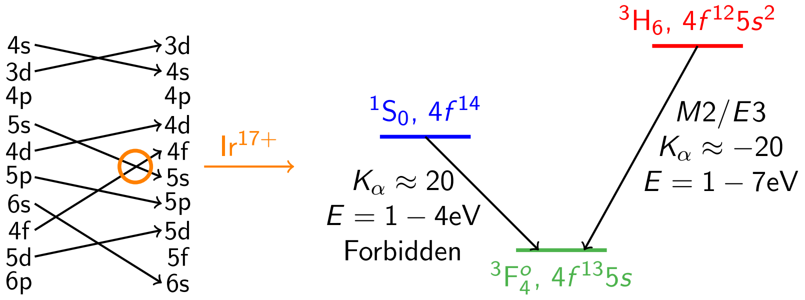

- Theory can not predict position of the clock line with required accuracy
- Identification of lines in spectrum is needed

J. C. Berengut *et al.*, PRL **106**, 210802 (2011)

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- ✓ $K_\alpha = \pm 20$
- ✓ Optical range
- ✓ $\Gamma \approx 10^{-10}$ Hz

A. Windberger, J. R. Crespo López-Urrutia, H. Bekker, NSO *et al.*, PRL **114** 150801 (2015)
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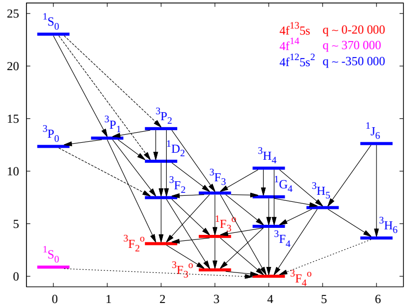
- Intraconfiguration $M1$ transitions: theory 0.1-1%



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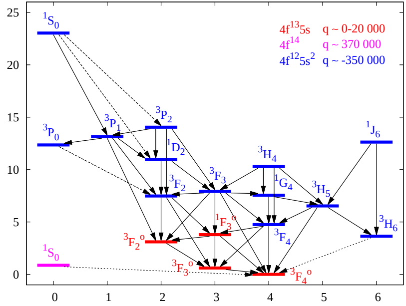

- Narrow lines: accurate theoretical predictions are needed

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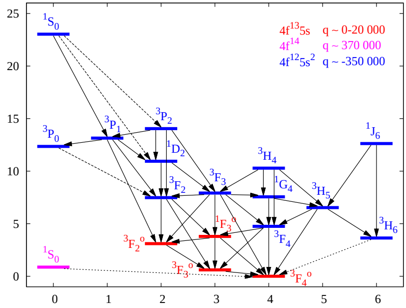

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  no theory predictions, no observations

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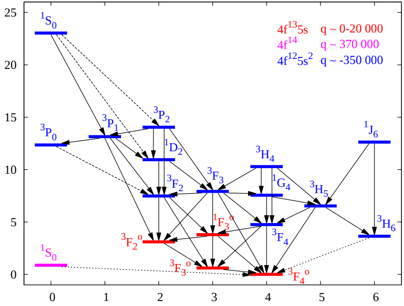

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✗ Configuration-crossing are determined by their atomic properties

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Hyperfine structure of H-like and Li-like ions

Talk of Valeriia Kosheleva

- interaction between nuclear magnetic dipole moment and the magnetic field generated by the electron(s)
- optical, accessible for the experiments
- simple atomic system with one or three electrons
- probing the magnetic sector of QED in strong field

$$E_\mu = \boxed{\frac{\alpha(\alpha Z)^3}{n^3(2l+1)j(j+1)}\frac{m_e}{m_p}\frac{\mu}{\mu_N}\frac{2I+1}{2I}m_e c^2}$$

Non-relativistic factor

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$$\times\Big[\boxed{A(\alpha Z)}$$

Relativistic factor

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Non-relativistic factor

$$\times \Big[A(\alpha Z) \cdot (1 - \delta)(1 - \varepsilon) \Big)$$

Relativistic factor

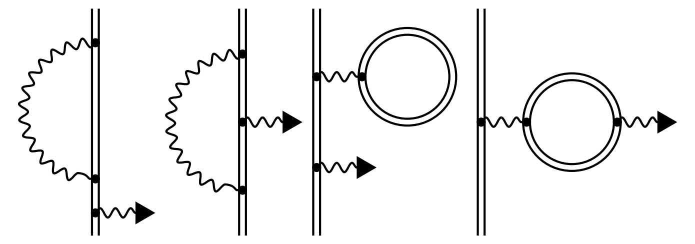
Finite nuclear size correction

Bohr-Weisskopf correction

Hyperfine structure of H-like and Li-like ions

Talk of Valeriia Kosheleva

QED correstions: self-energy and vacuum polarization



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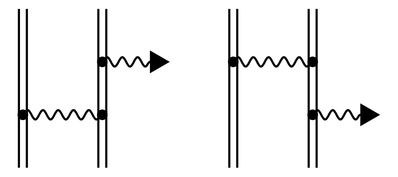
Talk of Valeriia Kosheleva

- interac … and the
magnetic
- optical
- simple
- probing

Electron-electron interaction



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Non-relativistic factor

$$\times\left[\boxed{A(\alpha Z)} \cdot (1-\boxed{\delta})(1-\boxed{\varepsilon}) + \boxed{\frac{1}{Z}B(\alpha Z)} + \boxed{x_{\rm rad}}\right].$$

Relativistic factor

Finite nuclear size correction

Bohr-Weisskopf correction

Electron-electron correlation

QED correction

HFS in simple HCI for VFC

Neglecting nuclear and QED effects, one can write an analytical expression for α-sensitivity enhancement factor:

$$K_\alpha \begin{bmatrix} 1s \\ 2s \end{bmatrix} \equiv \frac{dE}{d\alpha} \Big/ \frac{E}{\alpha} = 4 + \begin{bmatrix} 3 \\ 17/4 \end{bmatrix} (\alpha Z)^2 + \begin{bmatrix} 4 \\ 5 \end{bmatrix} (\alpha Z)^4 + \ldots$$

NSO *et al.*, PRA **96**, 030501(R) (2017)

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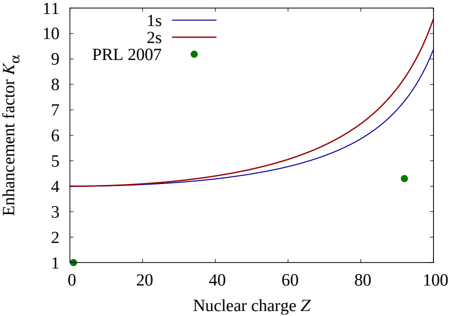

- FS: far beyond the optical and x-ray range
- HFS: optical, relativistic, common for theory and experiment
- $M1$ transition: suppressed for $Z = 1$

NSO *et al.*, PRA **96**, 030501(R) (2017)
S. Schiller, PRL **98**, 180801 (2007)

H-like & Li-like HFS-based clocks

H-like ions:

$^{113}In^{48+}$, $^{121}Sb^{50+}$,
$^{133}Cs^{54+}$, $^{139}La^{56+}$,
$^{153}Eu^{62+}$, $^{159}Tb^{64+}$,
$^{171}Yb^{69+}$, $^{189}Os^{75+}$,
$^{195}Pt^{77+}$, $^{197}Au^{78+}$,
$^{199}Hg^{79+}$, $^{207}Pb^{81+}$,
$^{235}U^{91+}$

Li-like ions:

$^{175}Lu^{68+}$, $^{185}Re^{72+}$,
$^{187}Re^{72+}$, $^{203}Tl^{78+}$,
$^{209}Bi^{80+}$, $^{231}Pa^{88+}$

NSO *et al.*, PRA **96**, 030501(R) (2017)

H-like & Li-like HFS-based clocks

- $K_\alpha = 4 - 7$
- $K_{e/p} = 1$
- $K_q < 0.1$, except for $^{159}Tb^{64+}$ and $^{197}Au^{78+}$ with $K_q = 1.165$
- Optical for a wide range of ions
- $\Gamma = 1 - 10$ Hz
- Quality factor is $Q \approx 10^{14} - 10^{15}$
- Systematic effects: decrease with $Z_{\rm eff}$, the accuracy of $10^{-19} - 10^{-20}$ can be achieved

<u>H-like ions:</u>
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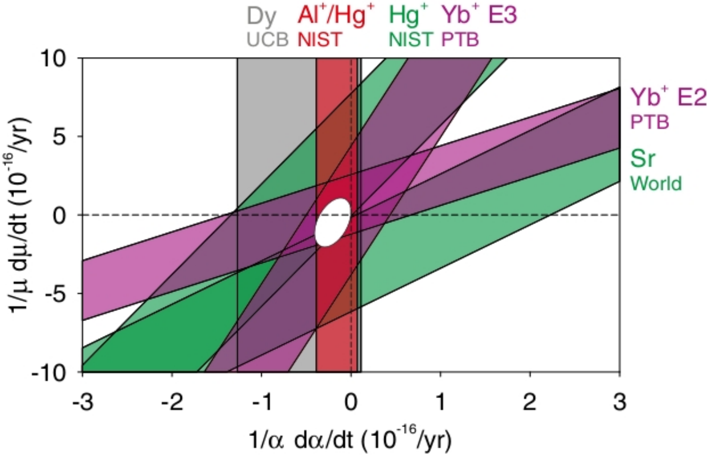
Current limits on the variation of fundamental constants



$$\frac{\delta\alpha}{\alpha} = -2.0(2.0) \times 10^{-17}/\text{yr} \qquad \frac{\delta(m_e/m_p)}{m_e/m_p} = 0.5(1.6) \times 10^{-16}/\text{yr}$$

N. Huntemann *et al.*, PRL **113**, 210802 (2014) A. D. Ludlow *et al.*, RMP **87**, 637 (2015)

Variation of fundamental constants

HFS with "insensitive" ($K_\alpha = K_{e/p} = 0$) atomic clocks
The conservative experimental accuracy of 10^{-17}/yr

$$\left|(K_\alpha^{\mathrm{HFS}} - K_\alpha^{\mathrm{insen.}} - 2)\frac{\delta\alpha}{\alpha} + \frac{\delta(m_e/m_p)}{m_e/m_p}\right| < 10^{-17}/\mathrm{yr}$$

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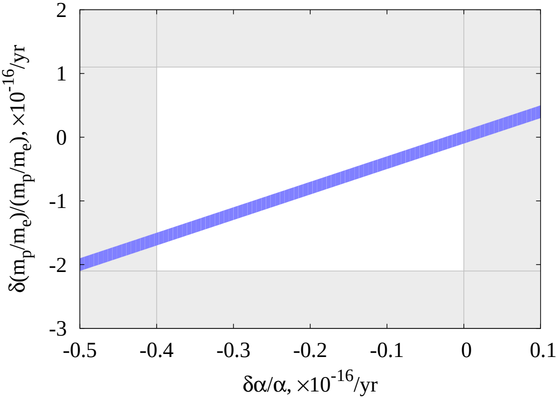

Gray area is excluded according to the current limitations on α and m_e/m_p

Multiple constraints!

NSO *et al.*, PRA **96**, 030501(R) (2017)

HFS: sensitivity on the potential variation of quark masses

In the single-particle approximation:

$$\frac{\mu}{\mu_N} = \begin{cases} [g_s + (2I-1)g_l]/2 & \text{for } I = L + 1/2, \\ \frac{I}{2(I+1)}[-g_s + (2I+3)g_l] & \text{for } I = L - 1/2. \end{cases}$$

V. V. Flambaum *et al.* PRD **69**, 1150006 (2004)

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Orbital g factors:

$g_l = 1$ for proton

$g_l = 0$ for neutron

Spin g factors:

$g_s = 5.586$ for proton

$g_s = -3.826$ for neutron

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Spin g factors:

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$m_q = (m_u + m_d)/2$ is the averaged mass of the up and down quarks

m_s is the strange quark mass

V. V. Flambaum *et al.* PRD **69**, 1150006 (2004)

HFS: quark masses variation sensitivity

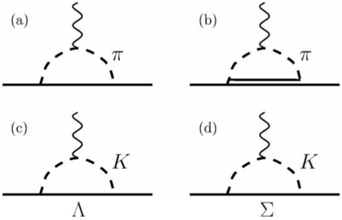

FIG. 1. Chiral corrections to the nucleon magnetic moments included in the present work.

$$\frac{\delta\mu_p}{\mu_p} = -0.087\frac{\delta m_q}{m_q}$$

$$\frac{\delta\mu_p}{\mu_p} = -0.013\frac{\delta m_s}{m_s}$$

$$\frac{\delta\mu_n}{\mu_n} = -0.118\frac{\delta m_q}{m_q}$$

$$\frac{\delta\mu_n}{\mu_n} = 0.0013\frac{\delta m_s}{m_s}$$

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Quark masses variation with HFS HCI

Ion	E_μ	$\Gamma/(2\pi)$	K_α	$K_{e/p}$	K_q	K_s
$^{153}Eu^{62+}$	0.6697(10)	0.967	4.78	1	-0.051	-0.008
$^{159}Tb^{64+}$	1.099(4)	3.79	4.85	1	1.165	0.174

$$K_\alpha[\mathrm{Tb}] - K_\alpha[\mathrm{Eu}] = 0.07$$

$$K_q[\mathrm{Tb}] - K_q[\mathrm{Eu}] = 1.216$$

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$$\left| 1.2 \times \frac{\delta(m_q/\Lambda_{\mathrm{QCD}})}{m_q/\Lambda_{\mathrm{QCD}}} \right| < 10^{-17}/\mathrm{yr}$$

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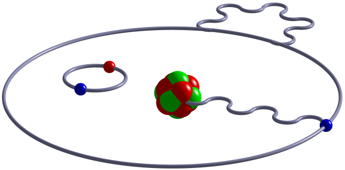
Ion	E_μ	$\Gamma/(2\pi)$	K_α	$K_{e/p}$	K_q	K_s
^{197}Au^{78+}	0.176(2)	0.0142	5.40	1	1.165	0.174
^{199}Hg^{79+}	0.931(12)	1.39	5.45	1	-0.118	0.001

Conclusions and Outlook

- HFS HCI transitions are sensitive to the potential α, m_e/m_p and $m_q/\Lambda_{\mathrm{QCD}}$ variation
- 1s, 2s states: no quadrupole shift



- Simple strong-single-line spectra
- On the 10^{-17} accuracy level, new limitations can be set
- On the 10^{-19} accuracy level, the anticipated variation of α can be observed

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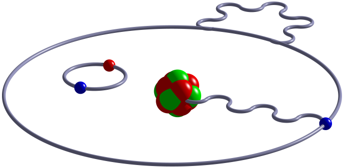

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Thank you for your attention!