Momentum-Space Quantum Walk of a Spinor Bose-Einstein Condensate

Sandro Wimberger

Complex Dynamics in Quantum Systems

Dipartimento di Scienze MFI – Parma University

INFN

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Many thanks to ...

Students at Heidelberg/Parma:

- Marcel Weiß
- Caspar Groiseau
- Michele Delvecchio

Collaborations:

- Andrea Alberti (Bonn, E)
- Raffaella Burioni (Parma, T)
- Gil Summy (Stillwater, E)

Why am I here?

PHYSICAL REVIEW LETTERS **121,** 070402 (2018)

Editors' Suggestion   Featured in Physics

Quantum Walk in Momentum Space with a Bose-Einstein Condensate

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We present a discrete-time, one-dimensional quantum walk based on the entanglement between the momentum of ultracold rubidium atoms (the walk space) and two internal atomic states (the "coin" degree

Quantum walks for what?

- **Universal quantum computation**:

PHYSICAL REVIEW A **81**, 042330 (2010)

Universal quantum computation using the discrete-time quantum walk

Neil B. Lovett,* Sally Cooper, Matthew Everitt, Matthew Trevers, and Viv Kendon†
School of Physics and Astronomy, University of Leeds, Leeds LS2 9JT, United Kingdom
(Received 19 October 2009; revised manuscript received 2 March 2010; published 30 April 2010)

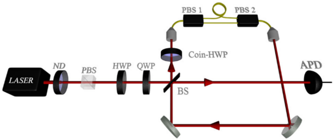
A proof that continuous-time quantum walks are universal for quantum computation, using unweighted graphs of low degree, has recently been presented by A. M. Childs [Phys. Rev. Lett. **102**, 180501 (2009)]. We present a version based instead on the discrete-time quantum walk. We show that the discrete-time quantum walk is able to implement the same universal gate set and thus both discrete and continuous-time quantum walks are computational primitives. Additionally, we give a set of components on which the discrete-time quantum walk provides perfect state transfer.

DOI: 10.1103/PhysRevA.81.042330 PACS number(s): 03.67.Ac, 05.40.Fb

Caveat: for N qubits, 2^N vertices needed in a physical realisation!

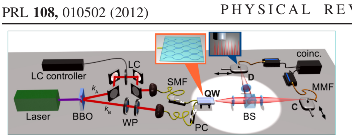
- **Quantum simulation**: Transport phenomena $\&$ topological phases (by intrinsic spin-orbit coupling)

Classical optics



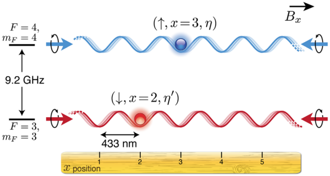
A. Schreiber, K. Cassemiro, V. Potocek, A. Gabris, P. Mosley, E. Andersson, I. Jex, C. Silberhorn PRL 2010

Photonics plus quantum statistics



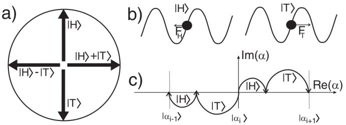
L. Sansoni, F. Sciarrino, G. Vallone, P. Mataloni, A. Crespi, R. Ramponi, R. Osellame PRL 2012

Single Ultracold atoms



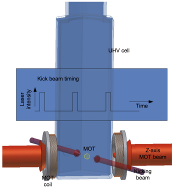
M. Karski, L. Förster, J. Choi, A. Steffen, W. Alt, D. Meschede, A. Widera Science 2009

Trapped ions



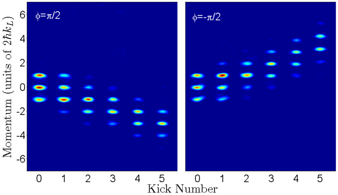
H. Schmitz, R. Matjeschk, C. Schneider, J. Glueckert, M. Enderlein, T. Huber, T. Schätz PRL 2009

Atom-optics kicked rotor



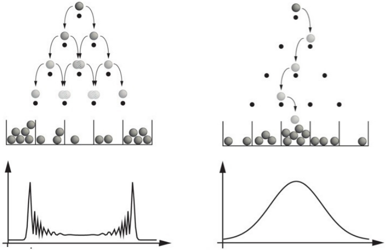
Adv. AMOP 2011

Quantum ratchet



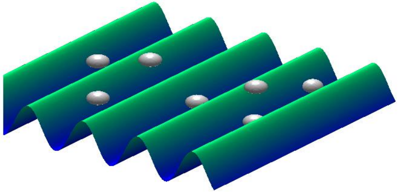
Ann. Phys. 2017

Quantum walks in momentum space

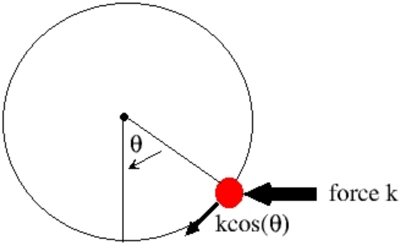


PRL 2018

- Experiments using BEC and optical lattices:
  Maarten Hoogerland and Gil Summy



- Rotator with pulsed gravity at period τ and strength k



- Quantum Hamiltionian consists of free motion and periodic kicks:

$$H(t) = \frac{P^2\tau}{2} + k\cos(X)\sum_{m=-\infty}^{\infty}\delta(t-m)$$

- reconstruction of a periodic problem:

$$P = n + \beta,\ n \in \mathbb{Z},\ \beta \,\epsilon\, [0,1]$$

Bloch's Theorem:

$$\psi(X) = e^{i\beta X}u(X),\ u(\pi) = u(-\pi) \quad \text{with } \theta = X \bmod 2\pi$$

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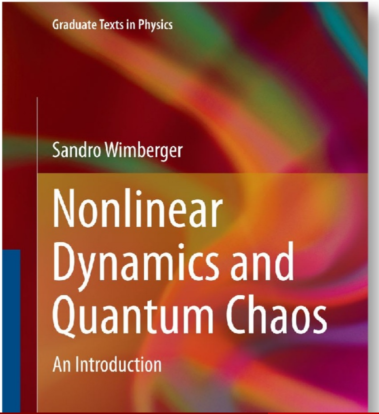


S. Wimberger

Nonlinear Dynamics and Quantum C

An Introduction

Series: Graduate Texts in Physics

- Carefully selected problems set for ea
- The balance between classical and qu among textbooks addressing the sam
- Endorsed by Giulio Casati, one of the l chaos

Quantum resonances

- Corresponding quantum map with evolution operator:

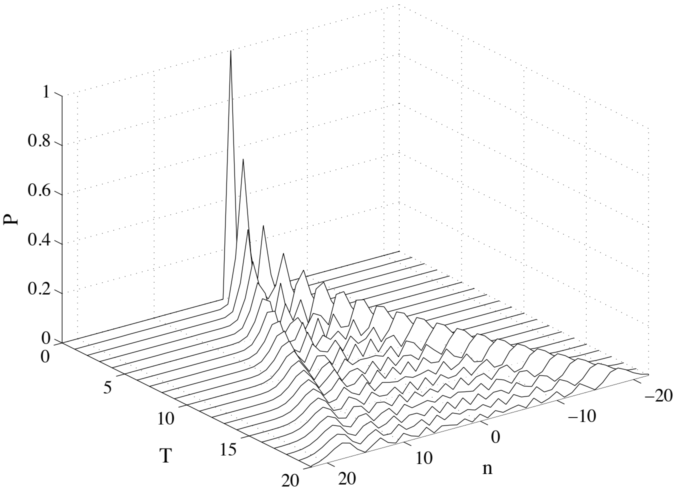
$$\widehat{U} = e^{-ik\cos(\hat{\theta})} \times e^{-i\tau\hat{P}^2/2}, \quad |\psi(t)\rangle = \widehat{U}^t\,|\psi_0\rangle$$

- Regime of ballistic energy growth: quantum resonances

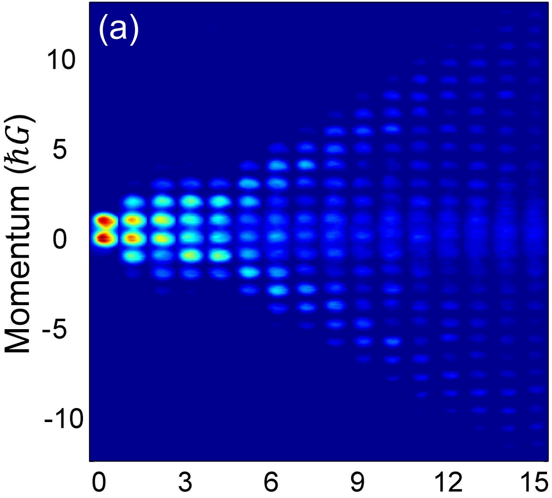
$$e^{-i\frac{\tau}{2}(n^2+2n\beta)} = 1.$$

review: Sadgrove, SW Adv. AMOP 2011

Measurement of momentum distribution

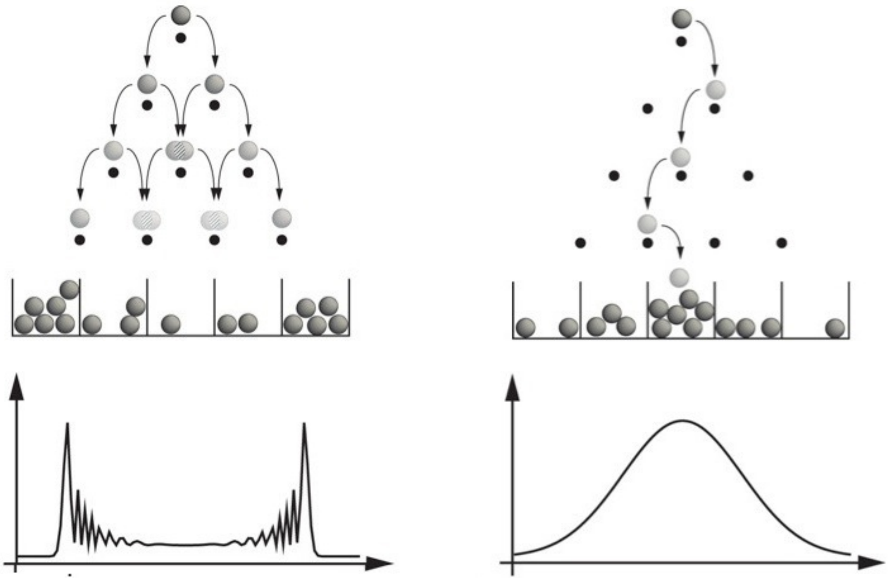


Measurement of momentum distribution

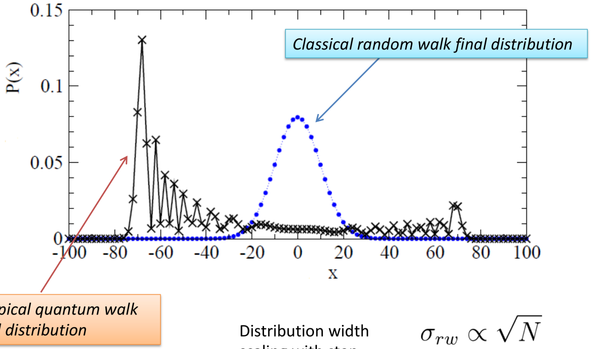


Courtesy of Gil Summy

QUANTUM VS. CLASSICAL WALK



What is a quantum walk?



Distribution width scaling with step number:

$$\sigma_{rw} \propto \sqrt{N}$$
$$\sigma_{qw} \propto N$$

Key differences to classical random walks:

- Interference between different paths leading to the same 'position' occurs
- The quantum evolution deterministic, true randomness only when the walker position is 'measured'

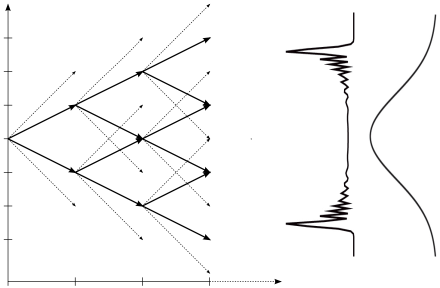
review: Kempe Contemp. Phys. 2003

Kick dynamics at resonance as Q-walk!

$$\widehat{U}(T,0) = \left(e^{-ik\cos(\hat{\theta})}\right)^{T} = e^{-iTk\cos(\hat{\theta})}$$

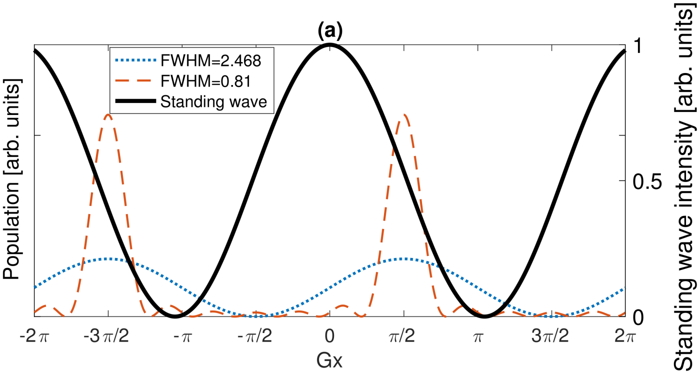
$$P(n,T|n_0,k) = J^2_{n-n_0}(kT) \qquad \text{using}$$

$$J_n(z) = \frac{1}{2\pi}\int_{\alpha}^{\alpha+2\pi} e^{-in\theta}\, e^{iz\cos(\theta)}\, d\theta$$



Usual ratchet: break spatial-temporal symmetry of potential!

BEC ratchet: asymmetric initial state with respect to the driving lattice:



QUANTUM RATCHET VIA SUPERPOSITION

Usual ratchet: break spatial-temporal symmetry of potential!

BEC ratchet: asymmetric initial state with respect to the driving lattice:

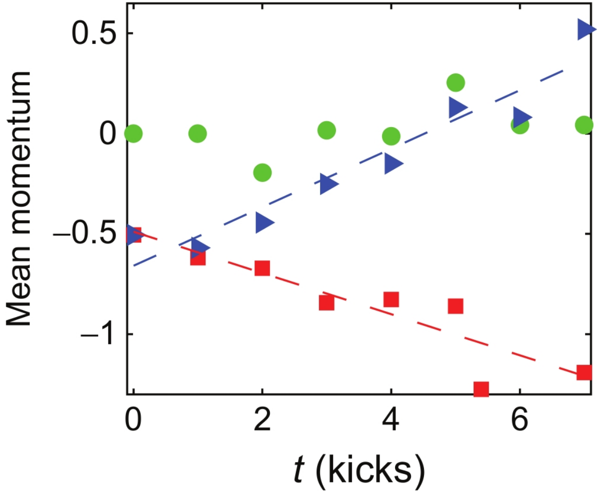
$$|\psi_2(P, t=0)\rangle \;=\; \frac{1}{\sqrt{2}}(|N=0\rangle + \exp(i\phi)\,|N=1\rangle)$$

$$|\psi(P, t=0)|^2 \;\approx\; \frac{1}{2}(\delta_{N,0} + \delta_{N,1}) \quad \text{or}$$

$$|\hat{\psi}(\theta, t=0)|^2 \;\approx\; \frac{1}{2\pi}(1 + \cos(\theta + \phi))$$

QUANTUM RATCHET VIA SUPERPOSITION

$$\langle P_{t,\mathrm{res}} \rangle = -\frac{kt}{2}\sin(\phi)$$

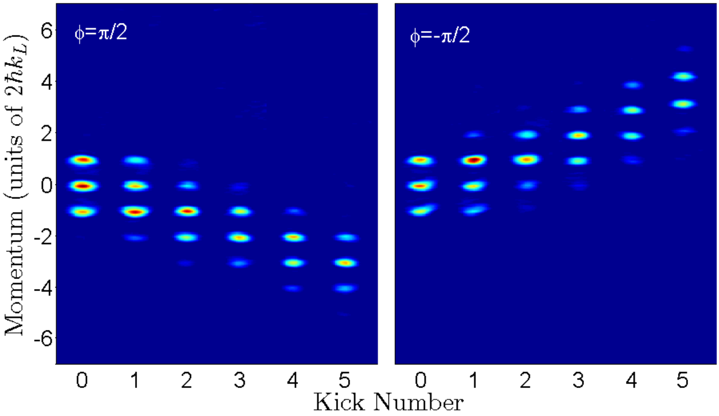


Sadgrove, SW Adv. AMOP 2011

New quantum ratchet

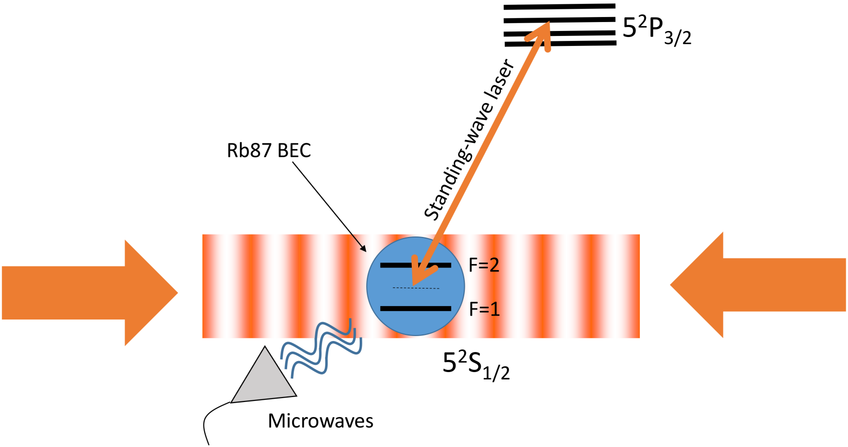
More complex initial states:

$$|\psi_3\rangle = (e^{-i\phi}|-1\rangle + |0\rangle + e^{i\phi}|1\rangle)/\sqrt{3}$$



Ni, Lam, Dadras, Borunda, SW, Summy PRA 2016

Experimental setup for quantum walk



Quantum evolution (for n steps):

$$|\psi_f\rangle = \hat{U}^n|\psi_0\rangle$$

Single step operator:

$$\hat{U} = \hat{S} \cdot \hat{C}$$

Walker shifting operator

"Coin tossing" operator

Our spinor Floquet operator

- Internal spin state mixing after each step by ‘coin’ matrix

$$\mathbf{C} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & \mathrm{i} \\ \mathrm{i} & 1 \end{pmatrix}$$

leads to a strong correlation (entanglement) between the internal and the external (momentum) degrees of freedom.

- External state:

$$\widehat{S} = e^{-ik\cos(\hat{\theta})} \rightarrow \widehat{S}_{\text{kick}} = \exp\left(-\mathrm{i}k\cos(\hat{\theta})\sigma_z\right)$$

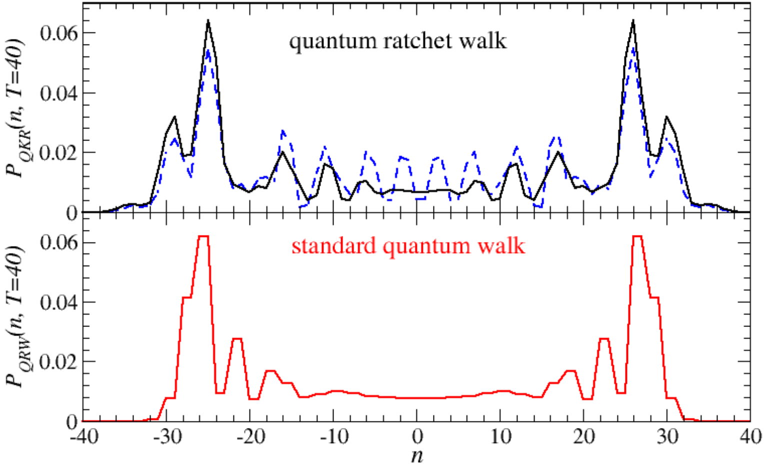
- Final (spin resolved) momentum distribution:

$$P_{-1/2}(n,T) = |c_{n,s=-1/2}(T)|^2 \qquad \text{and} \qquad P_{1/2}(n,T) = |c_{n,s=1/2}(T)|^2$$

for

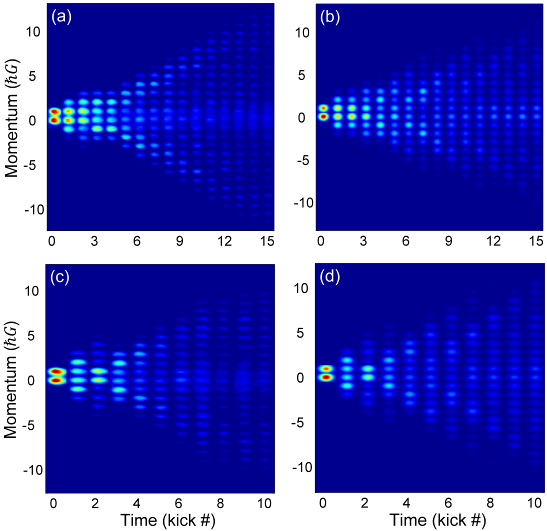
$$|\psi(T)\rangle_{\text{tot}} = \sum_{n,s} c_{n,s}(T)\,|n,s\rangle$$

Kicked-rotor walk vs. standard quantum walk



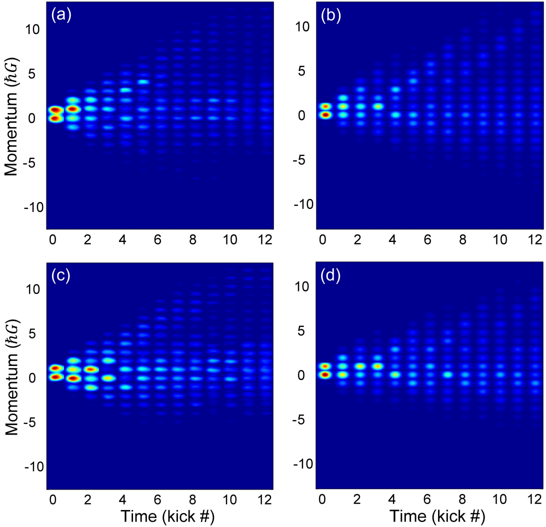
proposal: Summy & SW PRA 2016

Experiment (L) vs. Simulations (R)



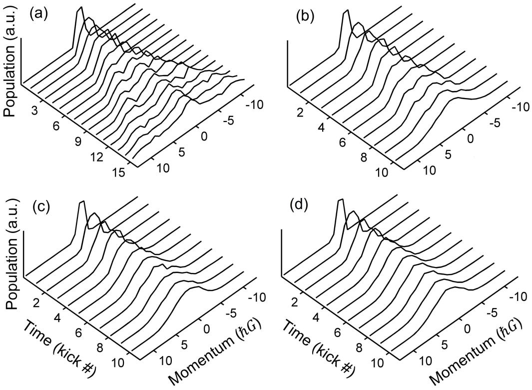
experiment: Dadras, Gresch, Groiseau, SW, Summy PRL 2018

Biased atom-optics kick rotor walk



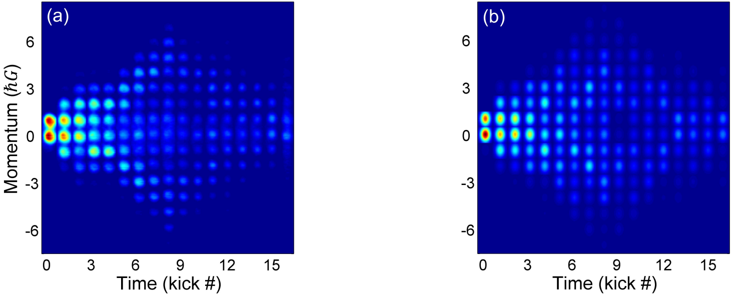
experiment: Dadras, Gresch, Groiseau, SW, Summy PRL 2018

QUANTUM-TO-CLASSICAL: COIN-PHASE NOISE



experiment: Dadras, Gresch, Groiseau, SW, Summy PRL 2018

Matter-wave interferometry by time reversal



experiment: Dadras, Gresch, Groiseau, SW, Summy PRL 2018

- Q-walks with a kicked BEC:
  - ⋆ Simulation of classical random walks, e.g. with power-law tails!
    Weiß, Groiseau, Lam, Burioni, Vezzani, Summy, SW PRA **92**, 033606 (2015)
  - ⋆ Realization of coherent quantum walks, with controlled bias and decoherence!

  Ongoing *Experiments* at Stillwater, OSU–USA!

- Q-walk of a BEC in quasimomentum space in static lattices
  Alberti, SW PRA **96**, 023620 (2017)
- To do: many-body (long-range) effects, quantum search, 2-3D walks, topological/Berry phases

Invitation to submit a paper

Many Body Quantum Chaos

Guest Editor:

Prof. Sandro Wimberger
Dipartimento di Scienze Matematiche, Fisiche e Informatiche, Università di Parma, Parco Area delle Scienze n. 7/A, I-43124 Parma, Italy

sandromarcel.wimberger@unipr.it

Deadline for manuscript submissions:
31 December 2019

Message from the Guest Editor

The field of chaos in many-body quantum systems has a long history, going back to Wigner's simple models for heavy nuclei. Quantum chaos is being investigated in a broad variety of experimental platforms such as heavy nuclei, driven (few-electron) atoms, ultracold quantum gases and photonic or microwave realizations. Quantum chaos nowadays plays a new and important role in many branches of physics, from condensed matter problems of many-body localization, including (pre)thermalization studies in closed and open quantum systems, and the question of dynamical stability relevant for quantum information and quantum simulation. This Special Issue addresses theory and experiment, methods from classical chaos, semiclassics, random matrix theory, as well as many-body condensed matter physics.

Special Issue on Many-body quantum chaos dedicated to Shmuel Fishman in *Condensed Matter*

- Summy, Wimberger, PRA **93**, 023638 (2016)
- Dadras, Gresch, Groiseau, Wimberger, Summy, PRL **121**, 070402 (2018)
- Alberti, Wimberger, PRA **96**, 023620 (2017)