Net-charge fluctuations as a probe of the chiral cross over transition

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Physikalisches Institut Universität Heidelberg

on behalf of ALICE Collaboration
and
ISOQUANT (A01)

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Anar Rustamov, Klaus Reygers, Alice Ohlson

Quantum Systems in Extreme Conditions (QSEC)
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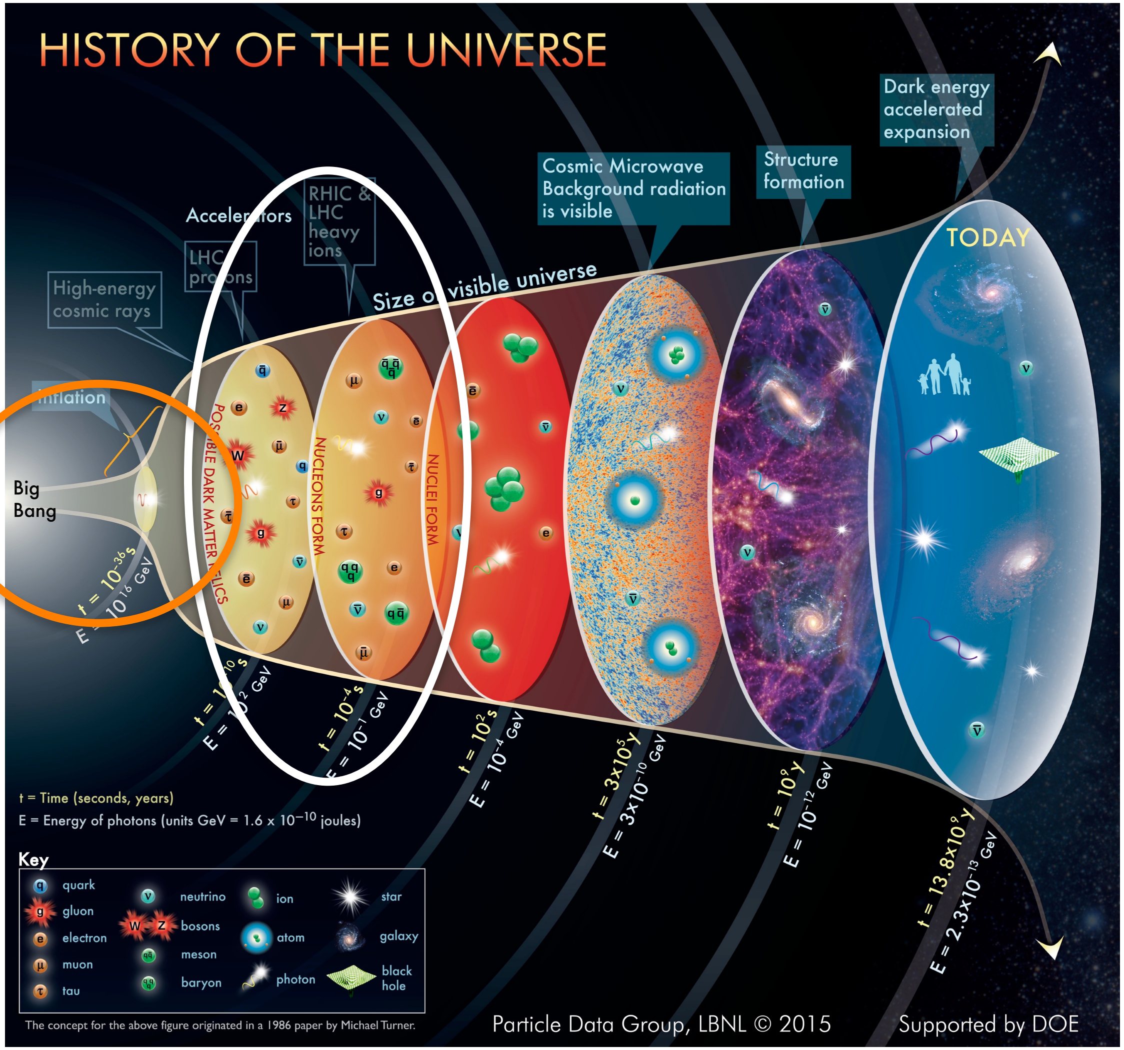
isoQuant SFB1225

BMBF Forschungsschwerpunkt ALICE Experiment 202

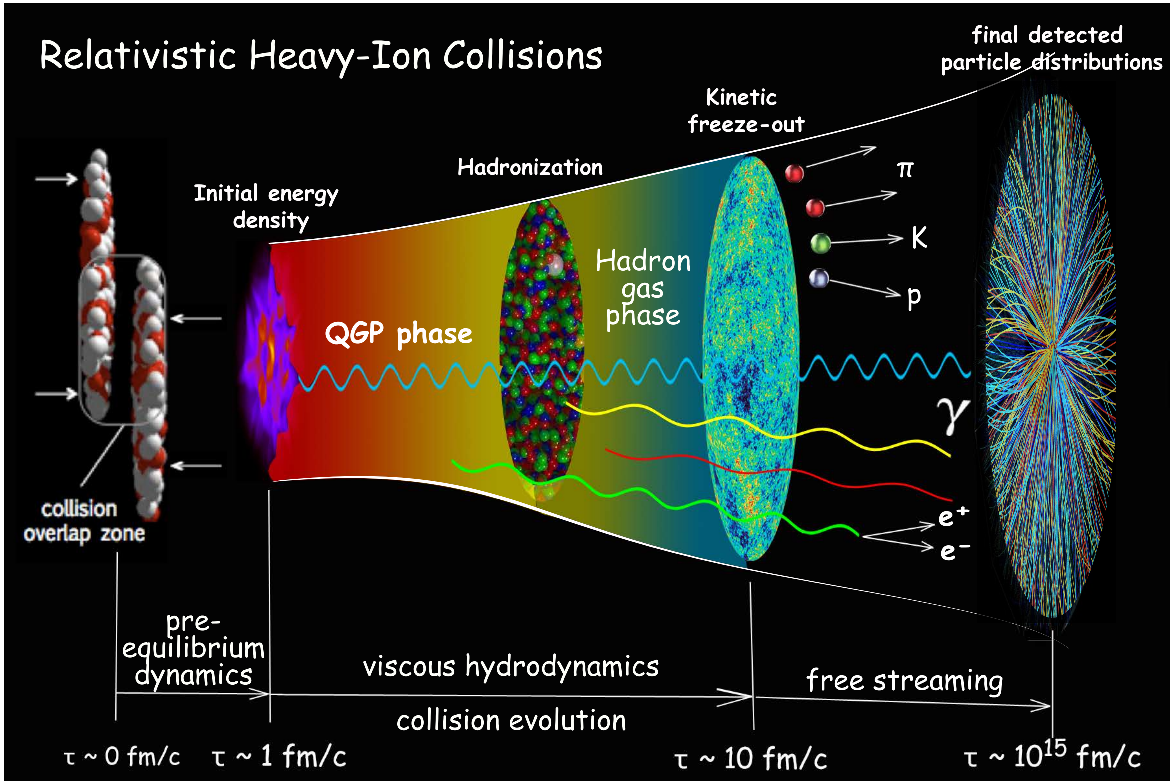
ALICE

"Philosophical GOAL": History of the Universe



Scientific GOAL: Hot QCD Matter in Lab

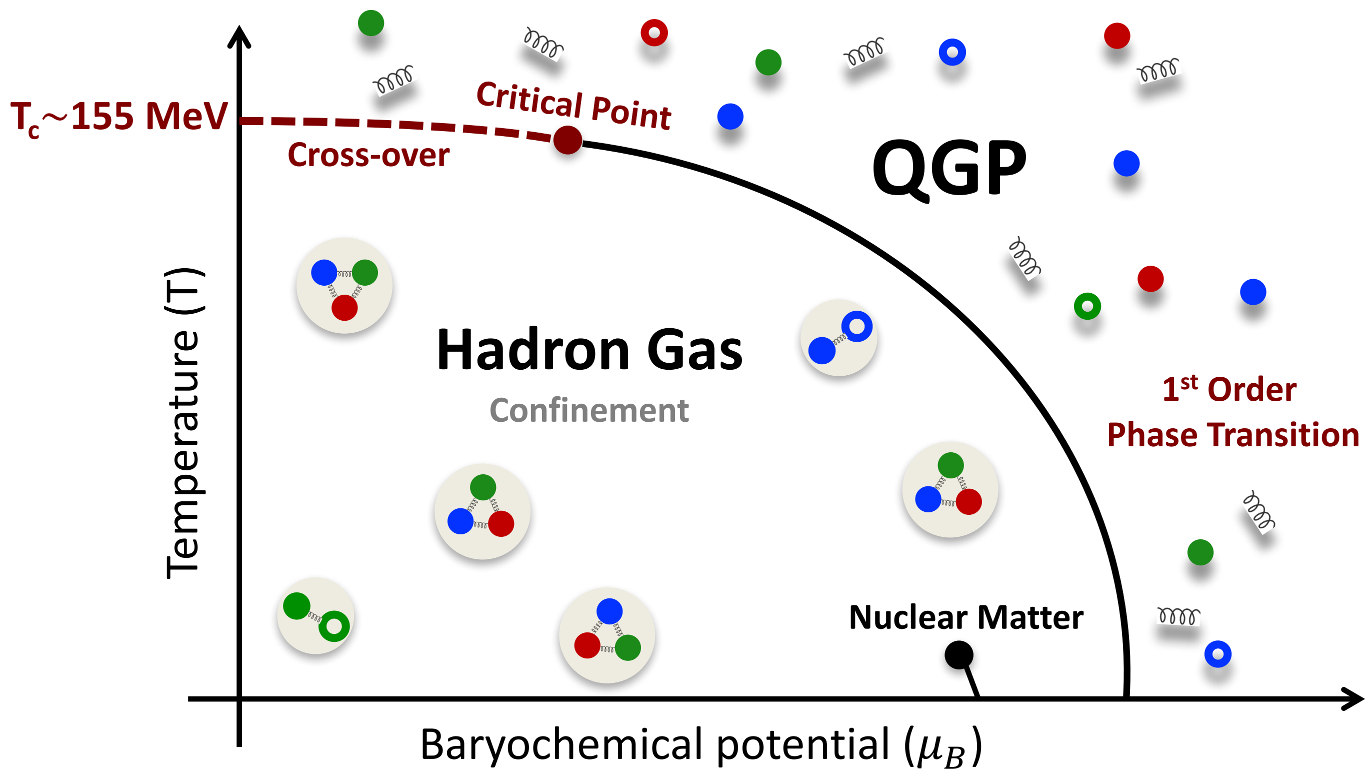
Little-Bang: Relativistic Heavy-ion Collisions



Quark-Gluon Plasma (QGP): A state of matter where the quarks and gluons are the relevant degrees of freedom, exist at few μs after the Big-Bang

- **Chiral symmetry**: $m_p \approx 937$ MeV $\longleftrightarrow$ $2m_u + m_d \approx 10$ MeV
- **Confinement:** No isolated quarks seen thus far

QCD phase diagram



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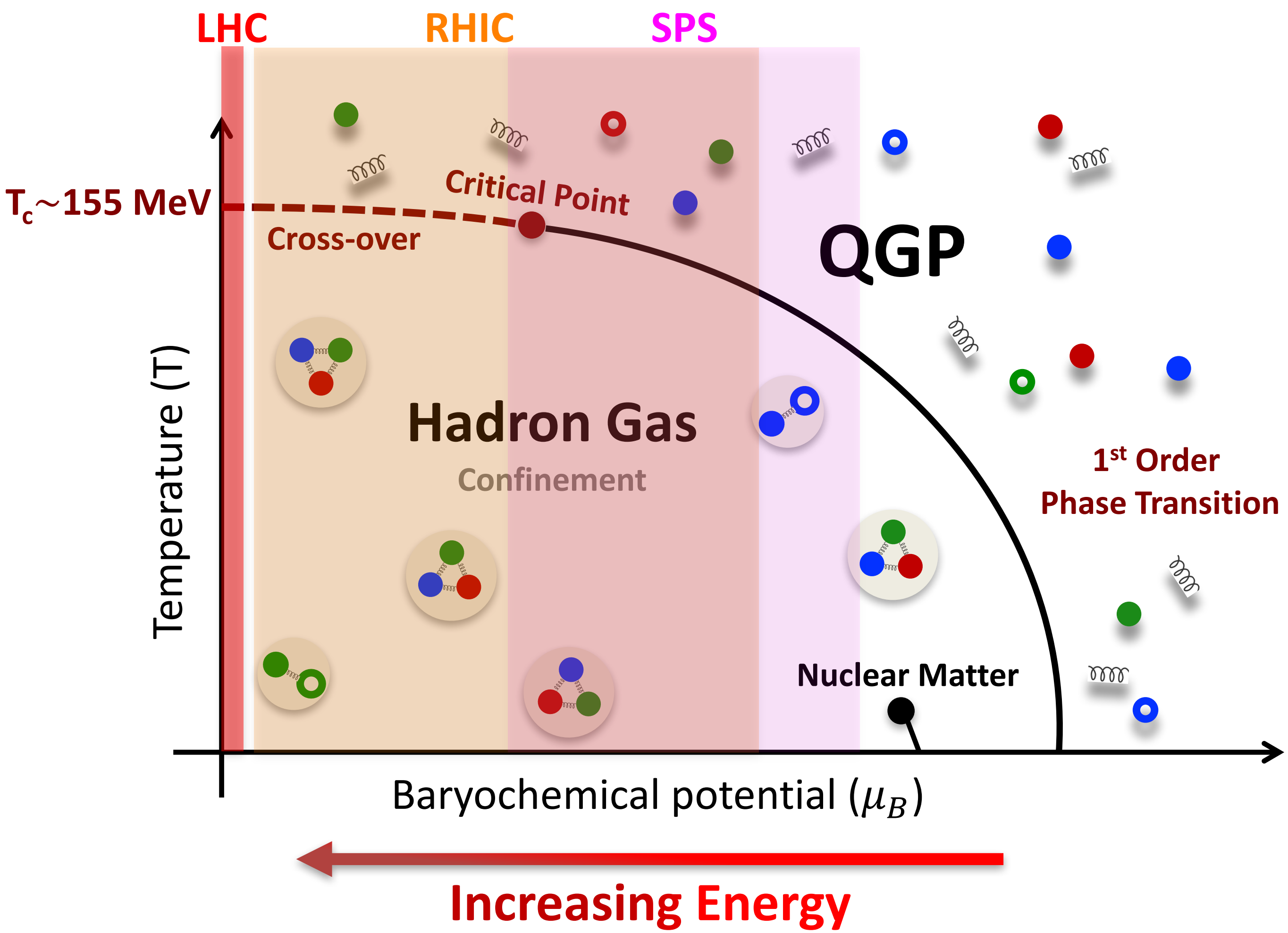
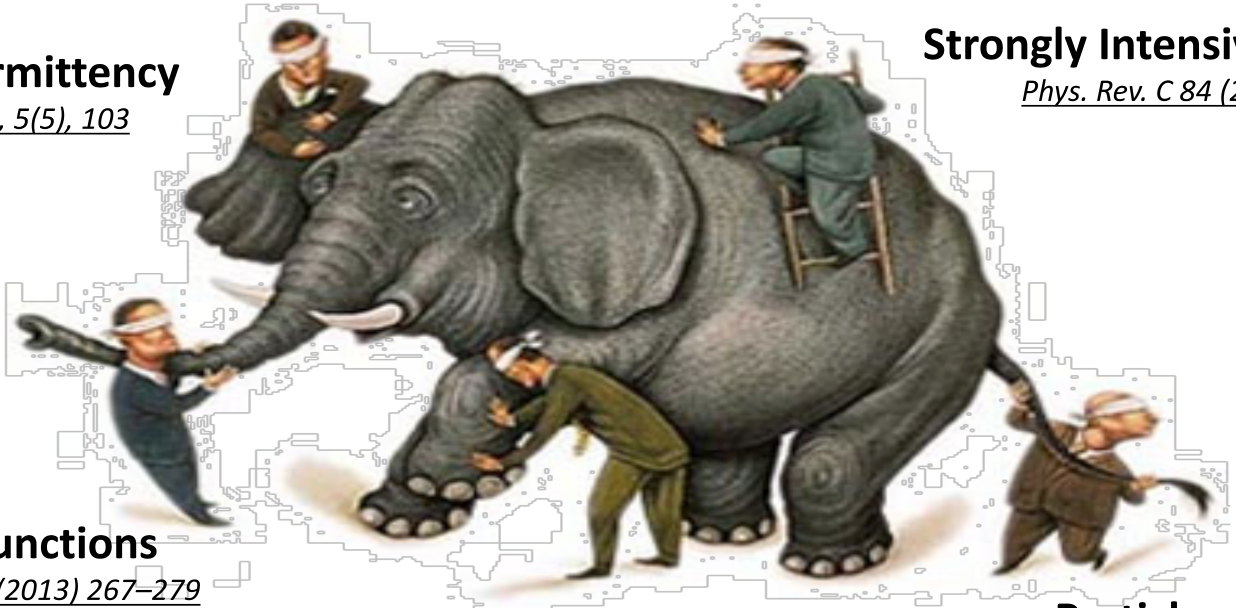


What to study? → Fluctuations

Mean-p_T fluctuations
Eur. Phys. J. C (2014) 74-3077

Proton Intermittency
Universe 2019, 5(5), 103

Strongly Intensive Quantities
Phys. Rev. C 84 (2011) 014904



Balance functions
Phys. Lett. B 723 (2013) 267–279

Particle ratio fluctuations
Eur. Phys. J. C (2019) 79: 236

Conserved-charge fluctuations
Anar Rustamov QM2017, Nucl.Phys. A967 (2017) 453-456

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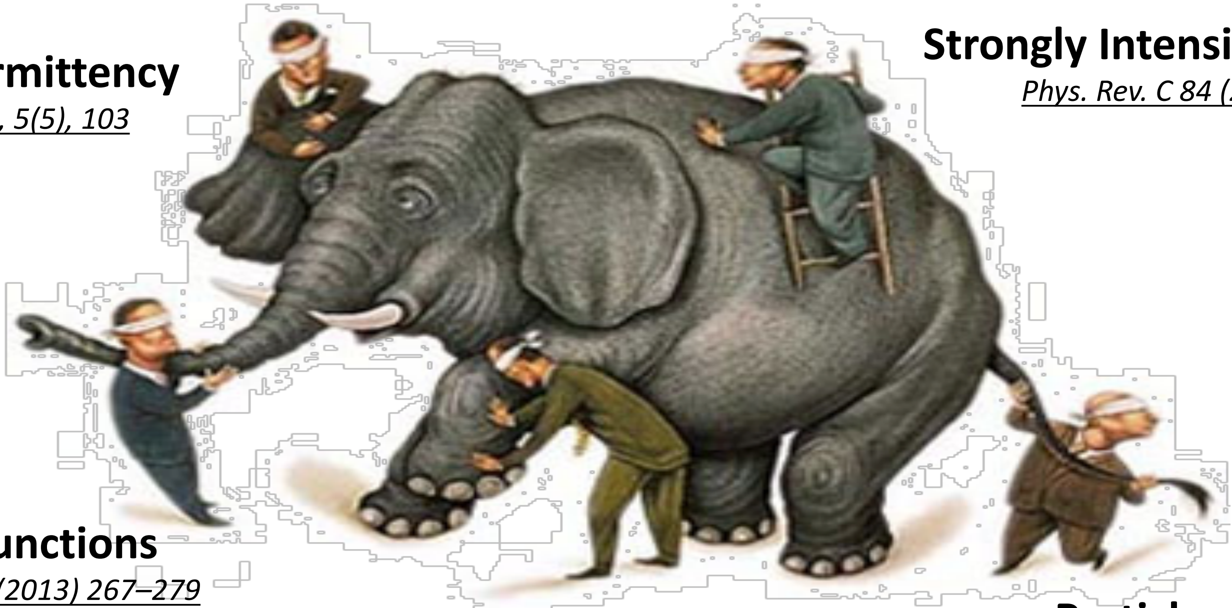

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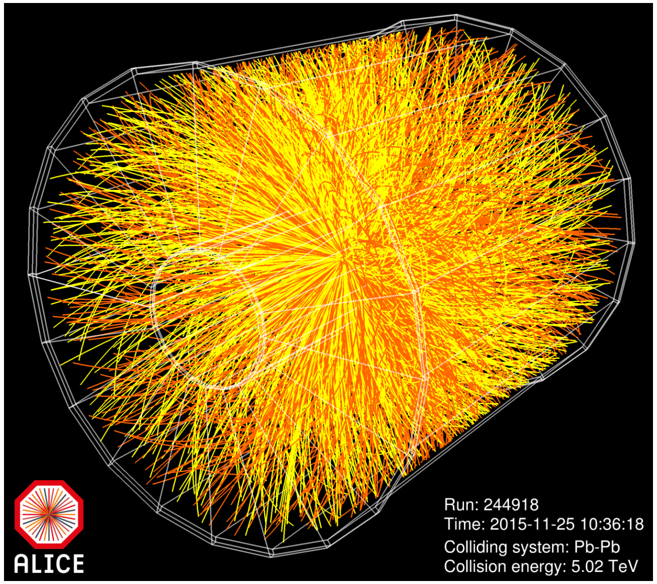
Keywords: Correlations, Criticality, Link to theory

Question #1:

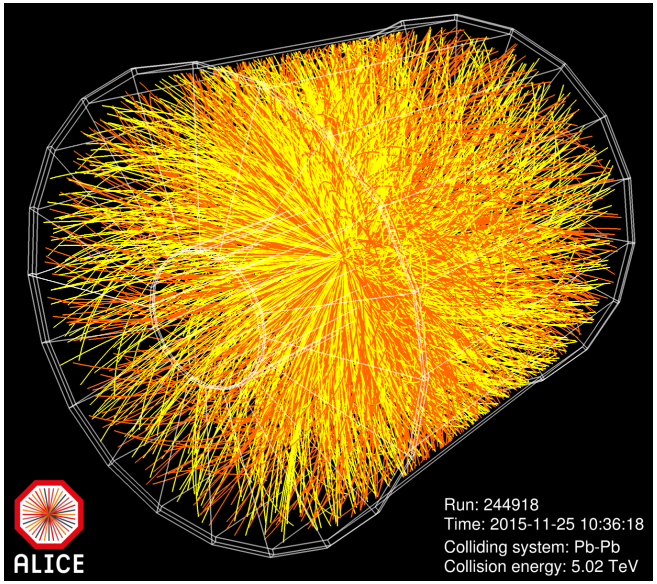
Why fluctuations?

Multiplicity distributions



~15000 charged particles are detected in one central Pb-Pb collision

Multiplicity distributions



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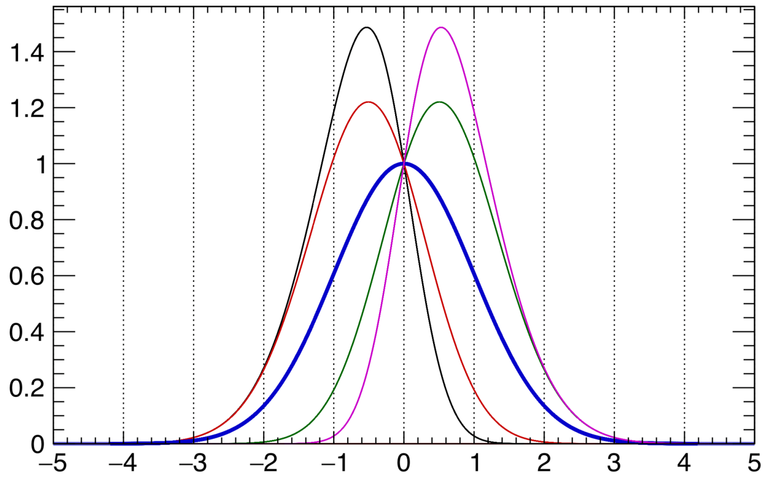
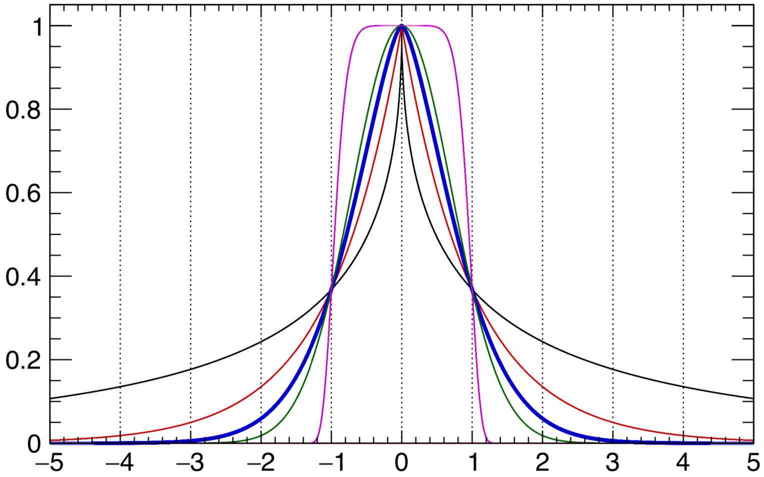
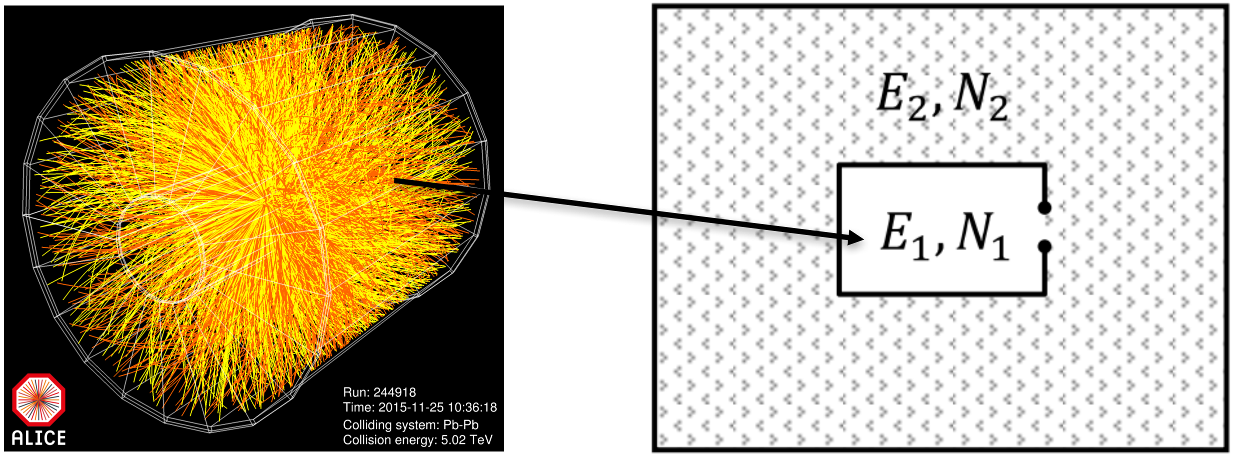
Moments of the multiplicity distributions





What kind of a system we are talking about?



<u>Grand canonical ensemble</u> where particles are in a thermal equilibrium

- Energy (*E*) and number of particles (*N*) are **not conserved** in each microstate

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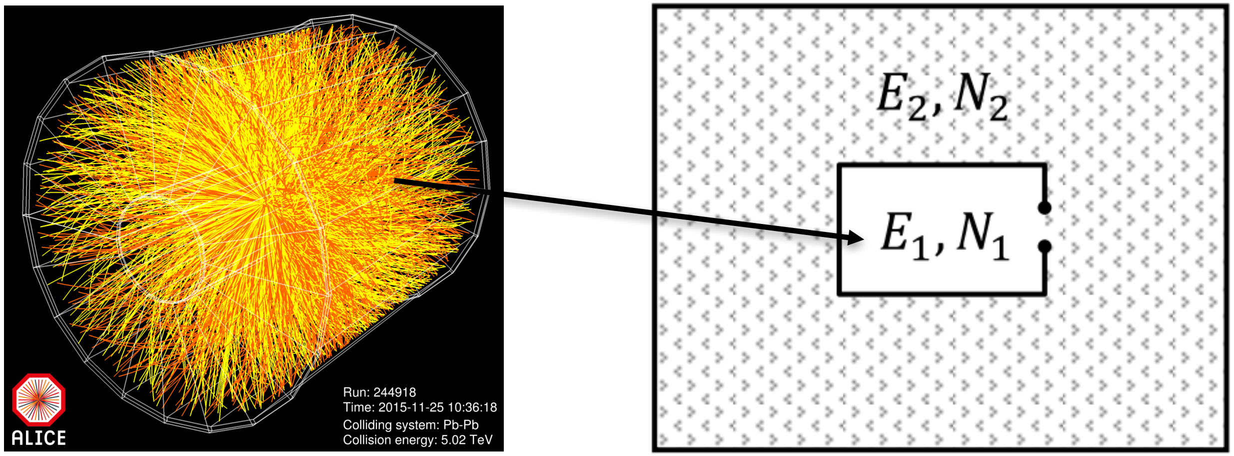

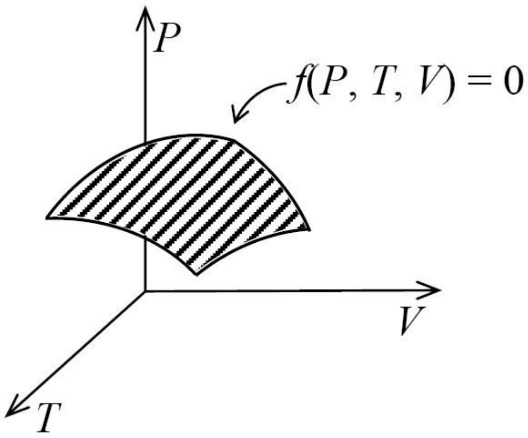

<u>Grand canonical ensemble</u> where particles are in a thermal equilibrium

- Energy (*E*) and number of particles (*N*) are **not conserved** in each microstate
- EOS can be represented **by a surface** in the state space spanned by P, V and T

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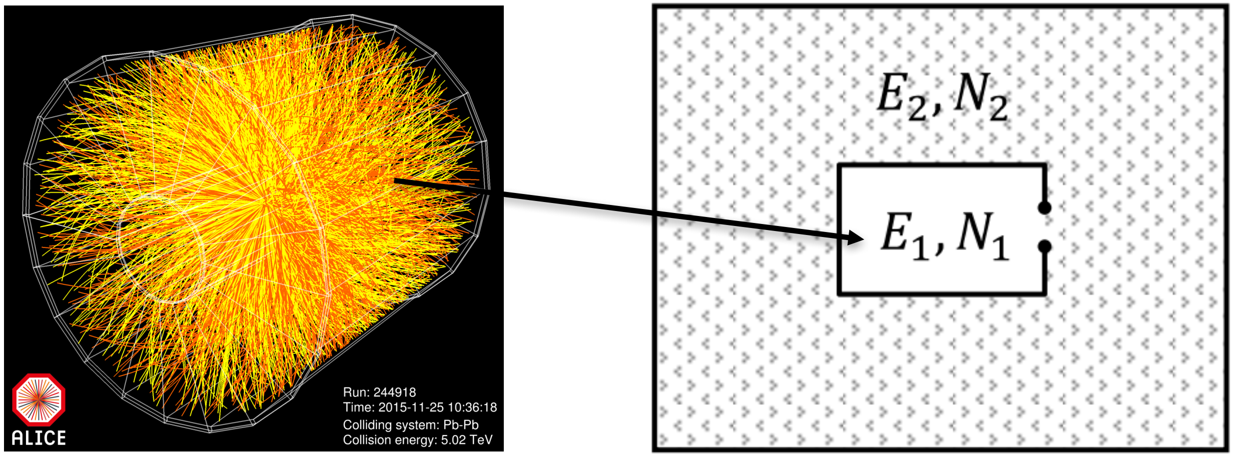
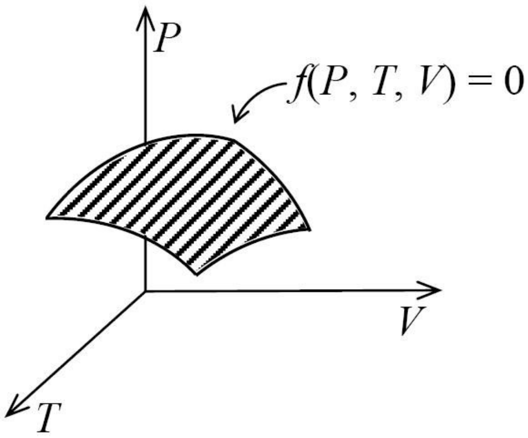




<u>Grand canonical ensemble</u> where particles are in a thermal equilibrium

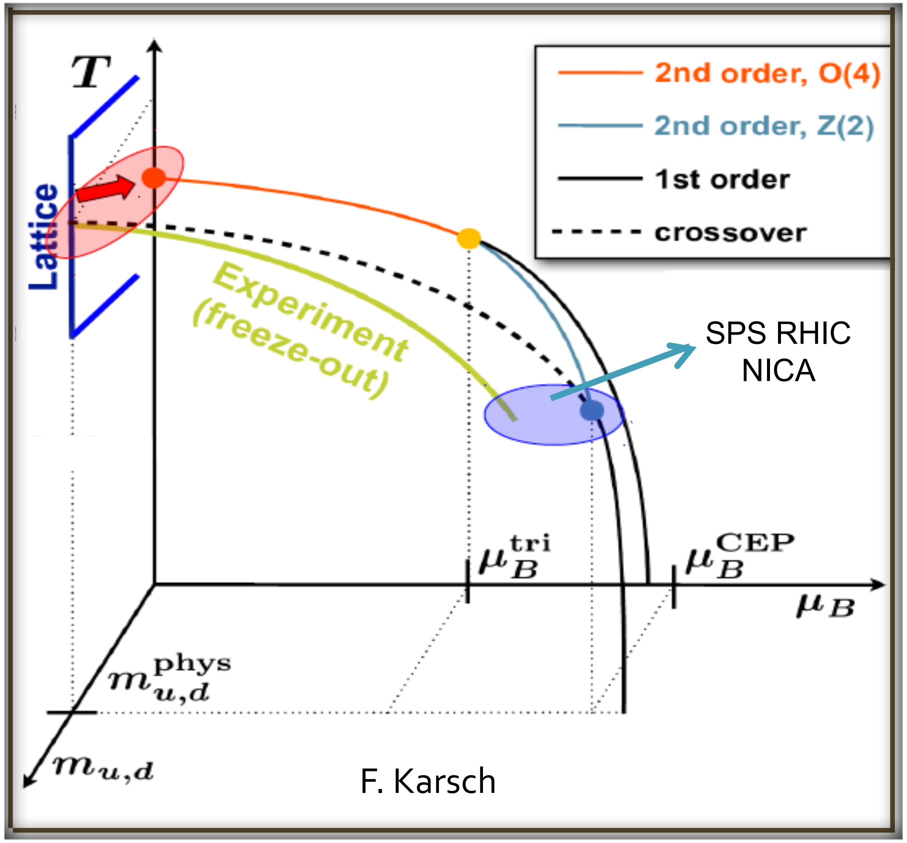
- Energy (*E*) and number of particles (*N*) are **not conserved** in each microstate
- EOS can be represented **by a surface** in the state space spanned by P, V and T
- Conservation laws are applied **on average**
- Chemical potential **(μ_B)**, Volume **(V)** and Temperature (T) are constant
- For a given state E_j and N_j grand canonical partition function

$$Z_{GCE}(T,V,\mu) = \sum_j \exp\left[-\frac{E_j - \mu N_j}{T}\right] \Rightarrow \langle N \rangle = \sum_j N_j p_j = T \frac{\partial \ln Z_{GCE}}{\partial \mu}\bigg|_V$$

Question #2:

How to link experiment to theory?

Closer look at QCD Phase diagram: **Nature of chiral phase transition**



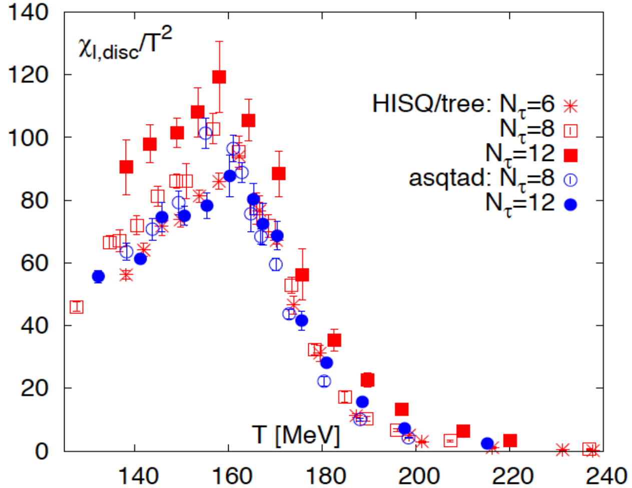
small u,d quark masses ↔ vicinity to O(4) criticality

pseudocritical features

Criticality & Link to Lattice QCD

HotQCD Collaboration

Phys.Rev. D85 (2012) 054503, arXiv:1812.08235



$$T_{pc} = 156.5 \pm 1.5 MeV$$

$$\langle\bar{\psi}\psi\rangle_l^{n_f=2} = \frac{T}{V}\frac{\partial \ln Z}{\partial m_l}$$

$$\chi_{m,l} = \frac{\partial}{\partial m_l}\langle\bar{\psi}\psi\rangle_l^{n_f=2}$$

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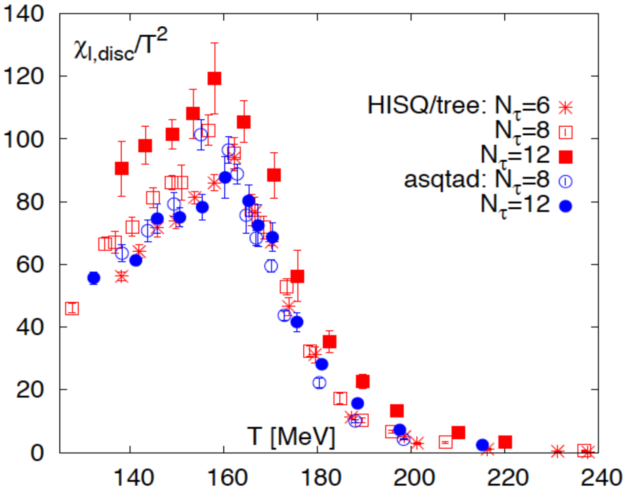
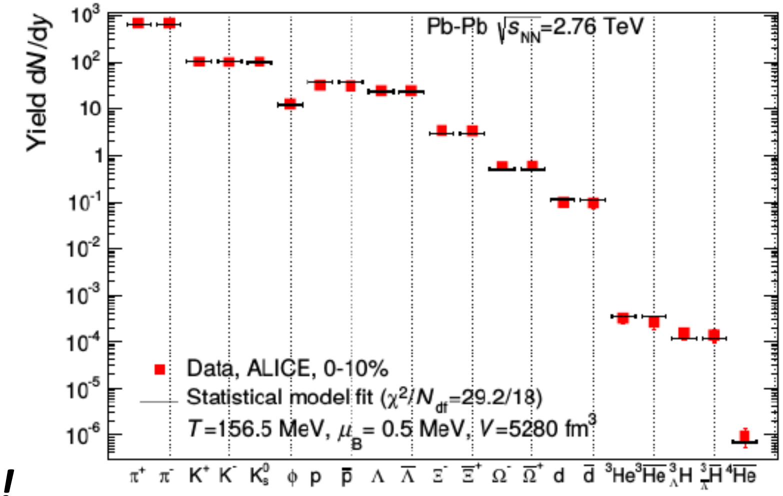


$$T_{pc} = 156.5 \pm 1.5 MeV$$

freeze-out at the phase boundary!

A. Andronic, P. Braun-Munzinger, J. Stachel and K. Redlich
Nature 561, 321–330 (2018), ALICE, PLB 726 (2013) 610



$$T_{fo}^{ALICE} = 156.5 \pm 3 MeV$$

$$\langle \bar{\psi}\psi \rangle_l^{n_f=2} = \frac{T}{V}\frac{\partial \ln Z}{\partial m_l}$$

$$\chi_{m,l} = \frac{\partial}{\partial m_l} \langle \bar{\psi}\psi \rangle_l^{n_f=2}$$

$$\langle N_i \rangle = V \frac{g_i}{2\pi^2} \int_0^\infty \frac{p^2 dp}{\exp\left[\left(E_i - \mu_i\right)/T\right] \pm 1}$$

$$\mu_i = \mu_B B_i + \mu_s S_i + \mu_I I_i$$

$$\chi^2 = \sum_{k=1}^{n} \frac{\left(\langle N_k^{\exp} \rangle - \langle N_k^{HRG} \rangle\right)^2}{\sigma_k^2}$$

Criticality & Link to Lattice QCD

LQCD

$$\frac{P}{T^4} = \frac{1}{VT^3} \ln Z\left(V, T, \mu_{B,Q,S}\right) \Rightarrow \hat{\chi}_n^{N=B,S,Q} = \frac{\partial^n P/T^4}{\partial\left(\mu_N/T\right)^n}$$

Susceptibilities

Criticality & Link to Lattice QCD

LQCD

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Susceptibilities

Experiment

$$\Rightarrow \hat{\chi}_2^B = \frac{\kappa_2\left(\Delta N_B\right)}{VT^3} \Rightarrow \frac{\kappa_4\left(\Delta N_B\right)}{\kappa_2\left(\Delta N_B\right)} = \frac{\hat{\chi}_4^B}{\hat{\chi}_2^B}$$

Cumulants **Higher orders**

P. Braun-Munzinger, A. Rustamov, J. Stachel
Nuclear Physics A 960 (2017) 114–130

Criticality & Link to Lattice QCD

LQCD

$$\frac{P}{T^4} = \frac{1}{VT^3}\ln Z\left(V,T,\mu_{B,Q,S}\right) \;\Rightarrow\; \hat{\chi}_n^{N=B,S,Q} = \frac{\partial^n P/T^4}{\partial\left(\mu_N/T\right)^n}$$

Susceptibilities

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$$\hat{\chi}_2^B = \frac{\kappa_2\left(\Delta N_B\right)}{VT^3}$$

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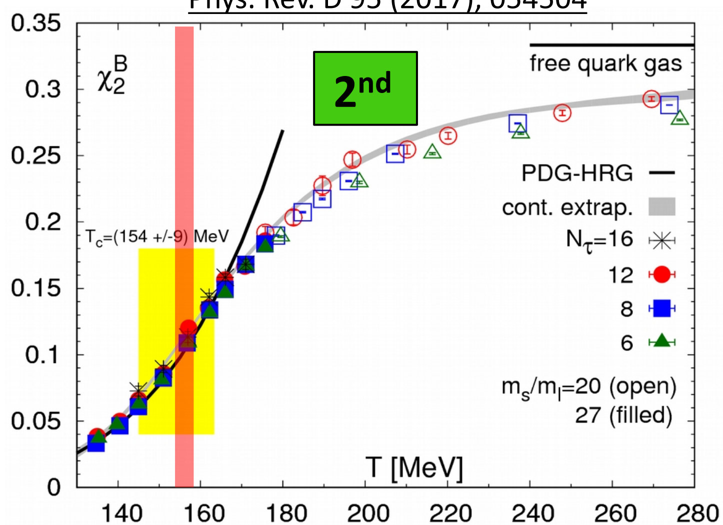
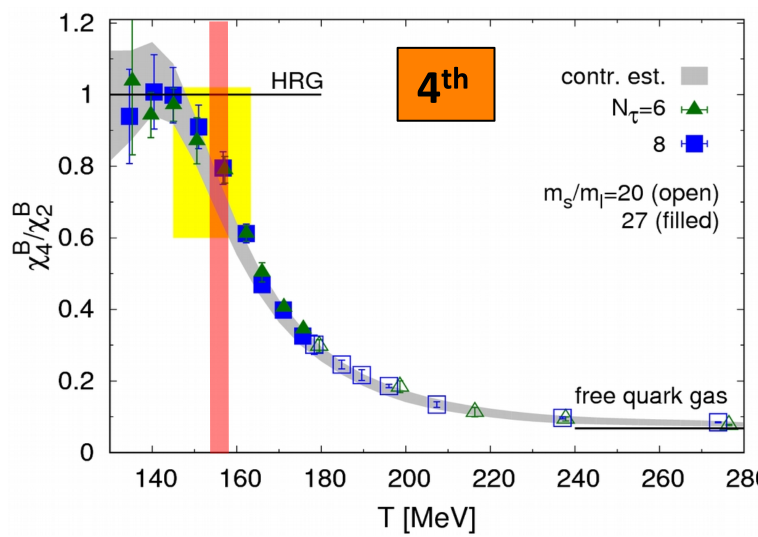
Phys. Rev. D 95 (2017), 054504





- **At 4th order** LQCD shows a **deviation** from Hadron Resonance Gas (HRG)

Question #3:

What is the baseline?

Skellam distribution

$$X = N_B - N_{\overline{B}}$$

- **r^th central moment:**

$$\mu_r \equiv \langle (X - \langle X \rangle)^r \rangle = \sum_X (X - \langle X \rangle)^r P(X)$$

- **First four cumulants**

$$\kappa_1 = \langle X \rangle, \quad \kappa_2 = \mu_2,$$

$$\kappa_3 = \mu_3, \quad \kappa_4 = \mu_4 - 3\mu_2^2$$

- **Uncorrelated Poisson limit:**

$$\langle N_B N_{\overline{B}} \rangle = \langle N_B \rangle \langle N_{\overline{B}} \rangle$$

Skellam distribution

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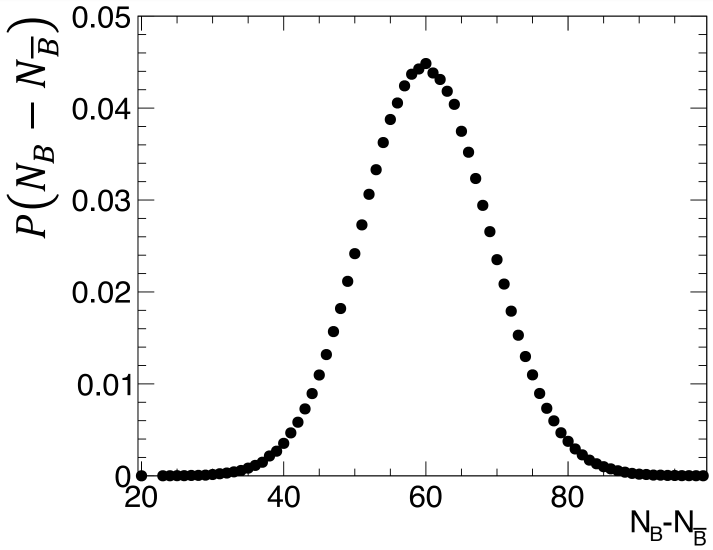


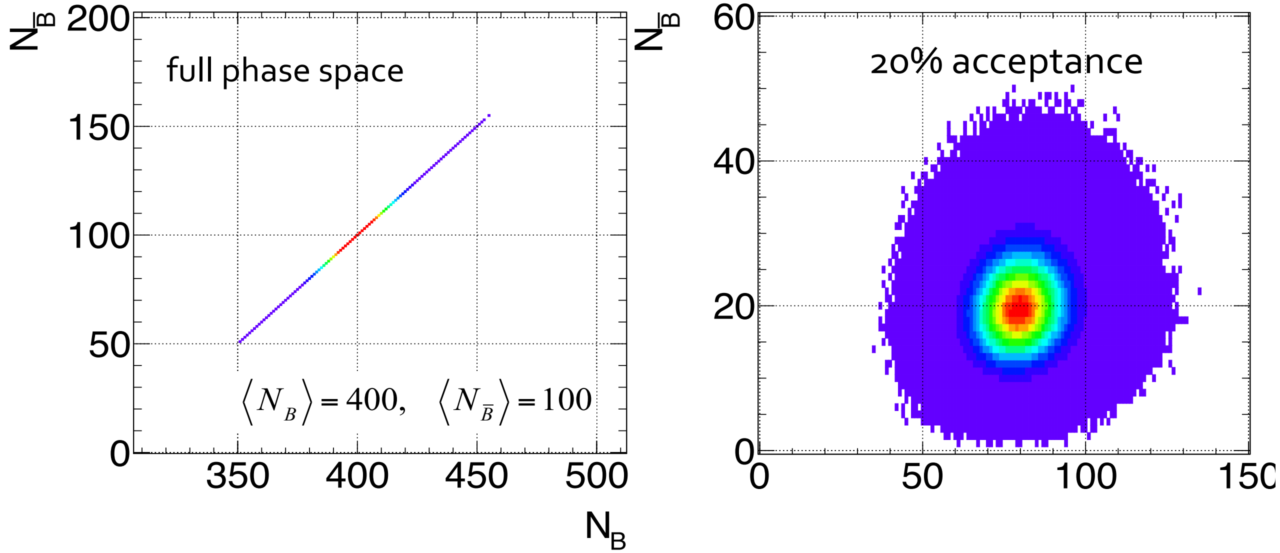
Difference between two independent Poissonian distributions

$$\kappa_n = \langle N_B \rangle + (-1)^n \langle N_{\overline{B}} \rangle$$

$$\frac{\kappa_{2n+1}}{\kappa_{2k}} = \frac{\langle n_B \rangle - \langle n_{\bar{B}} \rangle}{\langle n_B \rangle + \langle n_{\bar{B}} \rangle}$$

Importance of acceptance

- Fluctuations of net-baryons appear only inside **finite acceptance**
- **Baryon number conservation** imposes subtle correlations



P. Braun-Munzinger, A. Rustamov, J. Stachel, QM18, NPA 982 (2019) 307-310

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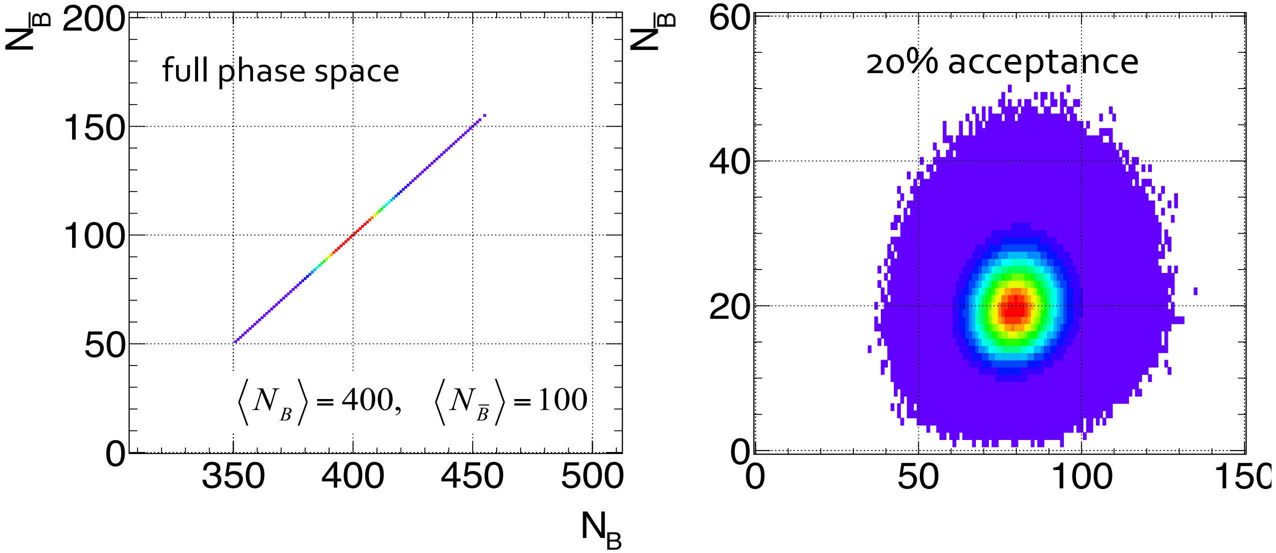

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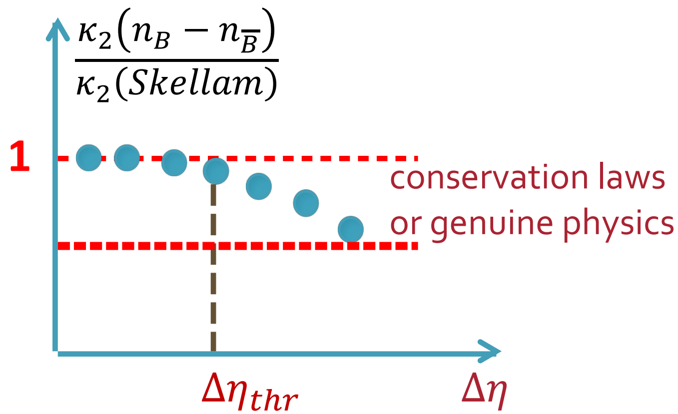


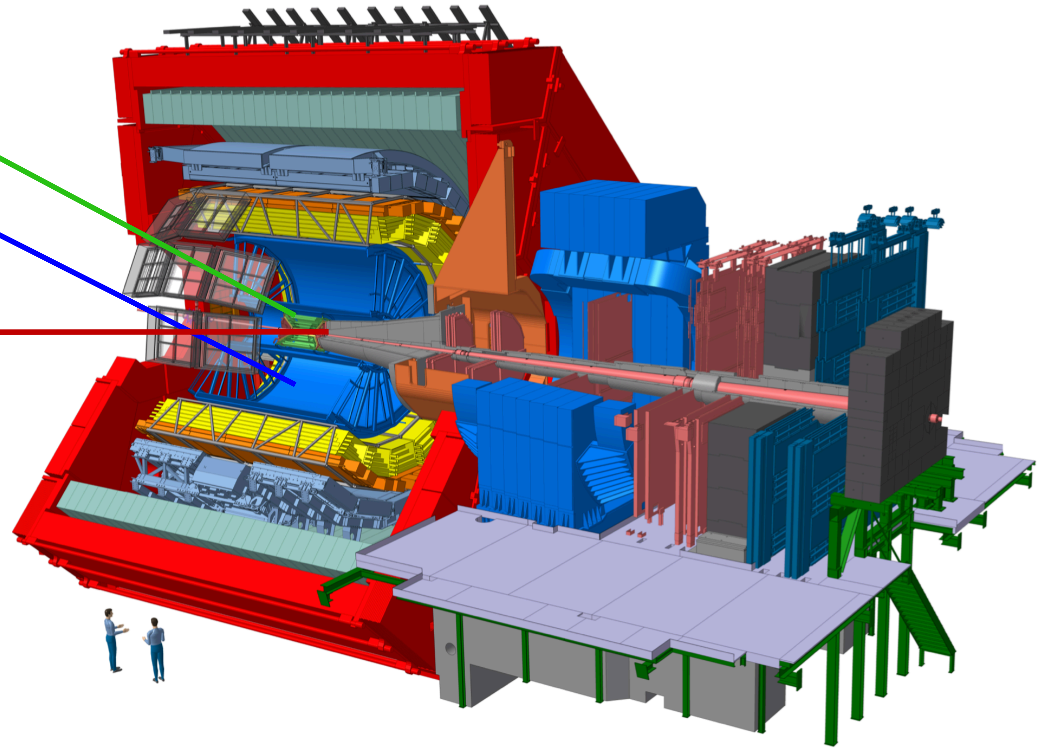
- **Limit of very small acceptance**
  - vanishing or invisible dynamical fluctuations
- **Acceptance has to be large enough**

From data to physics

A Large Ion Collider Experiment

<u>Main detectors used:</u>

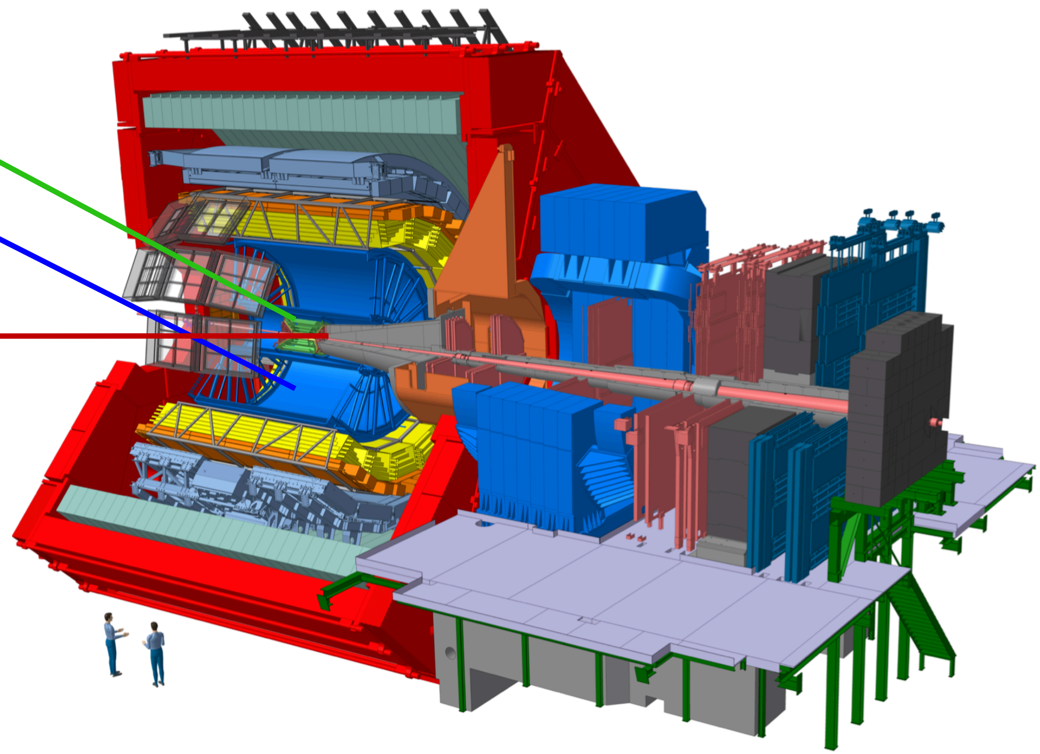
- **I**nner **T**racking **S**ystem **(ITS)**
  - Tracking and vertexing
- **T**ime **P**rojection **C**hamber **(TPC)**
  - Tracking and Particle identification (PID)
- **V**ertex **0** **(V0)**
  - Centrality determination



A Large Ion Collider Experiment

Main detectors used:

- Inner Tracking System **(ITS)**
  - Tracking and vertexing
- Time Projection Chamber **(TPC)**
  - Tracking and Particle identification (PID)
- Vertex **0 (V0)**
  - Centrality determination

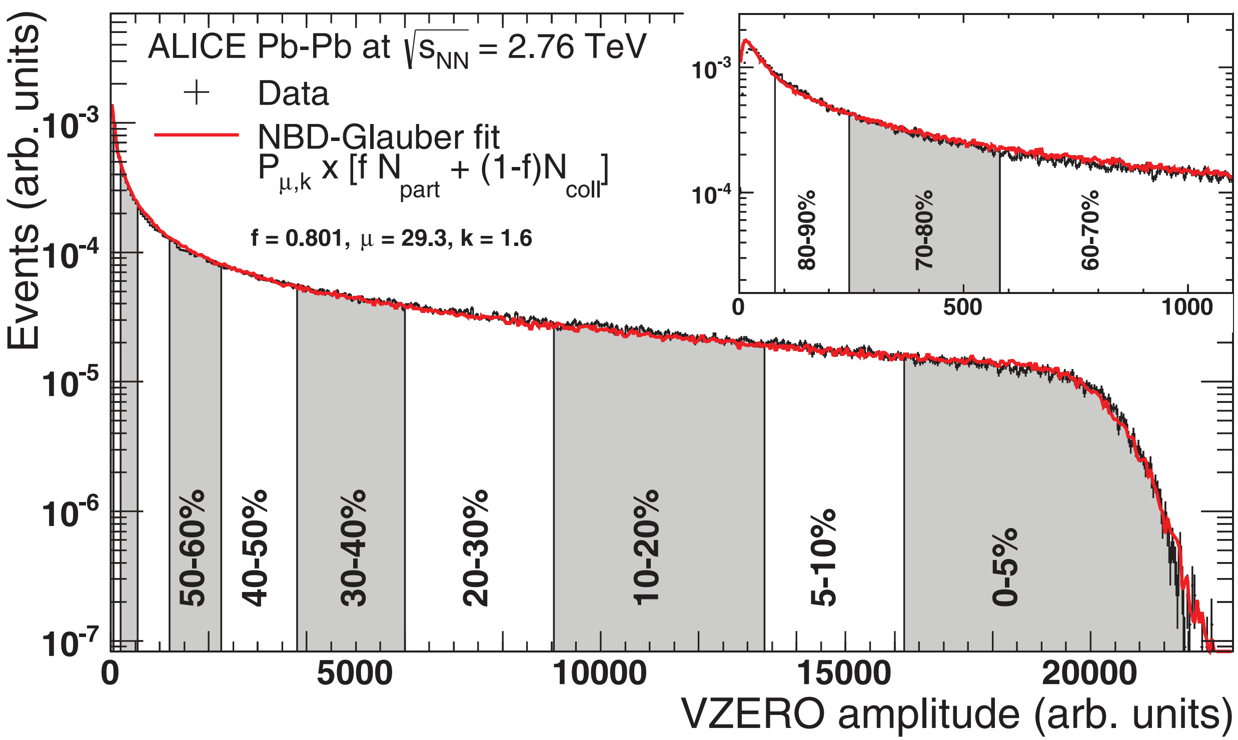


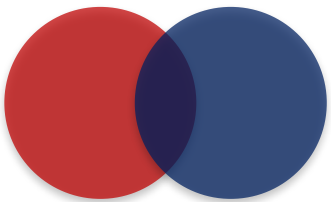
Data Set:

- **Pb-Pb collisions**
  - $\sqrt{s_{NN}} = 5.02$ TeV, ~60 M events
  - $\sqrt{s_{NN}} = 2.76$ TeV, ~12 M events
- **Model**
  - HIJING, ~6 M events
    (independent nucleus-nucleus collisions → **No QGP**)

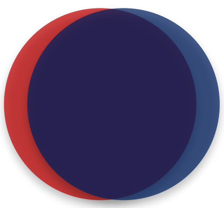
Volume in experiment? ⟶ "Centrality"

ALICE: Phys.Rev. C88 (2013) no.4, 044909





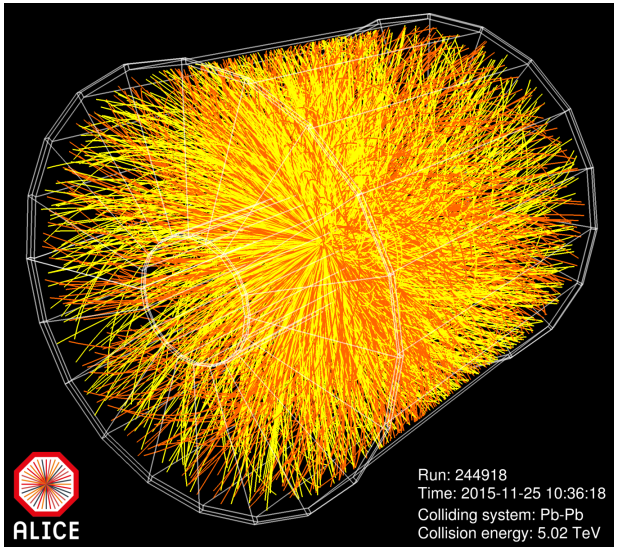
Peripheral collision

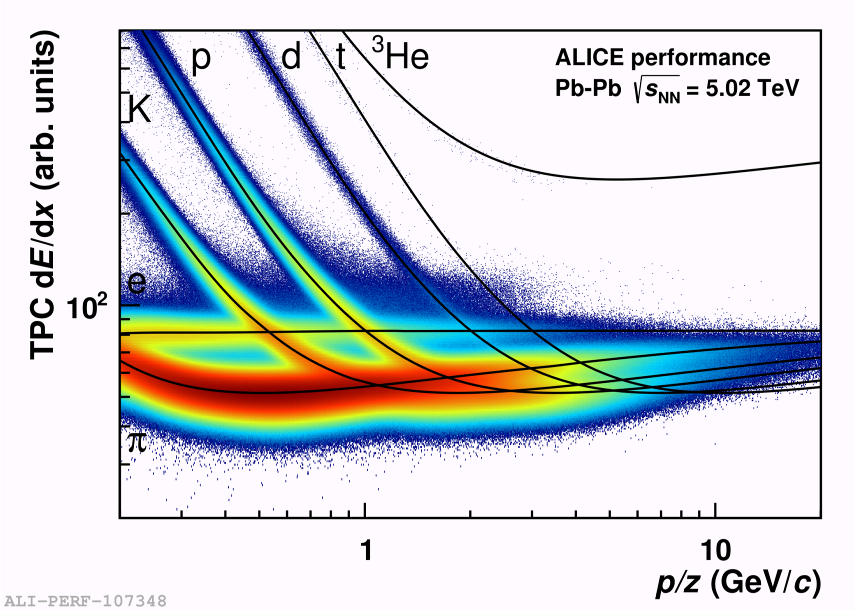


Central Collision

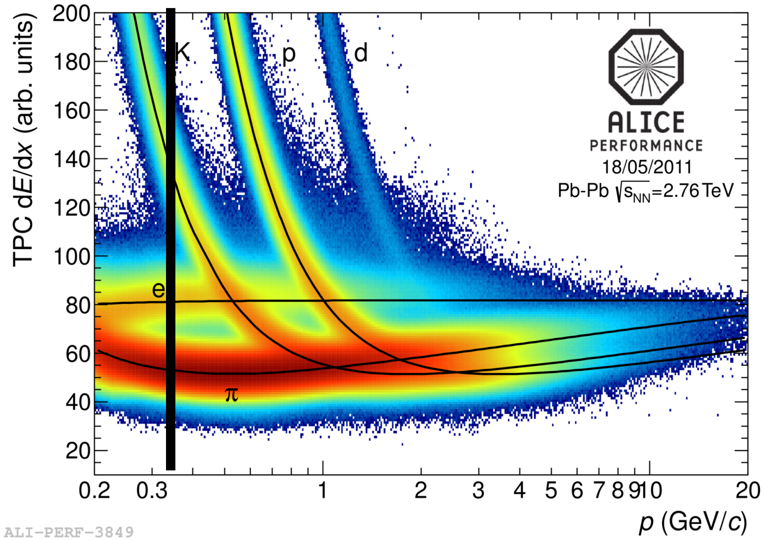
Particle Identification?

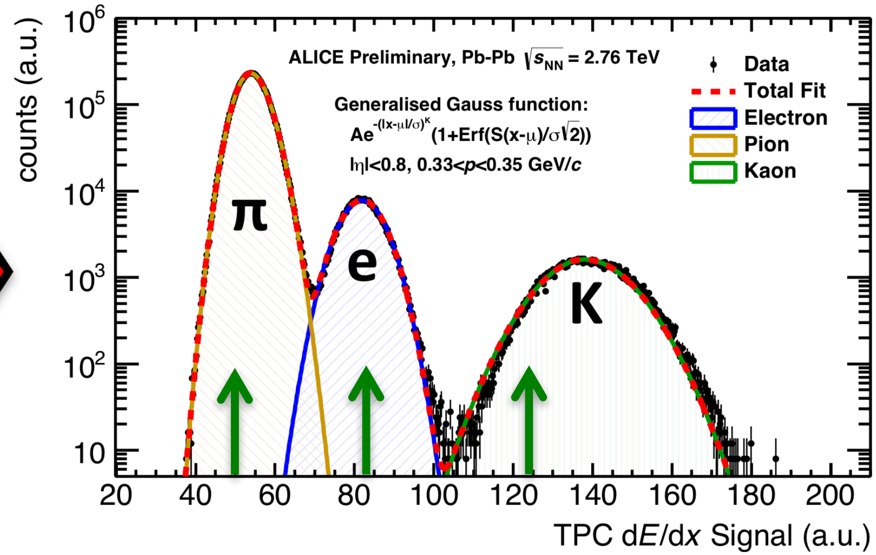
via specific energy loss as function of momentum in the TPC





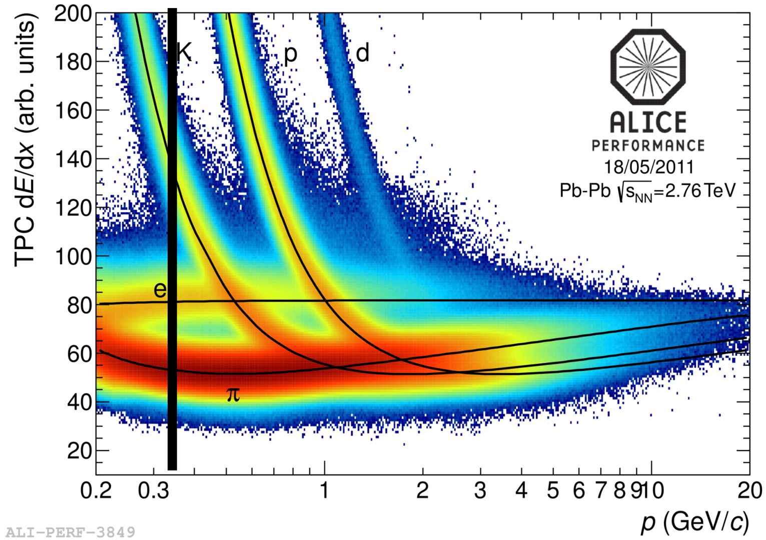
Identity method

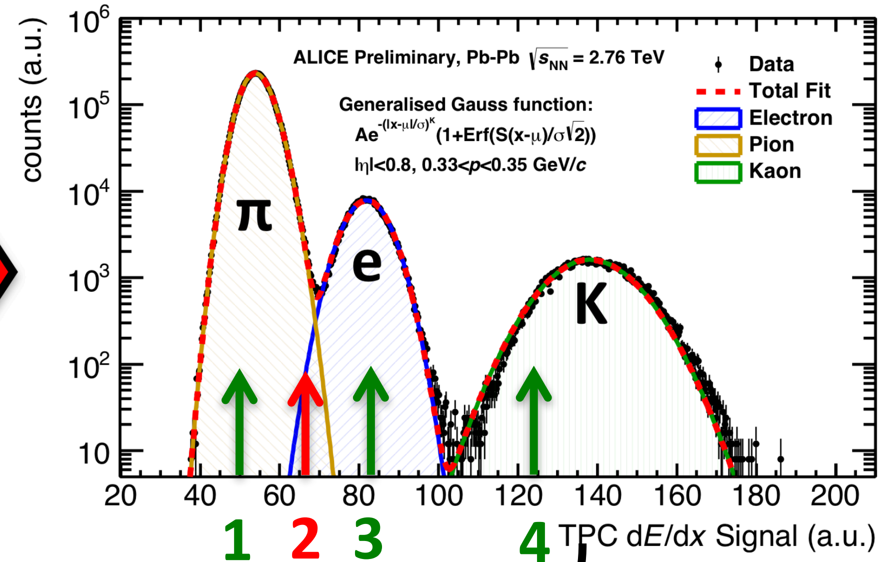




Identity method

Count probabilities to be of a given particle type





$$\omega_\pi^{(1)}=1,\quad \omega_\pi^{(2)}\cong 0.6,\quad \omega_\pi^{(3)}=0,\quad \omega_\pi^{(4)}=0 \quad\Rightarrow\quad W_\pi = 1.6 \neq N_\pi$$

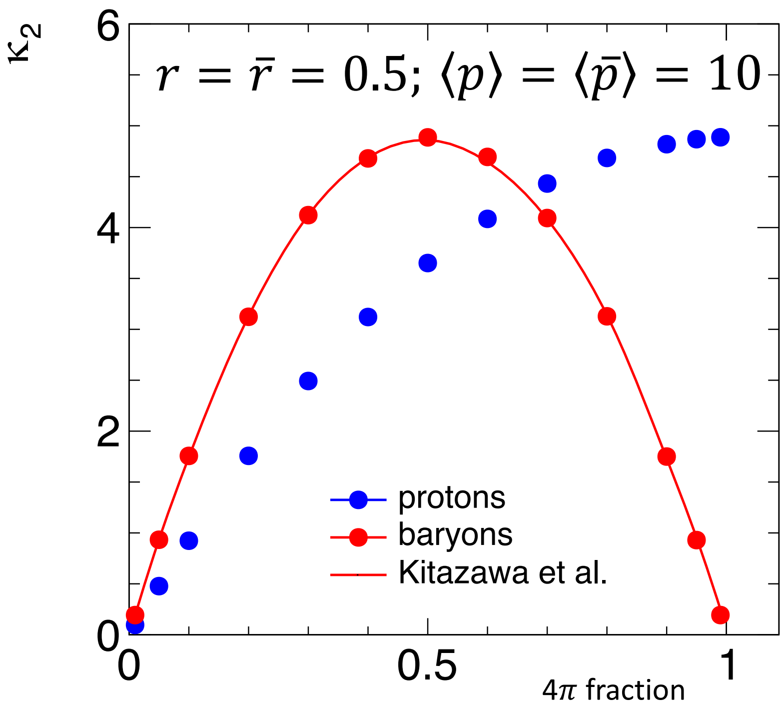
$$\langle N_j^n \rangle = \mathrm{A}^{-1} \langle W_j^n \rangle$$

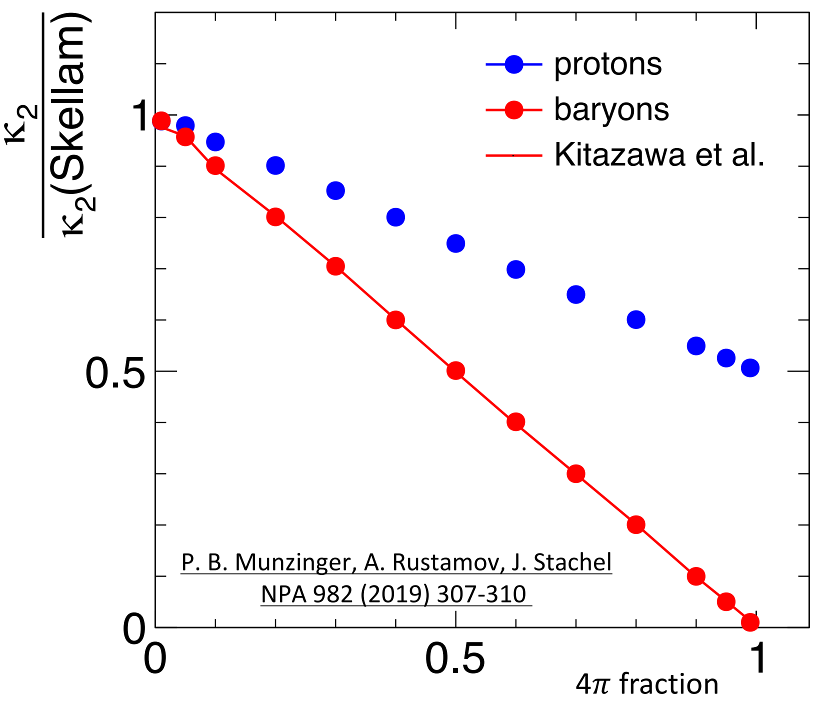
A. Rustamov, M. Gazdzicki, M. I. Gorenstein, PRC 86, 044906 (2012), PRC 84, 024902 (2011)

M. Arslandok, A. Rustamov, NIM A, 946, (2019), 162622

Which acceptance?

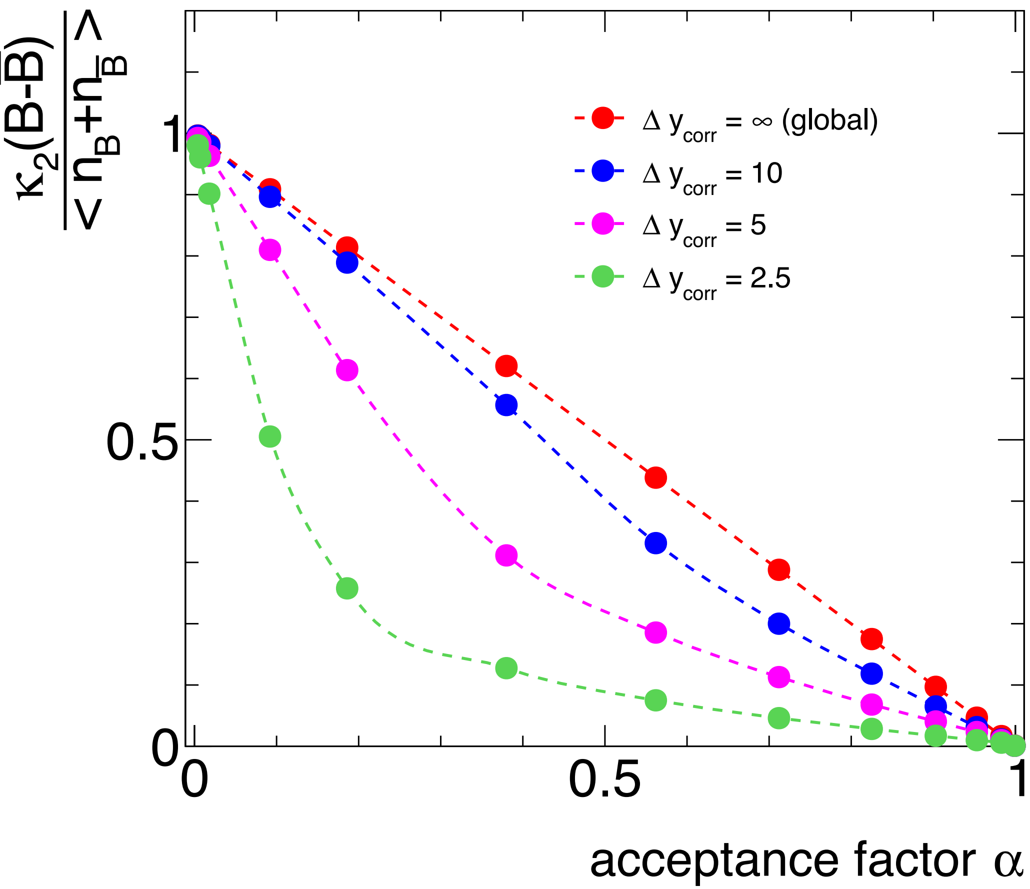
- Due to **isospin randomization,** at $\sqrt{s_{NN}}$ > 10 GeV **net-baryon** fluctuations can be obtained from corresponding **net-proton** measurements (M. Kitazawa, and M. Asakawa, Phys. Rev. C 86, 024904 (2012))





Global vs Local baryon number conservation

P. Braun-Munzinger, A. Rustamov, J. Stachel, arXiv:1907.03032



$$|y_{\bar{B}} - y_B| < \frac{\Delta y_{corr}}{2}$$

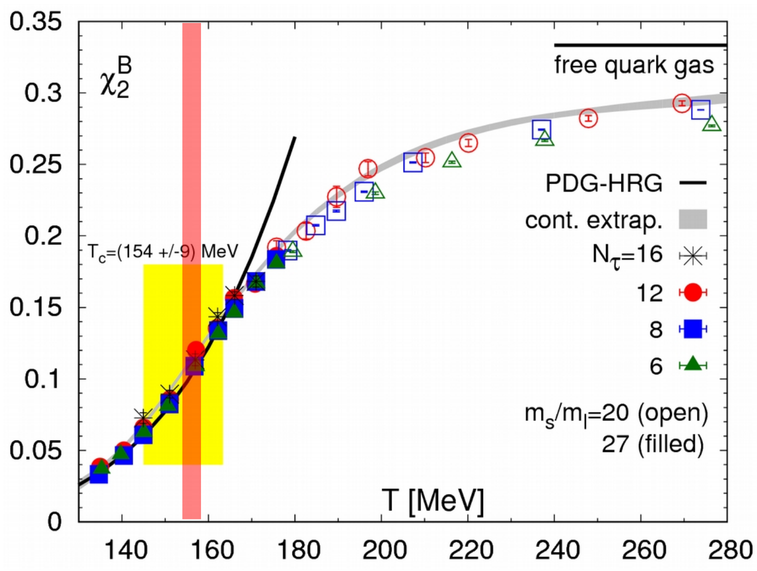
$$\alpha = \frac{\langle N_B^{acc} \rangle}{\langle N_B^{4\pi} \rangle}$$

$\langle n_B + n_{\bar{B}} \rangle \equiv \kappa_2(\text{skellam})$

1st and 2nd order cumulants at LHC

LQCD expectations:

- ✓ 1st moments → $T_{pc} = T_{freeze\text{-}out} = \sim 156$ MeV
- ✓ 2nd moments → No deviation from HRG at T_{pc}



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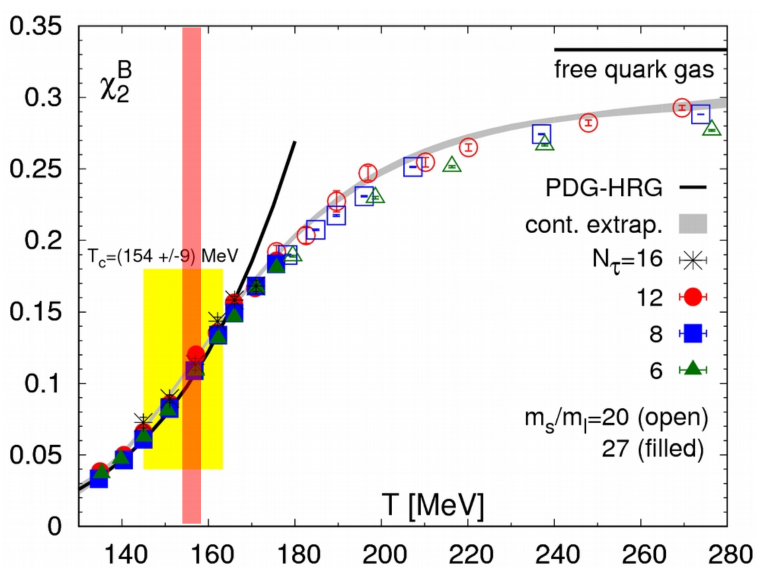
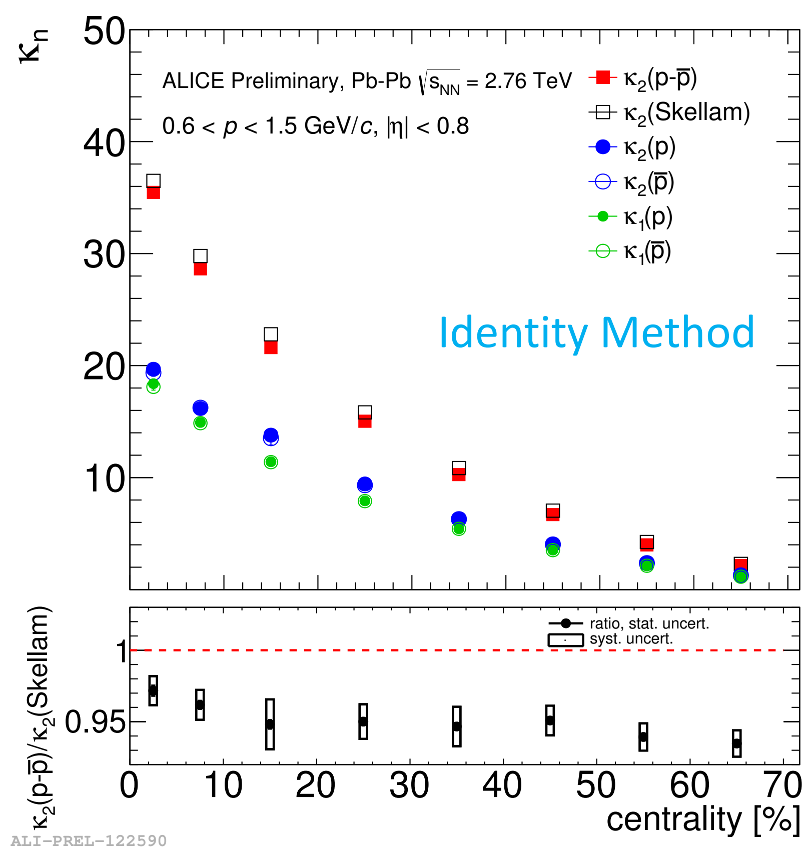


$$\kappa_2(\text{Skellam}) = \kappa_1(p) + \kappa_1(\bar{p})$$

$$\kappa_2(p-\bar{p}) = \kappa_2(p) + \kappa_2(\bar{p}) - 2\left(\langle p\bar{p}\rangle - \langle p\rangle\langle\bar{p}\rangle\right)$$

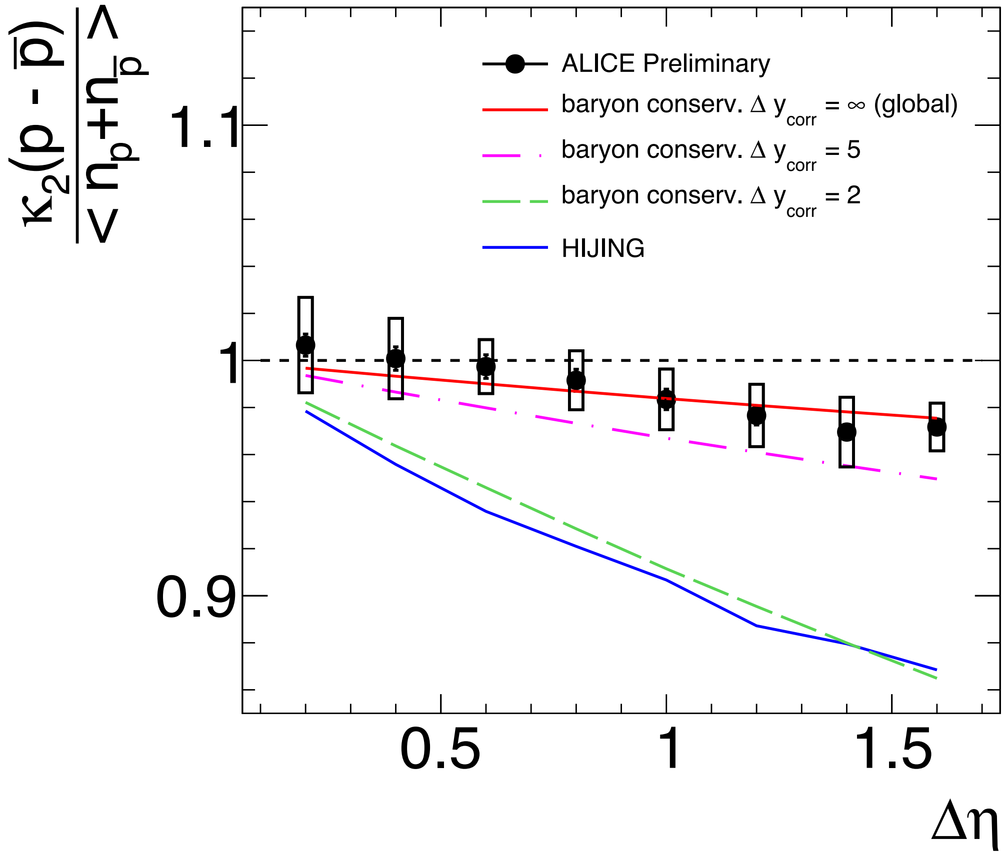


What is 3% deviation from baseline at most central collisions?

2nd order cumulants of net-p at LHC

P. Braun-Munzinger, A. Rustamov, J. Stachel, arXiv:1907.03032

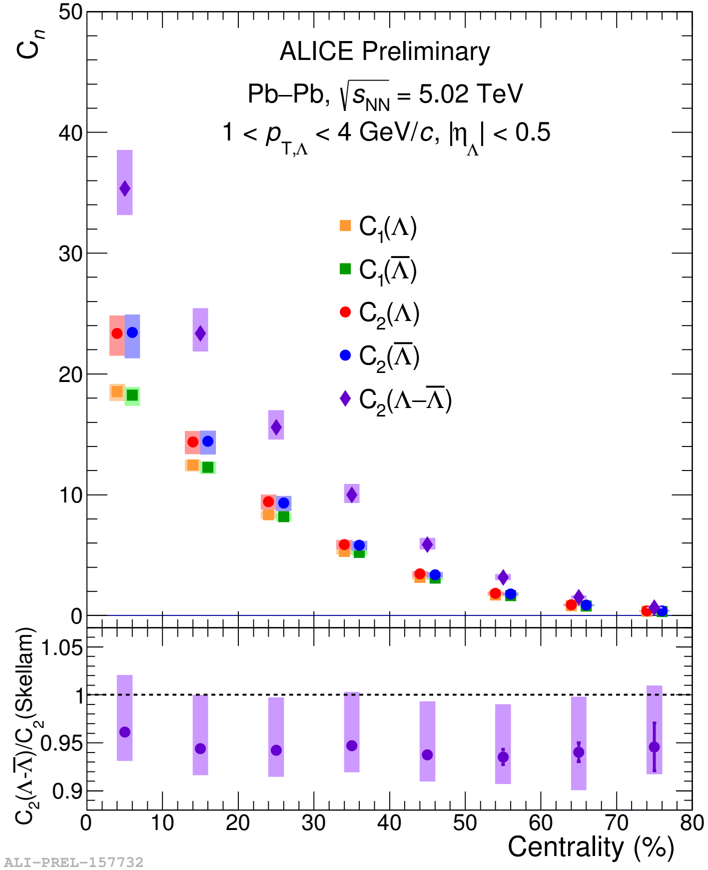
Anar Rustamov QM2017, Nucl.Phys. A967 (2017) 453-456



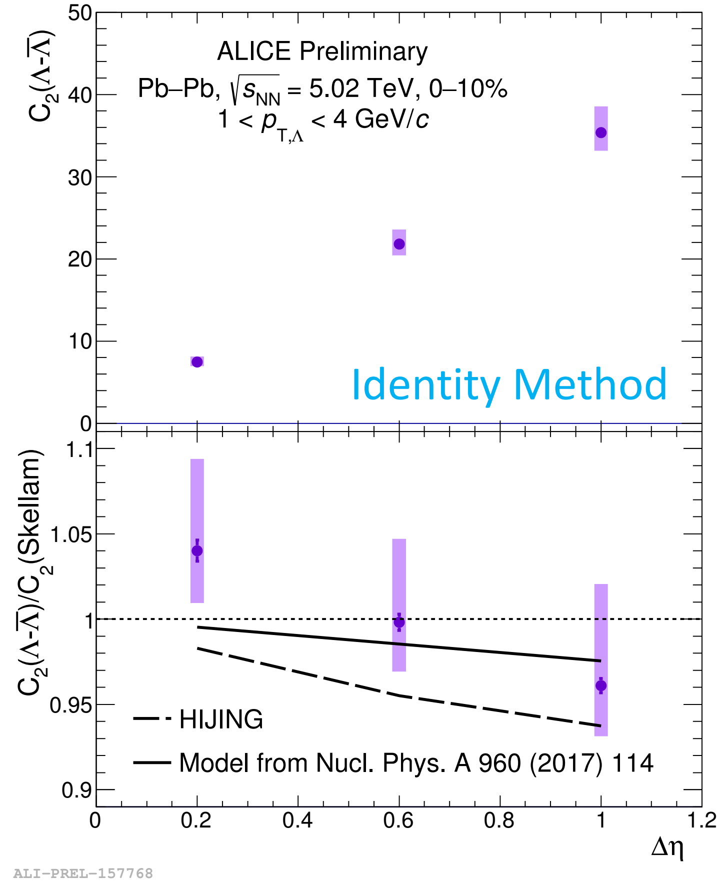
- ✓ **Data** is consistent with baryon number conservation over full solid angle
- ✓ **Event generators based on string fragmentation (HIJING)** conserve baryon number over a smaller interval

2^{nd} order cumulants of net-Λ at LHC

$$\kappa_2(n-\bar{n}) \equiv C_2(n-\bar{n})$$



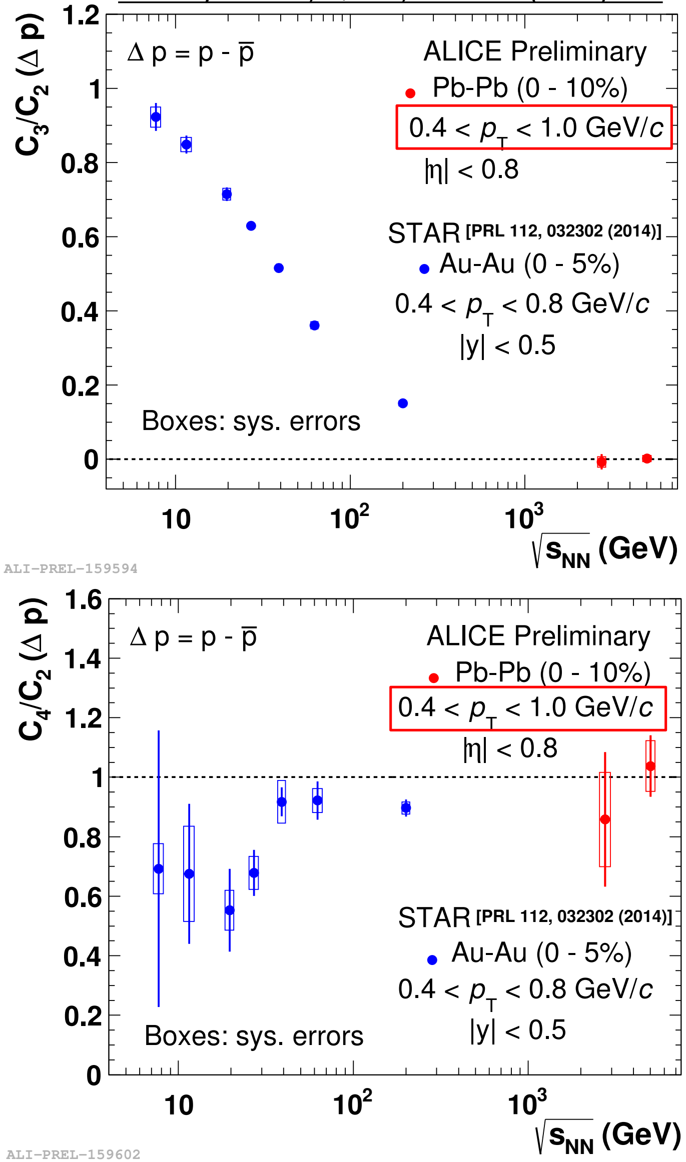
Alice Ohlson, QM2018, NPA 982 (2019) 299



- Similar trend as for **net-p**
- **Better precision** is needed to see the impact of strangeness conservation

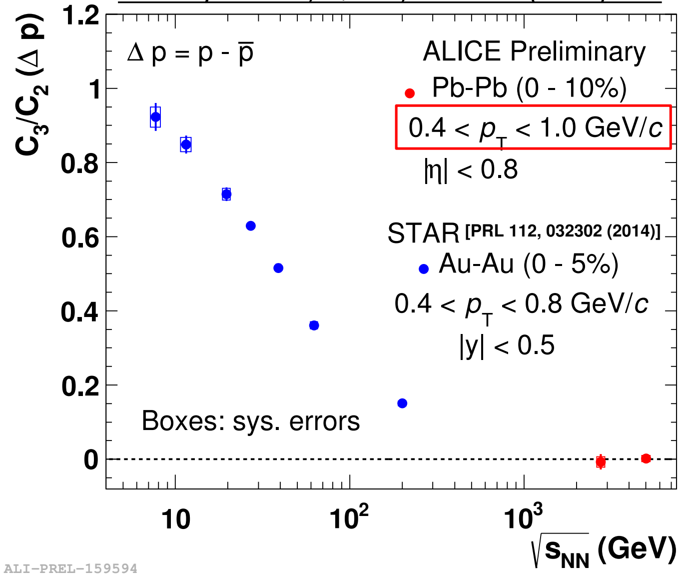
3rd and 4th order cumulants of net-p at LHC

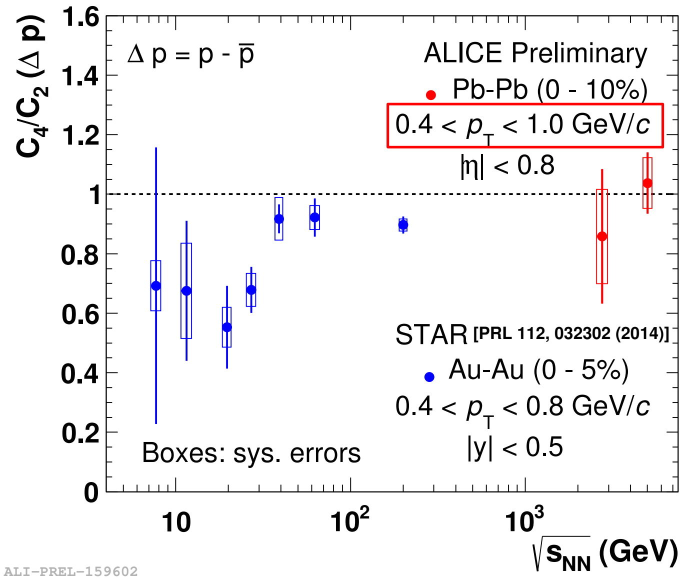
Nirbhay Kubera, QM18, NPA 982 (2019) 851



3rd and 4th order cumulants of net-p at LHC

Nirbhay Kubera, QM18, NPA 982 (2019) 851



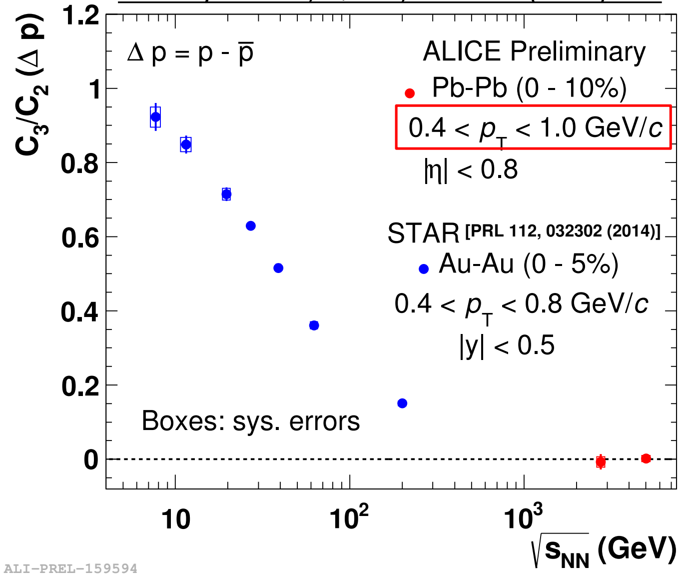


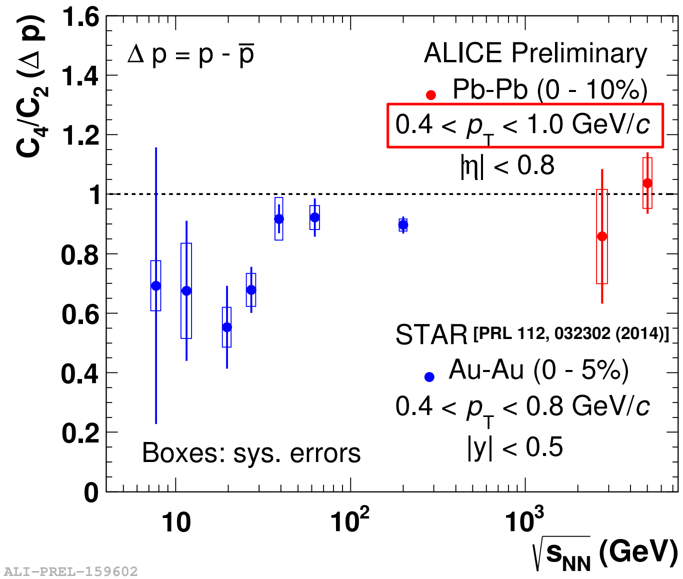
C_3/C_2 and C_4/C_2 agree with Skellam at LHC energies?

- Small acceptance
- Low statistics
- Cut-based approach for PID

3rd and 4th order cumulants of net-p at LHC

Nirbhay Kubera, QM18, NPA 982 (2019) 851





C_3/C_2 and C_4/C_2 agree with Skellam at LHC energies?

- Small acceptance
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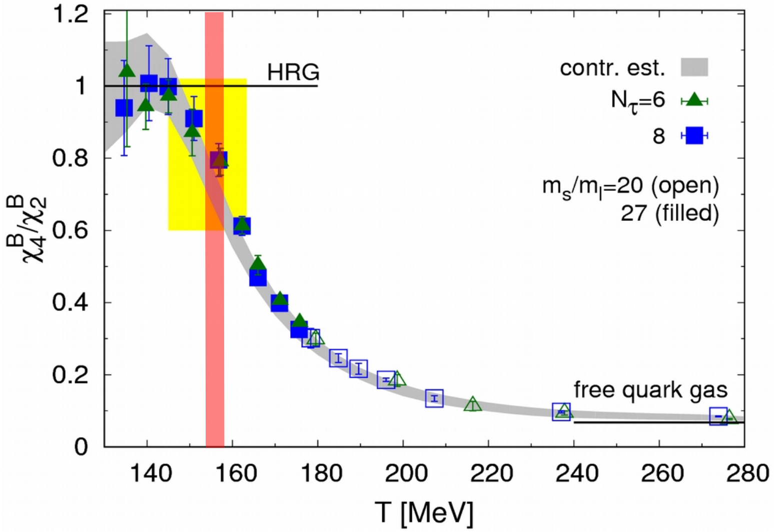
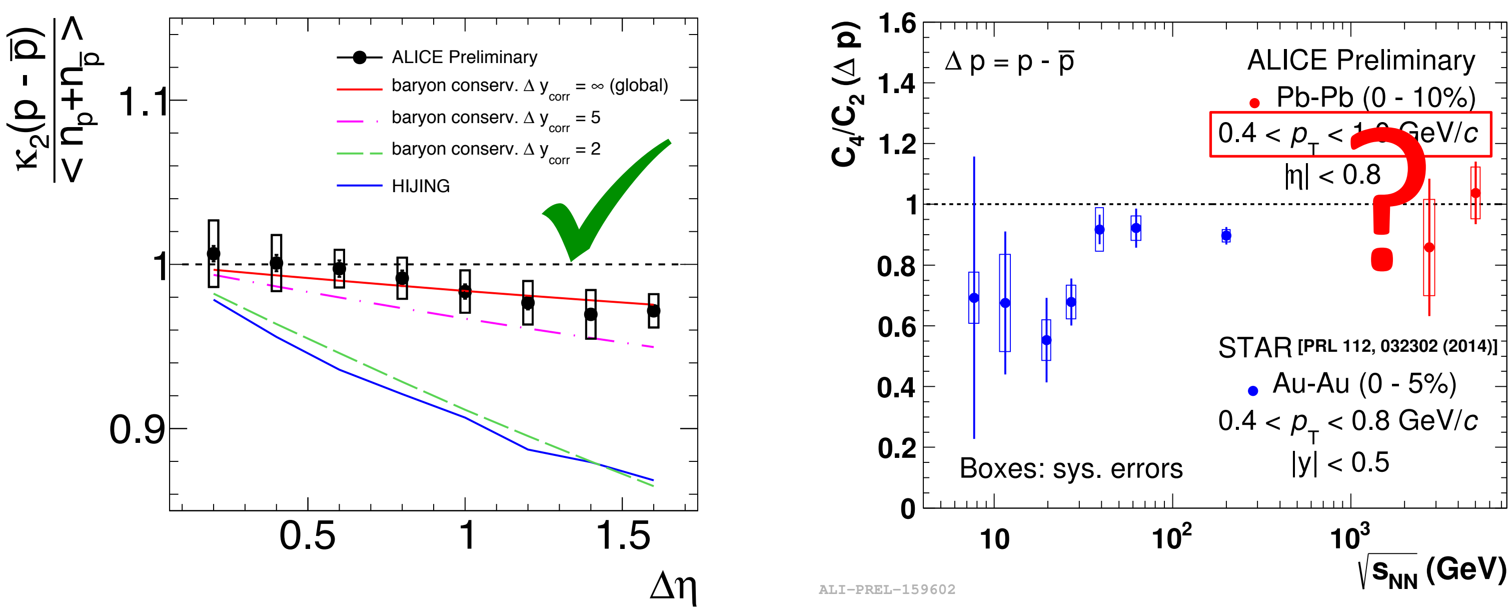


Analysis within a larger kinematic acceptance using
Identity Method is in progress

Summary

- ✓ **Fluctuations** are excellent tool to study QCD phase diagram
- ✓ Conserved charge fluctuations: **Link to LQCD**
  - ✓ **2nd second cumulants** of net-protons after accounting for baryon number conservation, **in agreement with the corresponding second cumulants of the Skellam distribution.**
    - LQCD predicts a Skellam behavior for the second cumulants of net-baryons at $T_{pc} \approx 156$
- ✓ Contributions due to **local baryon number conservation**, at LHC energies, are small if present at all in the second cumulants of net-protons.
- ✓ **Analysis of 3rd and 4th cumulants** within a larger kinematic acceptance using **Identity Method** is ongoing

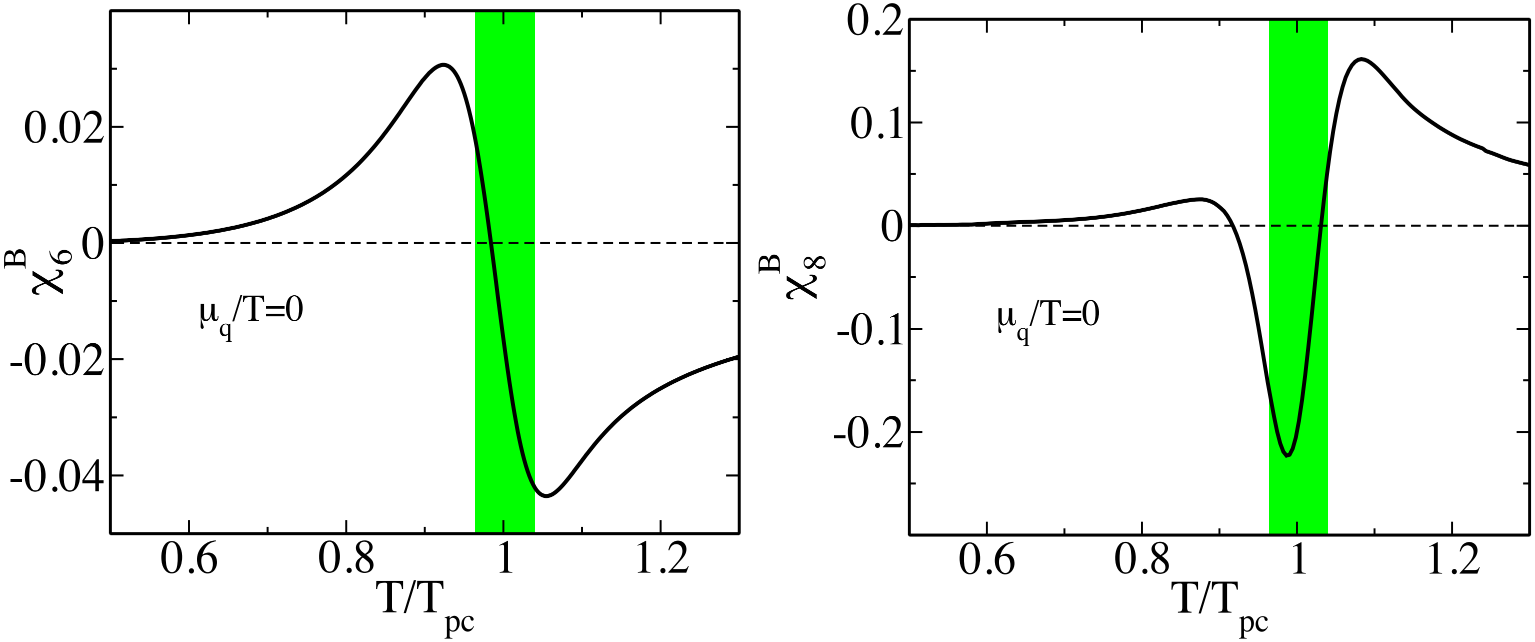


Outlook

Holy grail: see critical behavior in **6th and higher order** **cumulants**

→ Stay tuned for the RUN3 period of the LHC

B. Friman, F. Karsch, K. Redlich, V. Skokov Eur. Phys. J. C (2011) 71: 1694

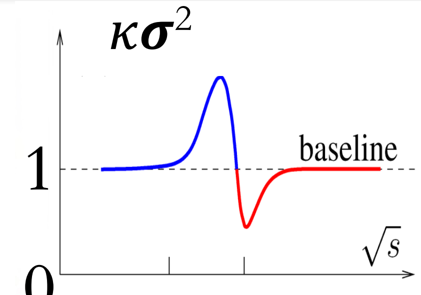


BACKUP

3rd and 4th order cumulants of net-p at RHIC

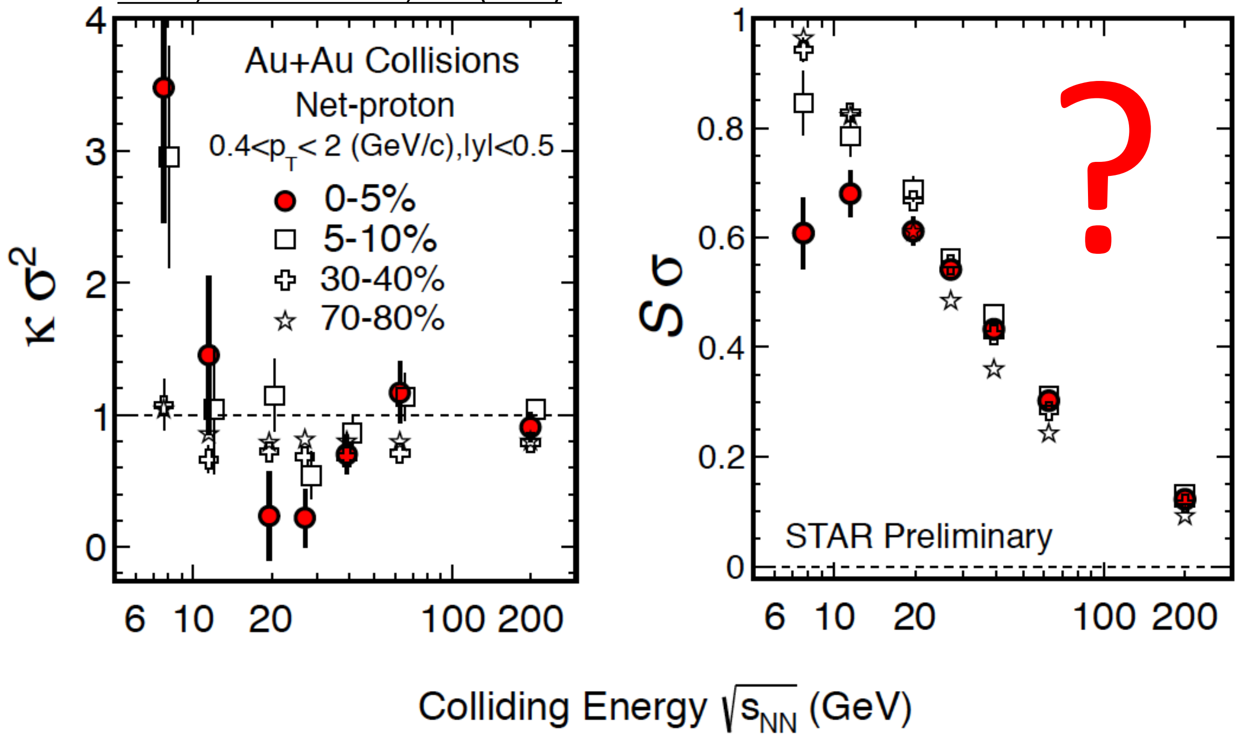
At RHIC:

Non-monotonic behavior as a function of energy



M. Stephanov
PRL102, 032301 (2009), PRL107, 052301 (2011)

X. Luo, PoS CPOD2014, 019 (2015)

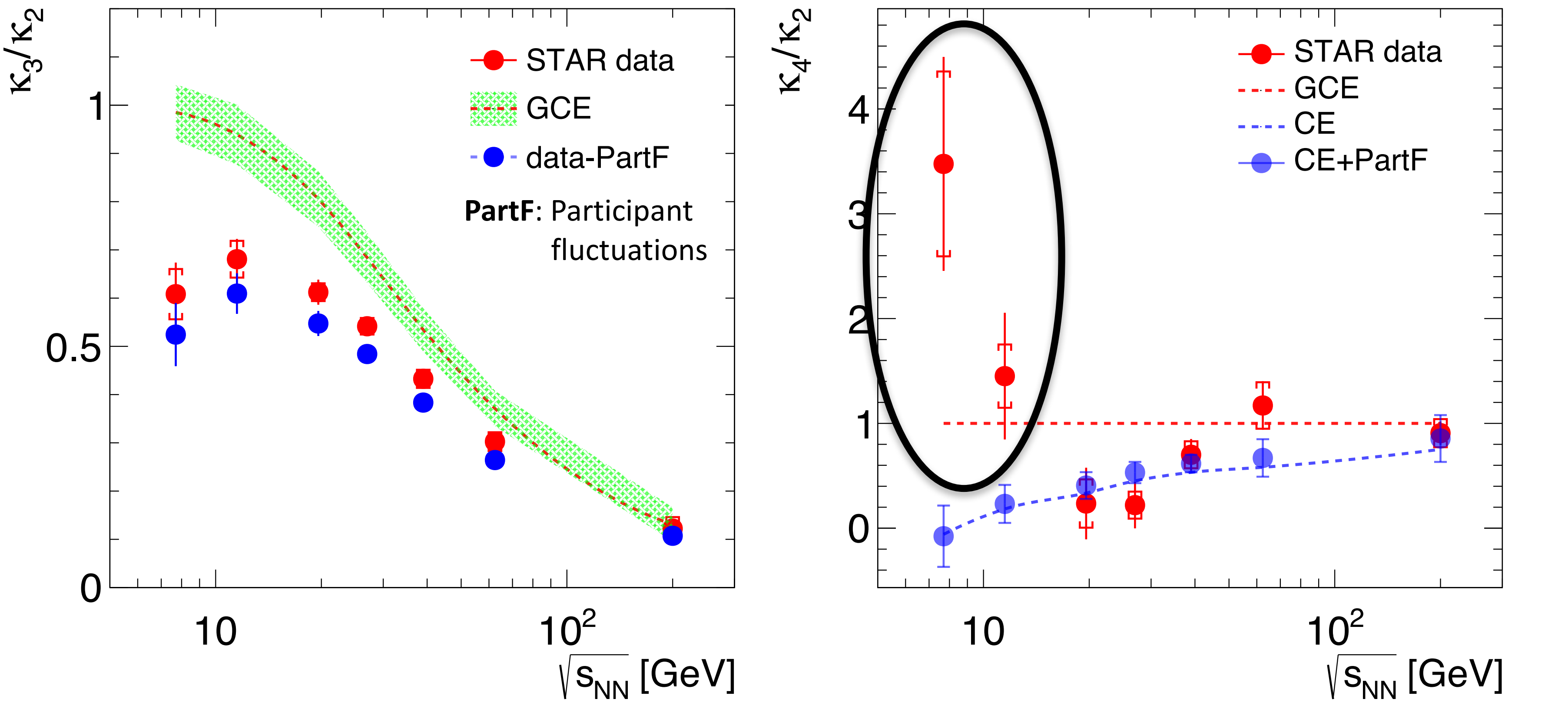


$$\frac{\kappa_4}{\kappa_2} = \kappa\sigma^2$$

$$\frac{\kappa_3}{\kappa_2} = S\sigma$$

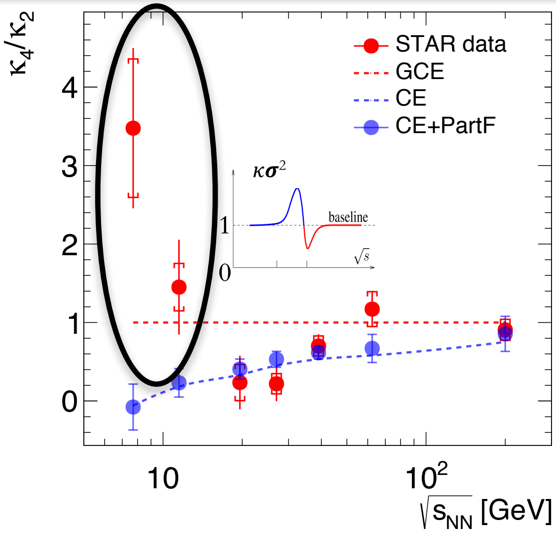
Effect of baryon number conservation

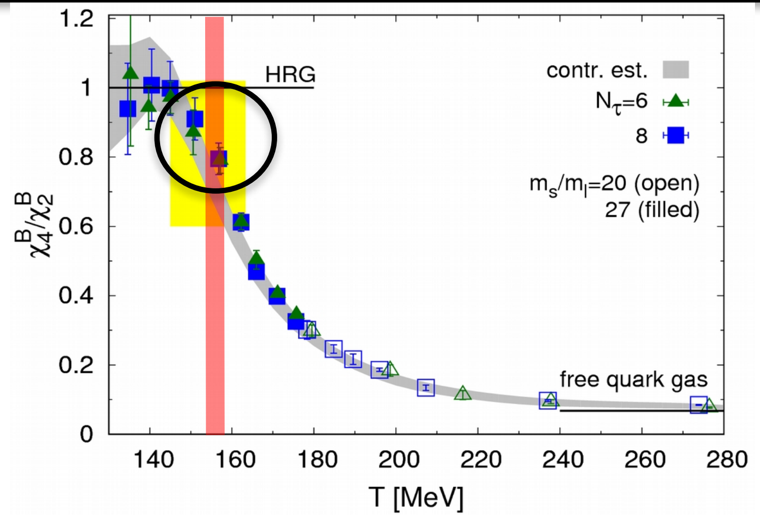
P. Braun-Munzinger, A. Rustamov, J. Stachel, NPA 982 (2019) 307-310



- ➢ κ_3/κ_2 and κ_4/κ_2 cannot be simultaneously explained for the lowest two energies
- ➢ Possible biases due to efficiency correction procedure and cut based approach

Open Questions





Experiment

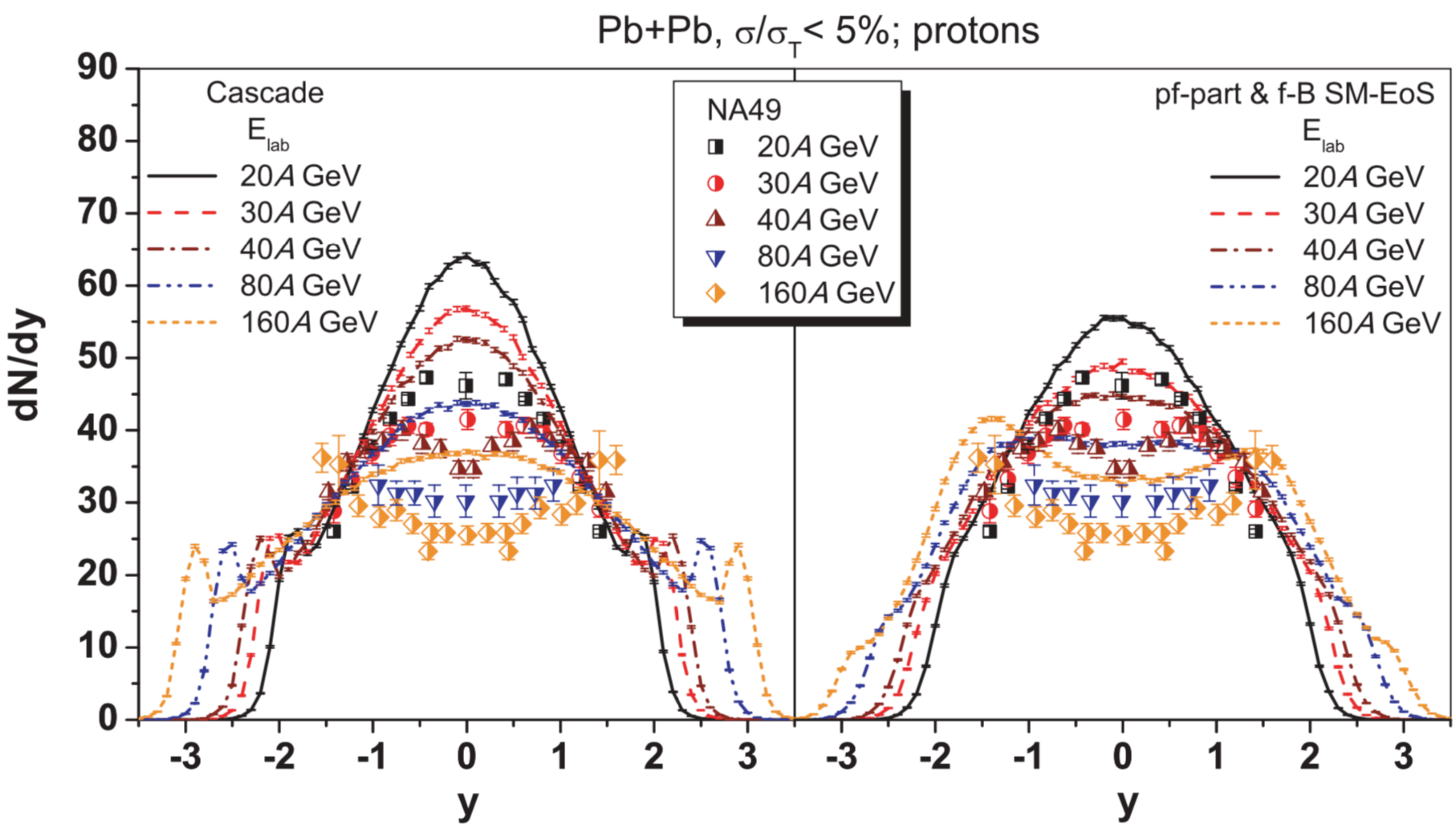
- Efficiency correction
  ⟶ realistic detector simulations
- Volume fluctuations
  ⟶ centrality resolution
- Effect of resonances
- Measurement at low energies
- Systematic uncertainties
- …

Theory

- Efficiency correction
  ⟶ unfolding or …
- Volume fluctuations
- Effect of resonances
- Measurement at low energies
  ⟶ baryon stopping, deuteron formation …
- Effect of hydrodynamic evolution
- …

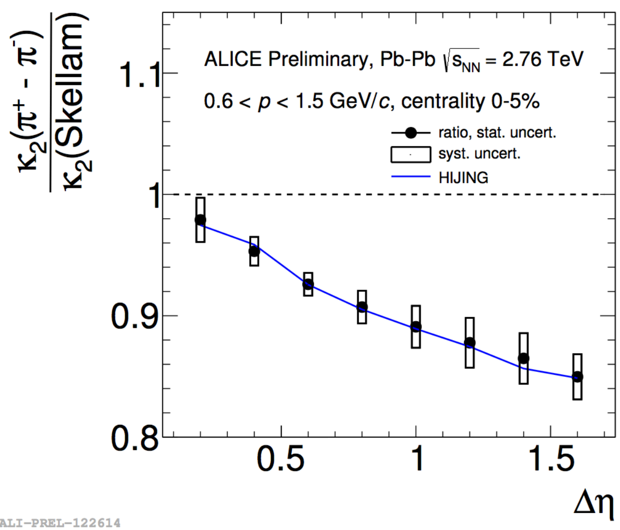
- Adam Bzdak et. al., arXiv:1906.00936
- Probing the Phase Structure of Strongly Interacting Matter: Theory and Experiment, https://indico.gsi.de/event/7994/overview

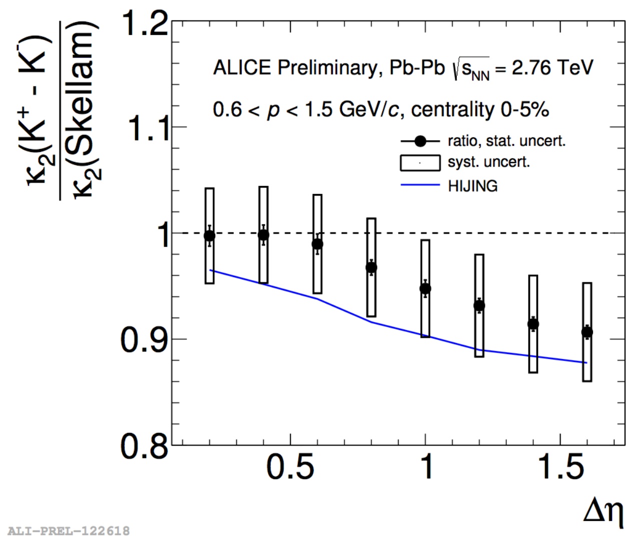
Stopping

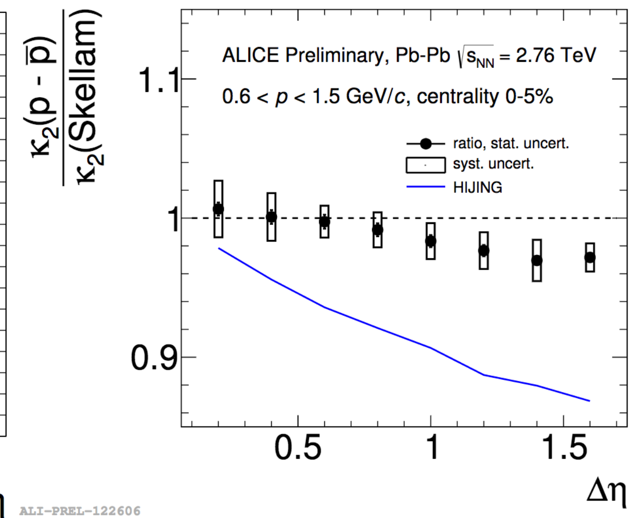


Pb+Pb, σ/σ_T< 5%; protons
Cascade
E_lab
20A GeV
30A GeV
40A GeV
80A GeV
160A GeV
NA49
20A GeV
30A GeV
40A GeV
80A GeV
160A GeV
pf-part & f-B SM-EoS
E_lab
20A GeV
30A GeV
40A GeV
80A GeV
160A GeV
dN/dy
y
y

2nd order cumulants of: π, K, p







- Effect of **Resonances** ?

Cross Cumulants

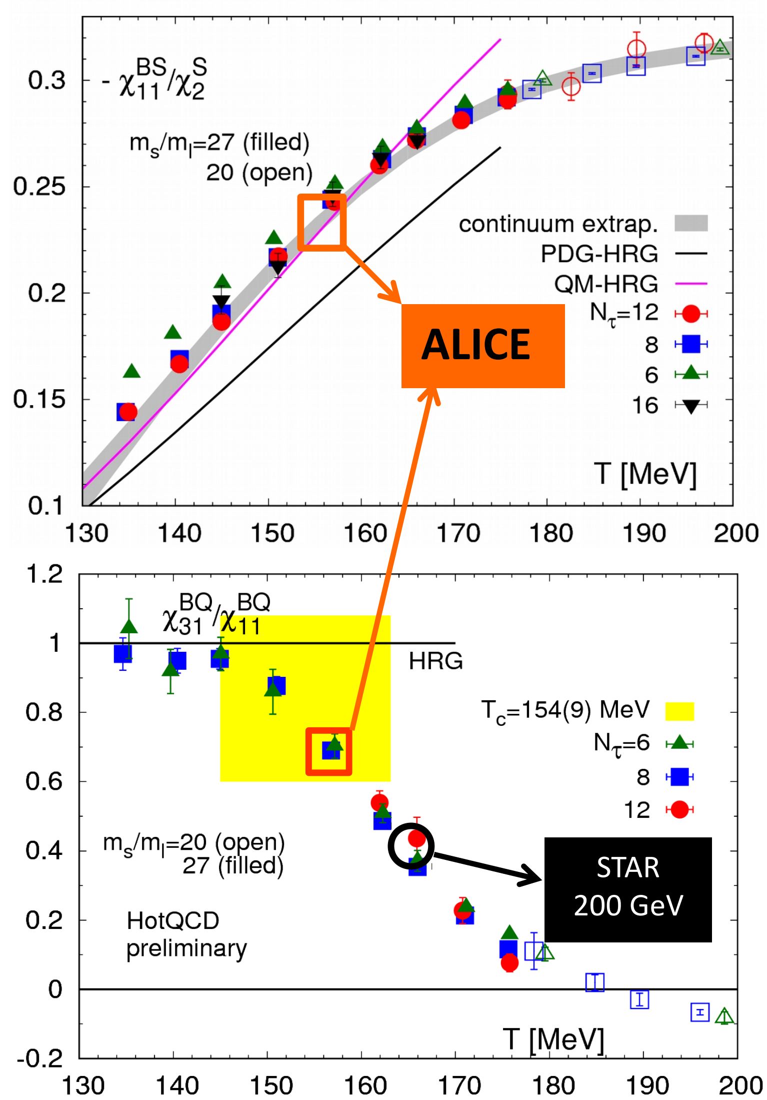
- Taylor expansion of the **QCD** pressure

$$\frac{P}{T^4} = \frac{1}{VT^3} \ln Z(T, V, \mu_B, \mu_Q, \mu_S)$$

$$\frac{P}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{ijk}^{BQS}(T) \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

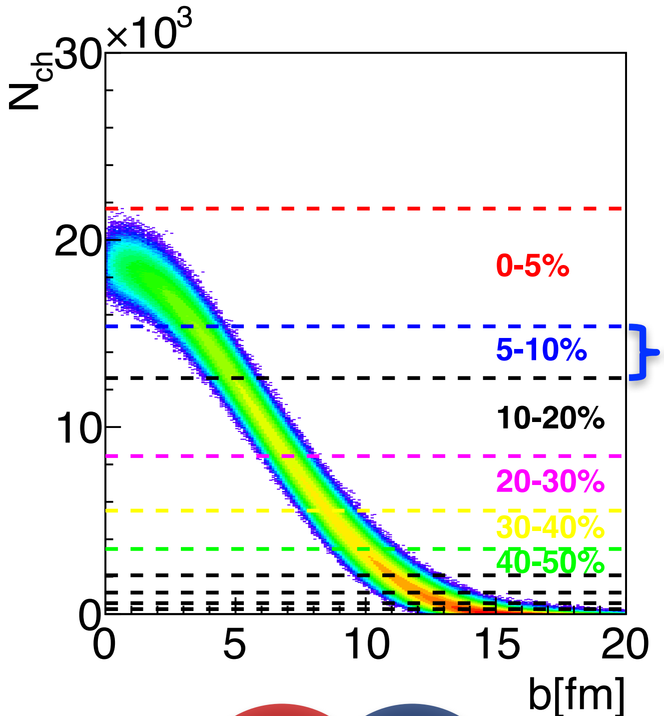
- Cumulants of **net-charge fluctuations and correlations**

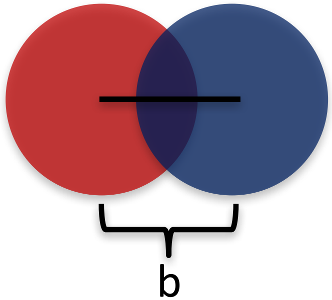
$$\chi_{ijk}^{BQS} = \left. \frac{\partial^{i+j+k} P/T^4}{\partial \hat{\mu}_B^i \partial \hat{\mu}_Q^j \partial \hat{\mu}_S^k} \right|_{\mu_{B,Q,S}=0} \quad , \quad \hat{\mu}_X \equiv \frac{\mu_X}{T}$$

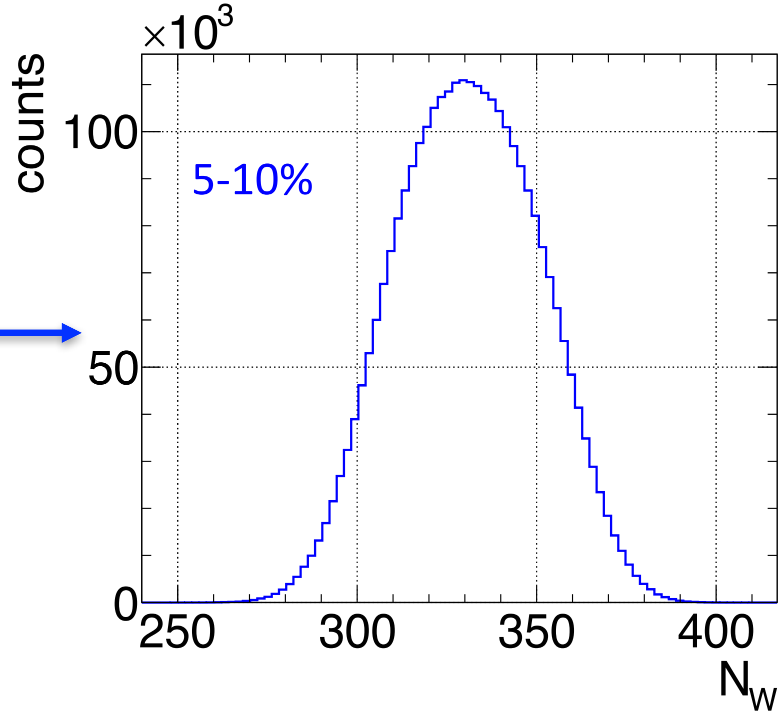


F. Karsch, QM17, arXiv:1706.01620

Volume Fluctuates



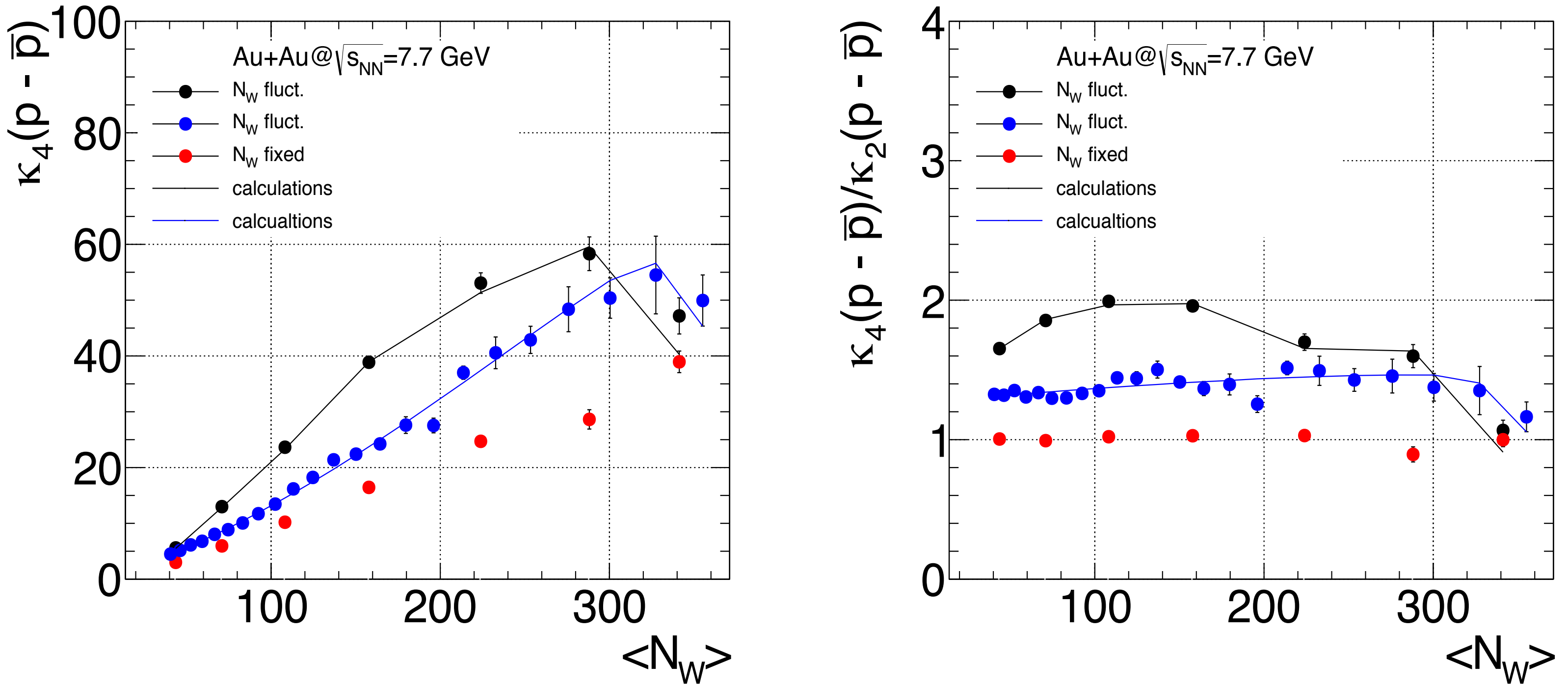




$$\frac{\kappa_4(\Delta N_B)}{\kappa_2(\Delta N_B)} \stackrel{?}{=} \frac{\hat{\chi}_4^B}{\hat{\chi}_2^B}$$

P. Braun-Munzinger, A. Rustamov, J. Stachel, Nuclear Physics A 960 (2017) 114–130

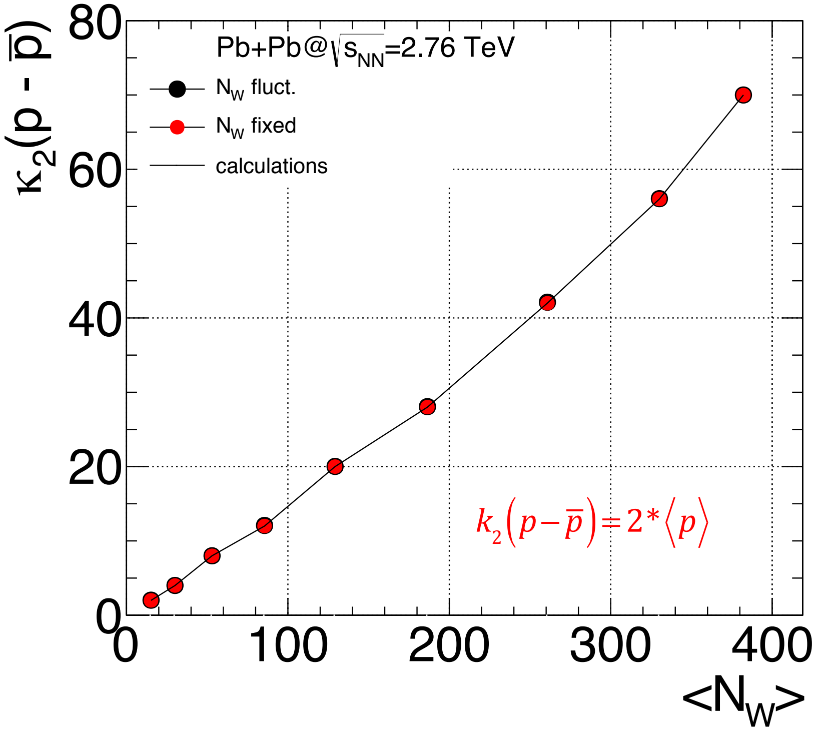
Volume Fluctuations at RHIC energies

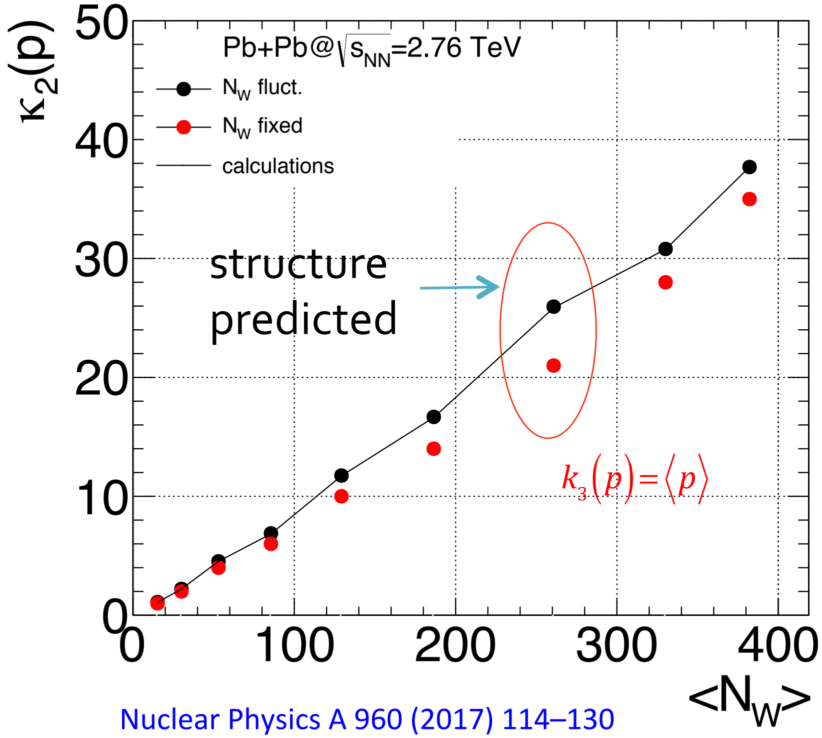


P. Braun-Munzinger, A. Rustamov, J. Stachel, Nuclear Physics A 960 (2017) 114–130

Volume Fluctuations: 2^{nd} order

$150*10^6$ Events





Nuclear Physics A 960 (2017) 114–130

$$k_2(p-\bar{p})=\langle N_w\rangle k_2(n-\bar{n})+\langle n-\bar{n}\rangle^2 k_2(N_w)$$

vanishes for ALICE

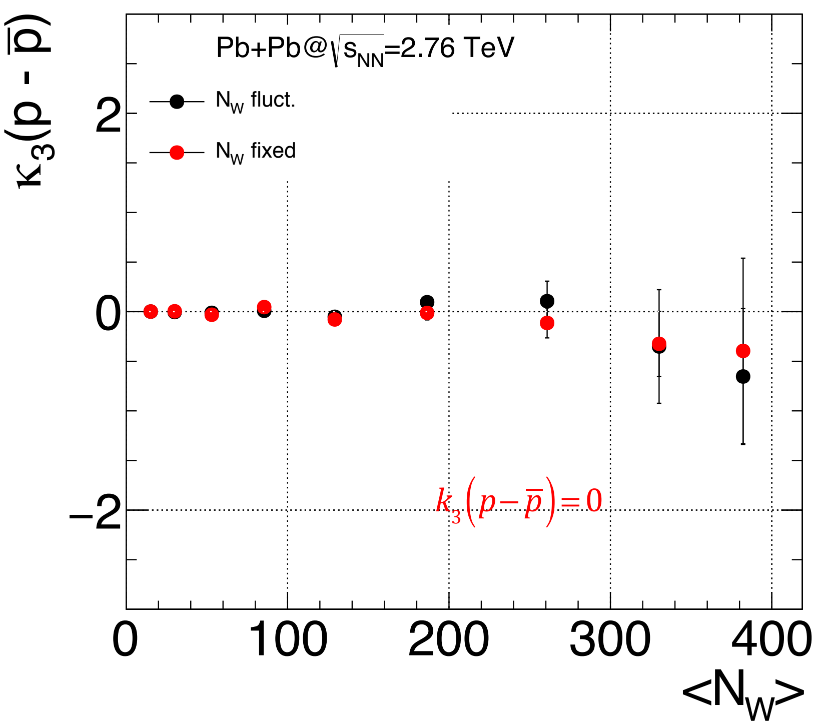
$$k_2(p)=\langle N_w\rangle k_2(n)+\langle n\rangle^2 k_2(N_w)$$

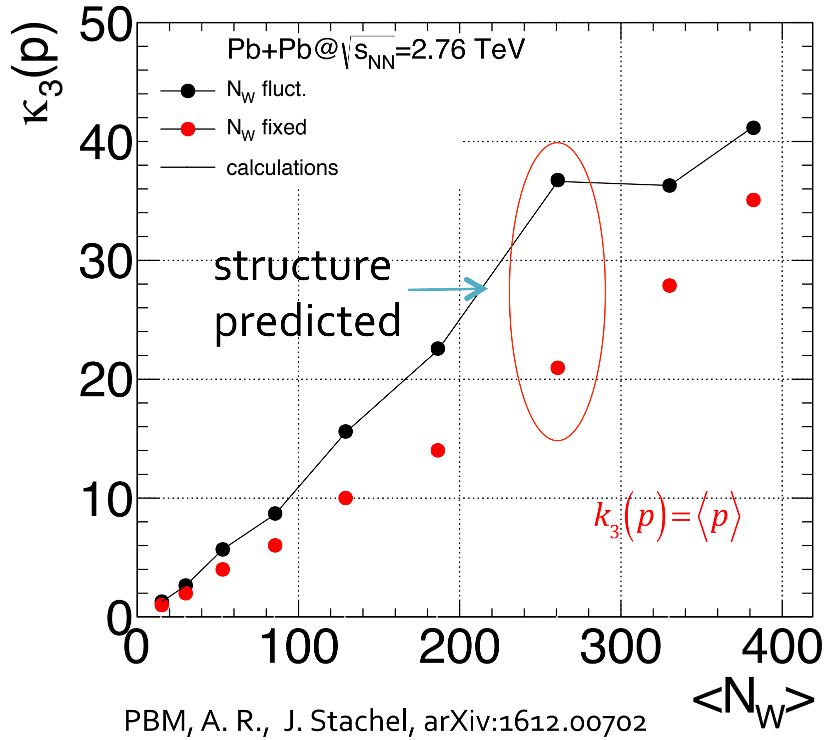
does not vanish

$n,\bar{n}$ from single wounded nucleon

Volume Fluctuations: 3rd order

150*10^6 Events





PBM, A. R., J. Stachel, arXiv:1612.00702

$$k_3(p-\bar{p}) = \langle N_w \rangle k_3(n-\bar{n}) + \langle n-\bar{n} \rangle (\ldots)$$

$\downarrow$ vanishes for ALICE

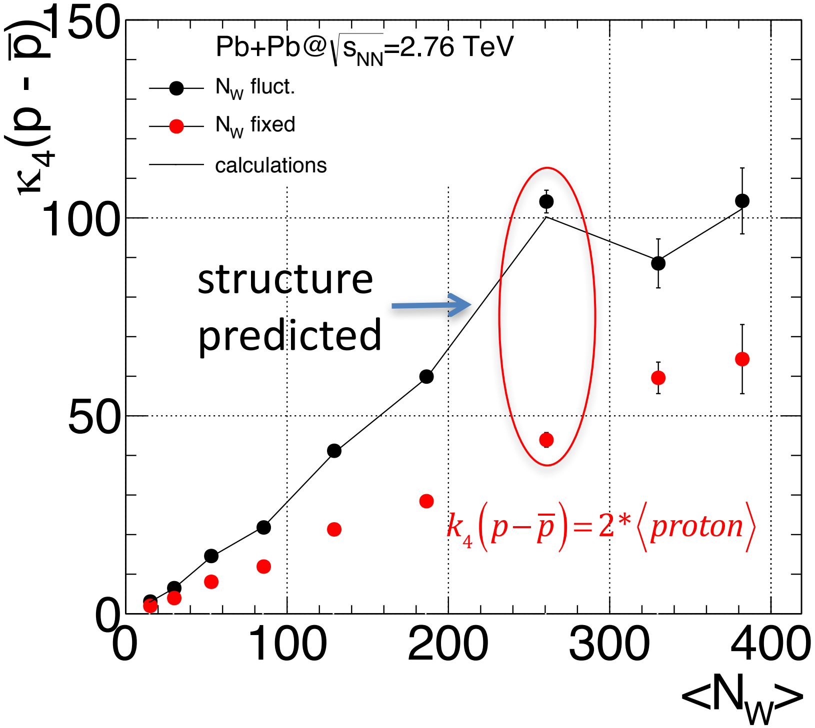
$$k_3(p) = \langle N_w \rangle k_3(n) + \langle n \rangle (\ldots)$$

$\downarrow$ does not vanish

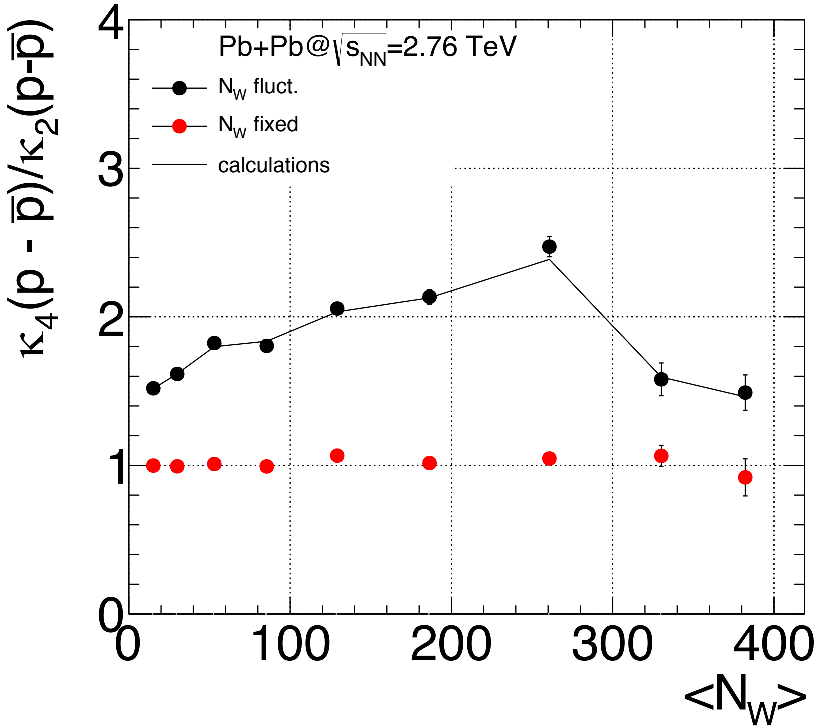
$n, \bar{n}$ from single wounded nucleon

Volume Fluctuations: 4th order

150*10^6 Events



P. Braun-Munzinger, A. Rustamov, J. Stachel
Nuclear Physics A 960 (2017) 114–130

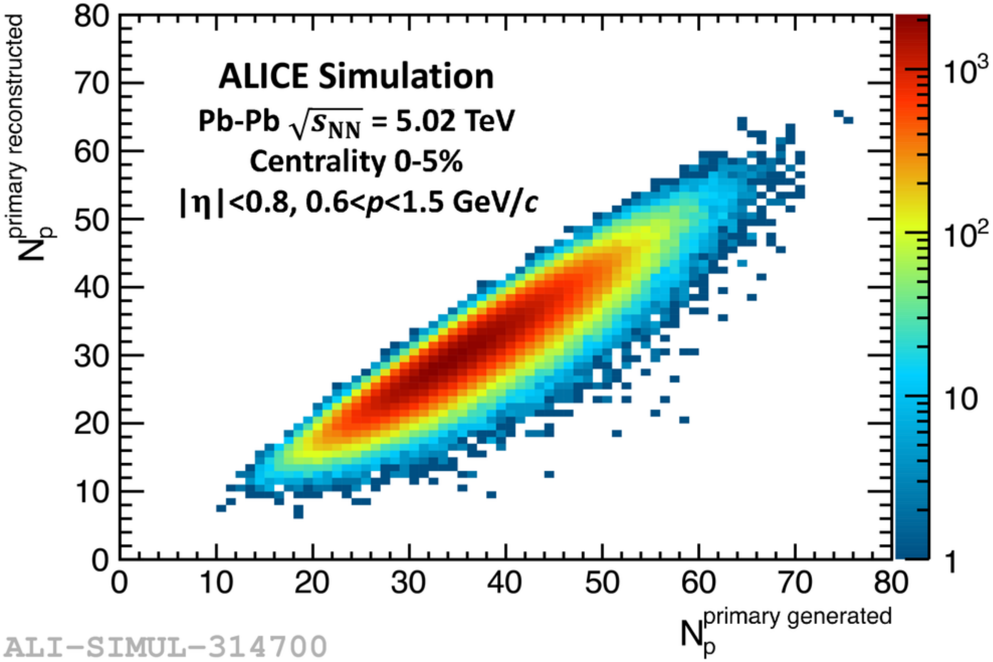


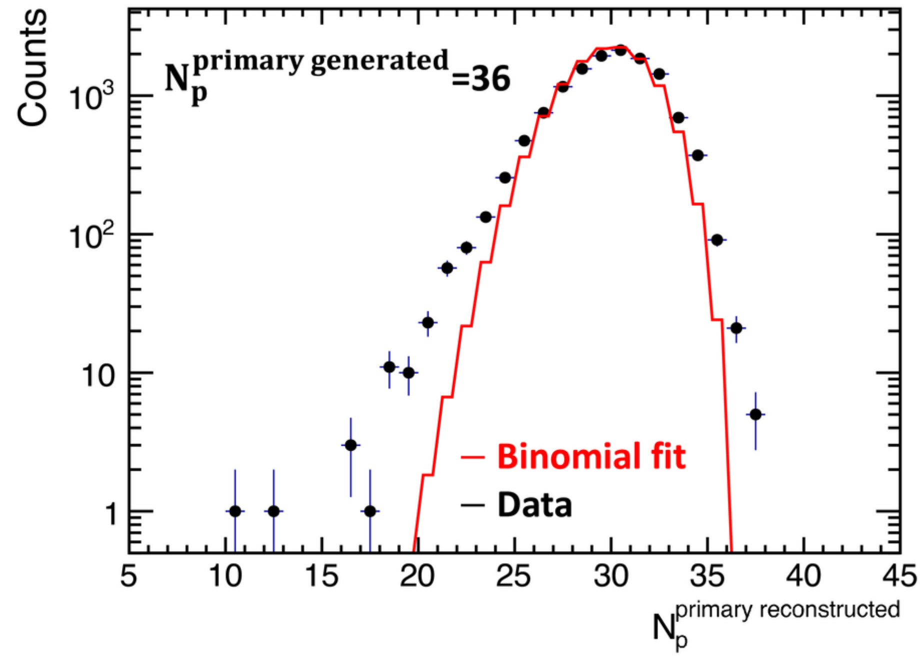
$$k_4(p-\bar{p}) = \langle N_w \rangle k_4(n-\bar{n}) + 3k_2(n-\bar{n})^2 k_2(N_w) + \langle n-\bar{n} \rangle (\ldots)$$

$n, \bar{n}$ → from single wounded nucleon

vanishes for ALICE

Is efficiency loss binomial in ALICE?

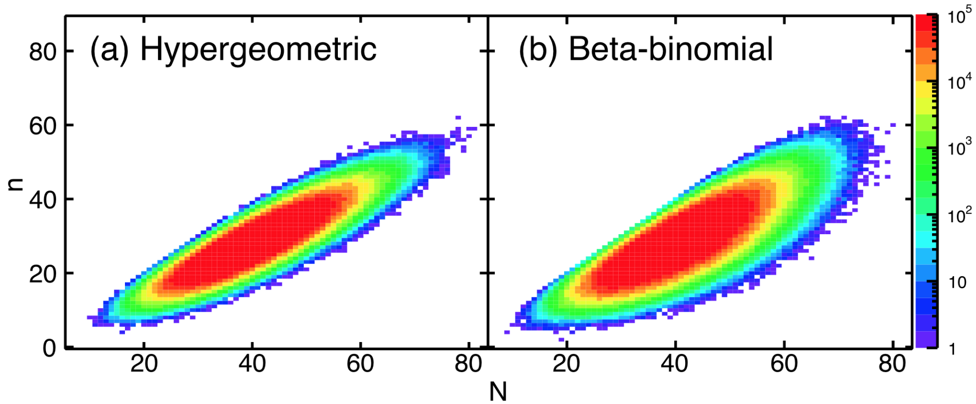




- Efficiency loss **deviates from binomial**
- How does it influence the efficiency correction of higher order cumulants?

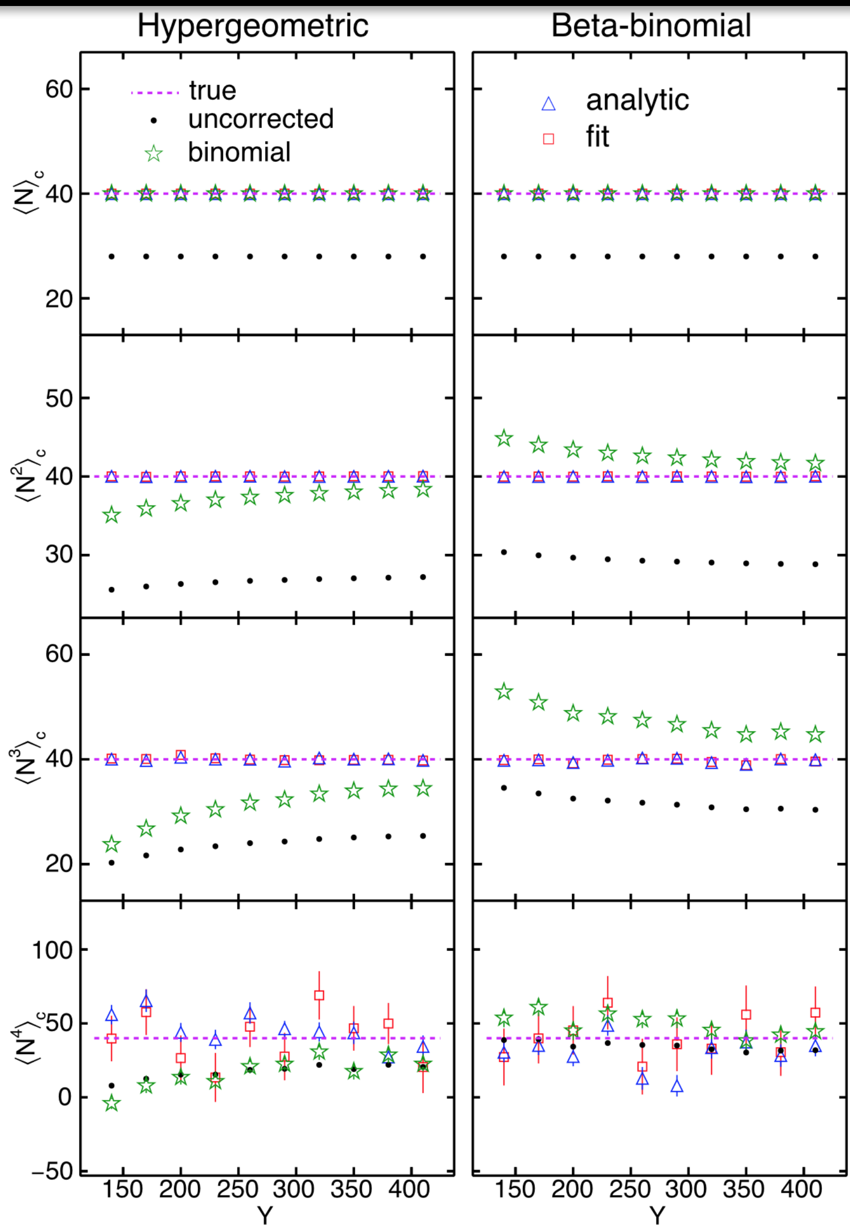
Efficiency correction

What if efficiency loss is not binomial?



Draw N balls from the urn without returning balls to the urn

In each draw, when one draws a white ball, two white balls are returned to the urn



T. Nonaka, M. Kitazawa, S. Esumi, Nucl.Instrum.Meth. A906 (2018) 10-17
T. Nonaka, M. Kitazawa, S. Esumi, Phys. Rev. C 95, 064912 (2017)
Adam Bzdak, Volker Koch, Phys. Rev. C86, 044904 (2012)

- Probability of measuring n_B baryons in the acceptance:

$$B\left(n_B; N_B, \alpha\right) = \frac{N_B!}{n_B!\left(N_B - n_B\right)!}\alpha^{n_B}\left(1-\alpha\right)^{N_B - n_B} \qquad \alpha = \frac{\langle N_B^{acc}\rangle}{\langle N_B^{4\pi}\rangle}$$

- Multiplicity distribution in the acceptance:

$$P(n_B) = \sum_{N_B} B(n_B; N_B, \alpha) P(N_B)$$

- The moments of the measured baryon distributions can be then calculated

$$\langle n_B \rangle = \sum_{n_B=0}^{\infty} n_B P(n_B) = \alpha \langle N_B \rangle \,,$$

$$\left\langle n_B^2 \right\rangle = \sum_{n_B=0}^{\infty} n_B^2 P(n_B) = \alpha^2 \left\langle N_B^2 \right\rangle + \alpha(1-\alpha)\langle N_B \rangle$$

0

$$\frac{\kappa_2\left(n_B - n_{\bar{B}}\right)}{\kappa_2\left(Skellam\right)} = \frac{\kappa_2\left(n_B - n_{\bar{B}}\right)}{\alpha\left(\langle N_B\rangle + \langle N_{\bar{B}}\rangle\right)} = \alpha\frac{\kappa_2\left(N_B - N_{\bar{B}}\right)}{\langle N_B\rangle + \langle N_{\bar{B}}\rangle} + 1 - \alpha$$

MC implementation of canonical ensemble

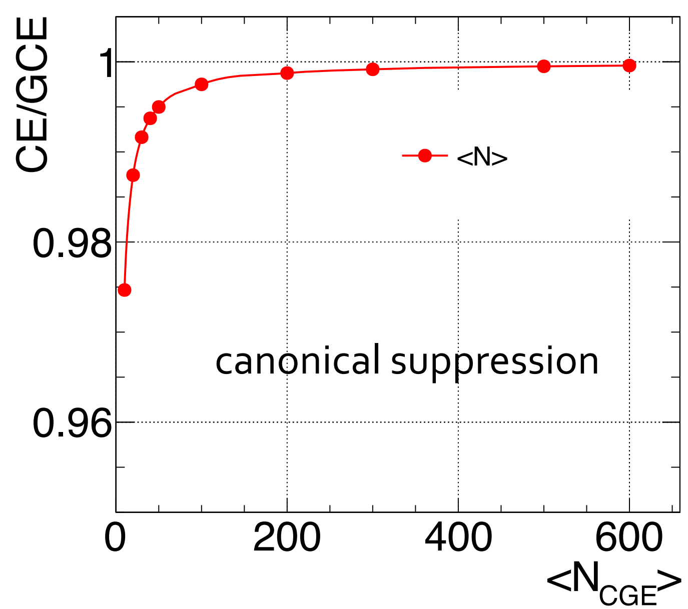
Two baryon species with the baryon numbers +1 and -1 in the ideal Boltzmann gas

$$Z_{GCE}(V,T,\mu)=\sum_{N_B=0}^{\infty}\sum_{N_{\bar{B}}=0}^{\infty}\frac{(\lambda_B z)^{N_B}}{N_B!}\frac{(\lambda_{\bar{B}} z)^{N_{\bar{B}}}}{N_{\bar{B}}!}=e^{2z\cosh\left(\mu/T\right)},\quad \lambda_{B,\bar{B}}=e^{\pm\frac{\mu}{T}}$$

$$Z_{CE}(V,T,B)=\sum_{N_B=0}^{\infty}\sum_{N_{\bar{B}}=0}^{\infty}\frac{(\lambda_B z)^{N_B}}{N_B!}\frac{(\lambda_{\bar{B}} z)^{N_{\bar{B}}}}{N_{\bar{B}}!}\delta\left(N_B-N_{\bar{B}}-B\right)=I_B(2z)\Big|_{\lambda_B=\lambda_{\bar{B}}=1}$$

$$\langle N_{B,\bar{B}}\rangle_{GCE}=\lambda_{B,\bar{B}}\frac{\partial\ln Z_{GCE}}{\partial\lambda_{B,\bar{B}}}=e^{\pm\frac{\mu}{T}}z,\quad z=\sqrt{\langle N_B\rangle_{GCE}\langle N_{\bar{B}}\rangle_{GCE}}$$

$$\langle N_{B,\bar{B}}\rangle_{CE}=\sqrt{\langle N_B\rangle_{GCE}\langle N_{\bar{B}}\rangle_{GCE}}\frac{I_{B\mp1}\left(2\sqrt{\langle N_B\rangle_{GCE}\langle N_{\bar{B}}\rangle_{GCE}}\right)}{I_B\left(2\sqrt{\langle N_B\rangle_{GCE}\langle N_{\bar{B}}\rangle_{GCE}}\right)}$$



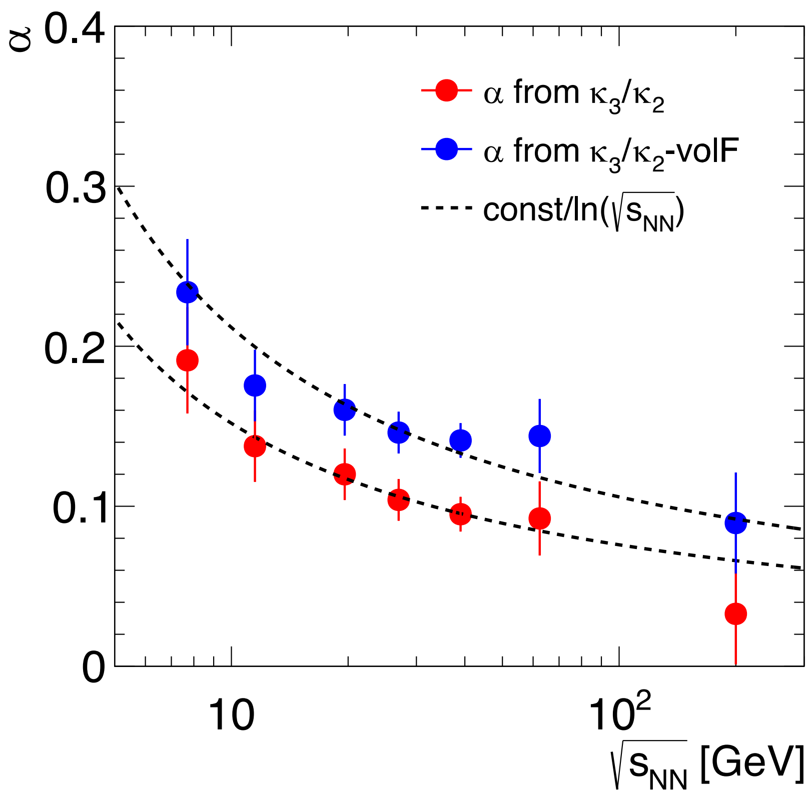
R. Hagedorn, K. Redlich Z. Phys. 27, 1985
V.V. Begun, M. I. Gorenstein, O. S. Zozulya, PRC 72 (2005) 014902
P. Braun-Munzinger, B. Friman, F. Karsch, K. Redlich, V. Skokov, NPA 880 (2012)
A. Bzdak, V. Koch, V. Skokov, PRC87 (2013) 014901

P. Braun-Munzinger, A. Rustamov, J. Stachel, NPA 982 (2019) 307-310

Results from STAR vs Our predictions

Acceptances: $\alpha_{\sqrt{s}=7.7GeV} = 0.19 \pm 0.03, \quad \alpha_{\sqrt{s}=19.6GeV} = 0.12 \pm 0.016$

$$\frac{\kappa_3}{\kappa_2} = \frac{\langle n_B - n_{\bar{B}} \rangle_{CE}}{\langle n_B + n_{\bar{B}} \rangle_{CE}}(1-2\alpha), \quad \frac{\kappa_4}{\kappa_2} = 1 - 6\alpha(1-\alpha)\left(1 - \frac{2}{\langle n_B + n_{\bar{B}} \rangle_{CE}}\left[\langle n_B \rangle_{GCE} \langle n_{\bar{B}} \rangle_{GCE} - \langle n_B \rangle_{CE} \langle n_{\bar{B}} \rangle_{CE}\right]\right)$$



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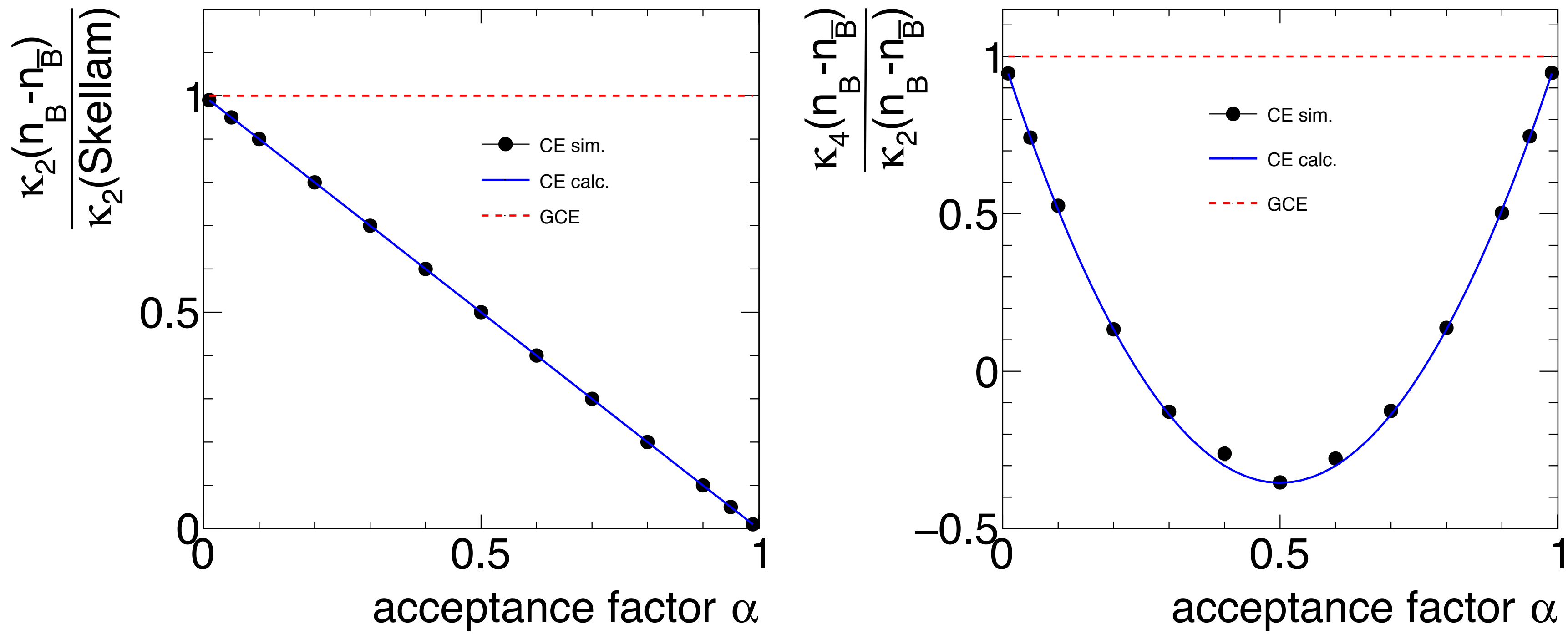
Effect of baryon number conservation at 4th order?

CE:Canonical ensemble

GCE: Grand canonical ensemble

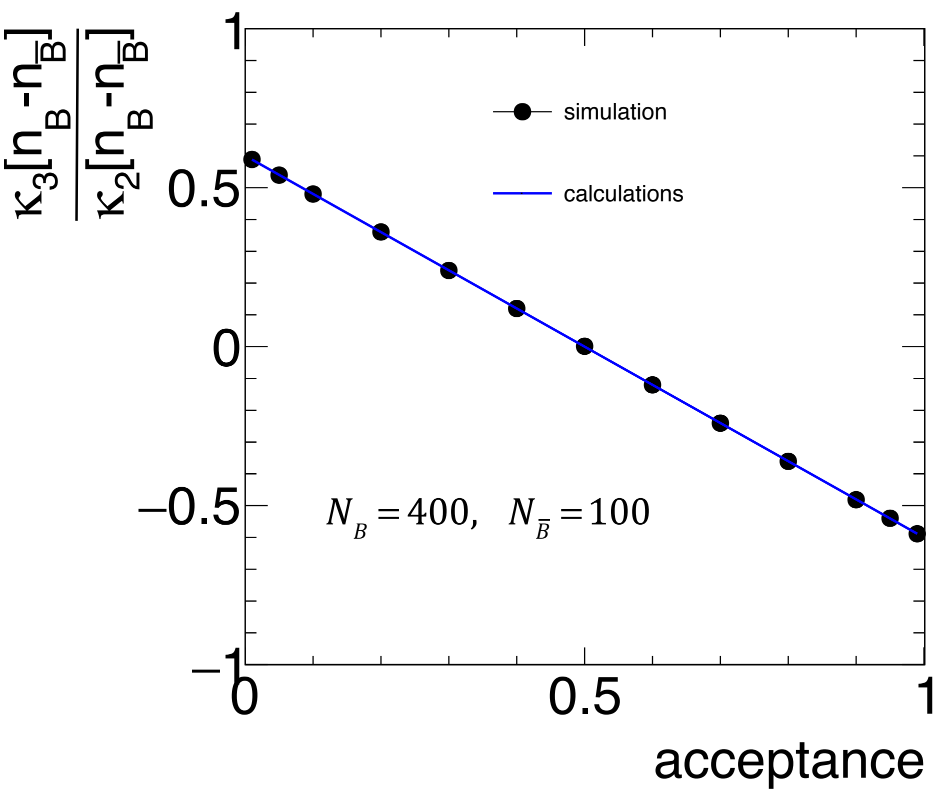


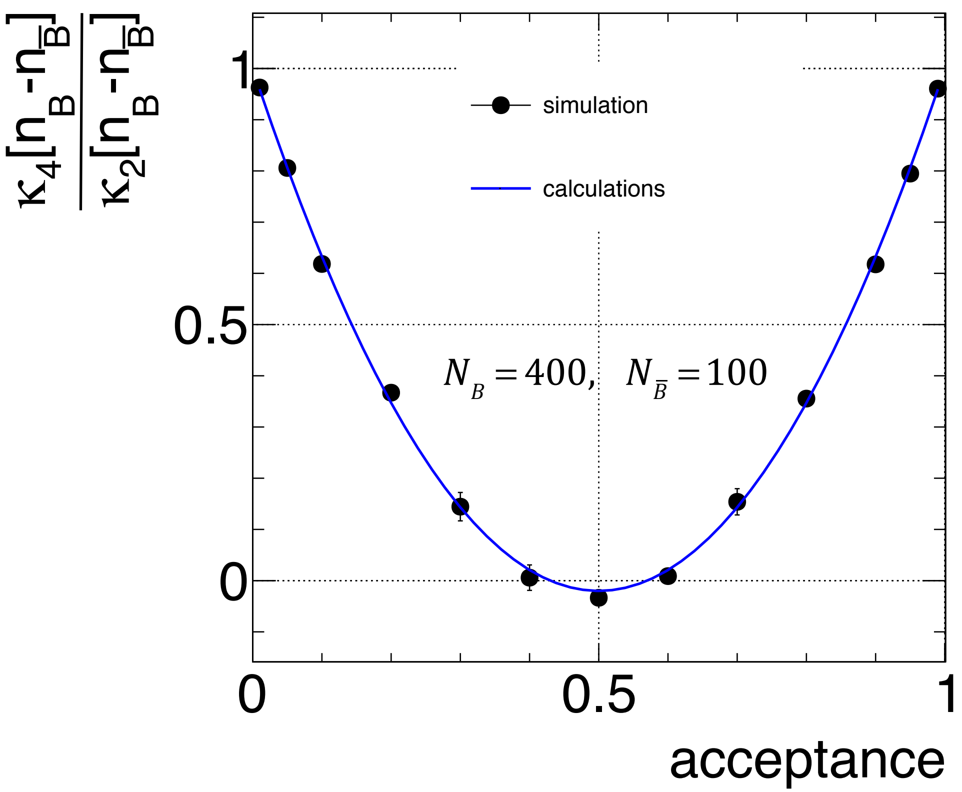
- **Small acceptance** → small multiplicities → approach to Poissonian limit
- Acceptance is more crucial for the **4th cumulant**

P. Braun-Munzinger, A. Rustamov, J. Stachel, NPA 982 (2019) 307-310

3rd and 4th cumulants

$$\frac{\kappa_3}{\kappa_2} = \frac{\langle n_B - n_{\bar{B}} \rangle_{CE}}{\langle n_B + n_{\bar{B}} \rangle_{CE}} (1-2\alpha) \xrightarrow{\langle n_{\bar{B}} \to 0 \rangle} (1-2\alpha)$$





$$\frac{\kappa_4}{\kappa_2} = 1 - 6\alpha(1-\alpha)\left(1 - \frac{2}{\langle n_B + n_{\bar{B}} \rangle_{CE}} \left[\langle n_B \rangle_{GCE} \langle n_{\bar{B}} \rangle_{GCE} - \langle n_B \rangle_{CE} \langle n_{\bar{B}} \rangle_{CE} \right] \right)$$

Free energy density

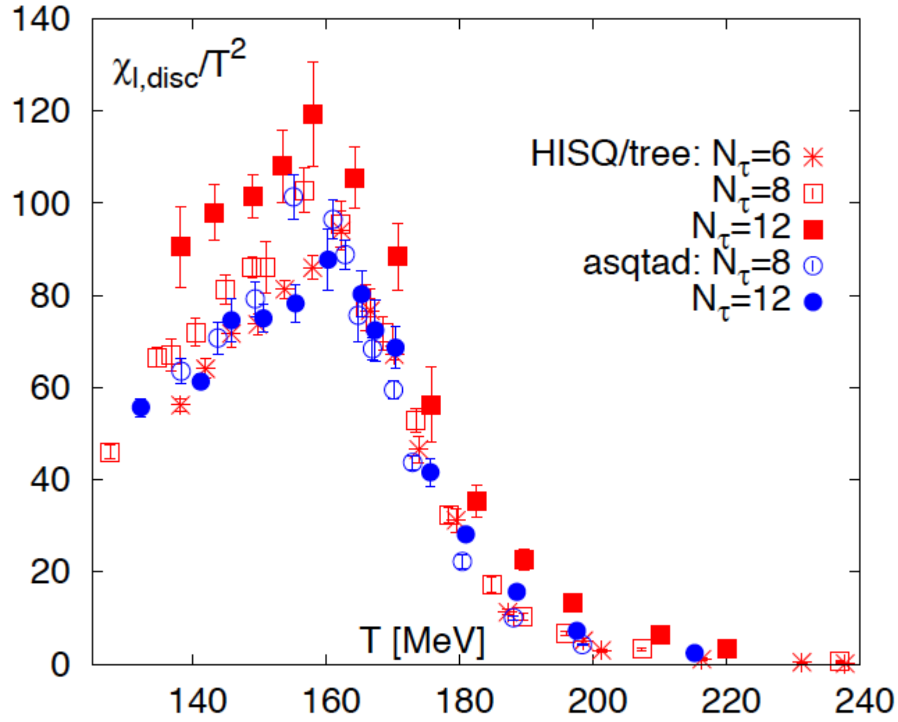
$$f = -\frac{T}{V}\ln Z$$

2-flavor light quark chiral condensate

$$\langle\bar{\psi}\psi\rangle_l^{n_f=2} = \frac{T}{V}\frac{\partial \ln Z}{\partial m_l}$$

Chiral susceptibility (sum of connected and disconnected Feynman diagrams)

$$\chi_{m,l} = \frac{\partial}{\partial m_l}\langle\bar{\psi}\psi\rangle_l^{n_f=2}$$



"The disconnected part of the light quark susceptibility describes the fluctuations in the light quark condensate"