Chirality, Helicity, Anomaly in High-Energy Nuclear Physics

Kenji Fukushima

The University of Tokyo

— Quantum Systems in Extreme Conditions (QSEC2019) —

Formal:

Anomaly

=

Phase Ambiguity in the Partition Func.

Very powerful theoretical tool

Formal:

Anomaly

=

Phase Ambiguity in the Partition Func.

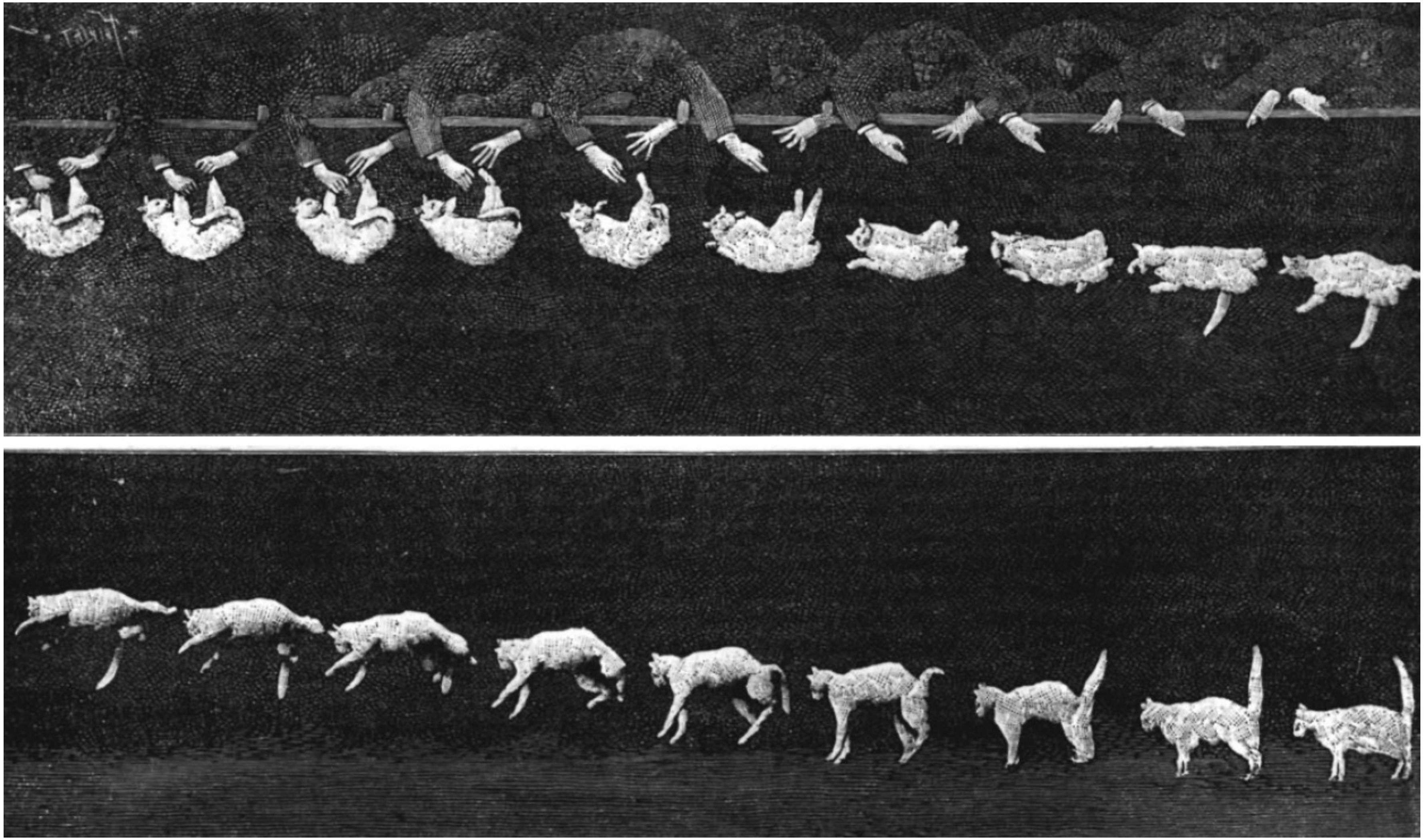
Very powerful theoretical tool

Practical:

Something counterintuitive?

Very interesting phenomena

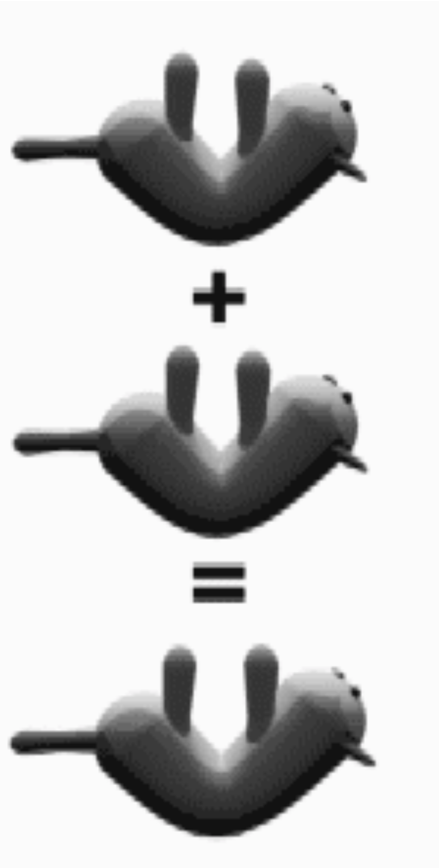
Falling Cat Problem



[Wikipedia]

Falling Cat Problem

Cats can survive picking up a geometrical phase



For further reading:

"*Gauge kinematics of deformable bodies*"
by A. Shapere, F. Wilczek
American Journal of Physics (1989)

Berry Phase

Larmor precession of spin

$$\frac{\partial \boldsymbol{S}}{\partial t} = \frac{e}{m_e} \boldsymbol{B} \times \boldsymbol{S}$$

How to derive this? $\leftarrow$ Commutator

$$[\hat{S}^i, \hat{S}^j] = i\epsilon^{ijk} S^k$$

Quantum Physics

Berry Phase

Larmor precession of spin

$$\frac{\partial \boldsymbol{S}}{\partial t} = \frac{e}{m_e} \boldsymbol{B} \times \boldsymbol{S}$$

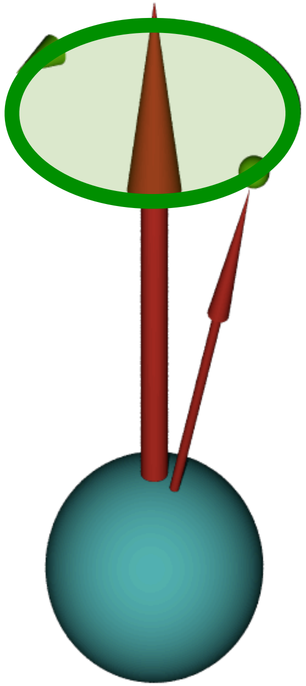
How to derive this classically? (Euler-Lagrange eq.)
Lagrangian is needed for the path integral!

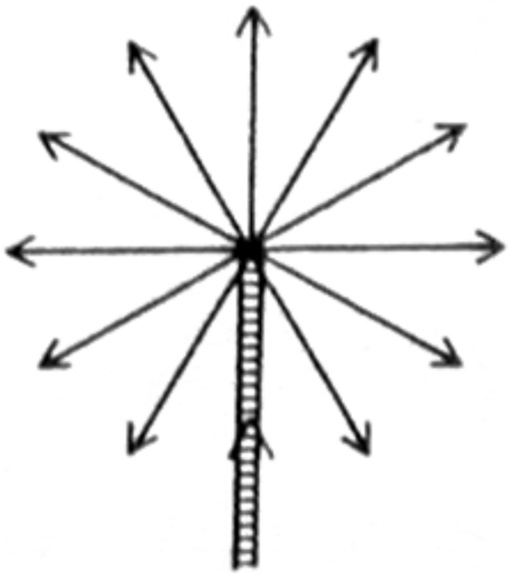
$$L = \underbrace{-S\dot{\phi}(\cos\theta - 1)}_{\text{Geometrical (Berry) Phase}} + \frac{e}{m_e} \boldsymbol{B} \cdot \boldsymbol{S}$$

Berry Phase

$$L = \underline{-S\dot{\phi}(\cos\theta - 1)} + \frac{e}{m_e}\boldsymbol{B}\cdot\boldsymbol{S}$$

Geometrical (Berry) Phase





Dirac monopole

Berry Phase in High-Energy QCD

Side Remark

In the Color Glass Condensate (Talk by Venugopalan) the color source is "classical" (dense).

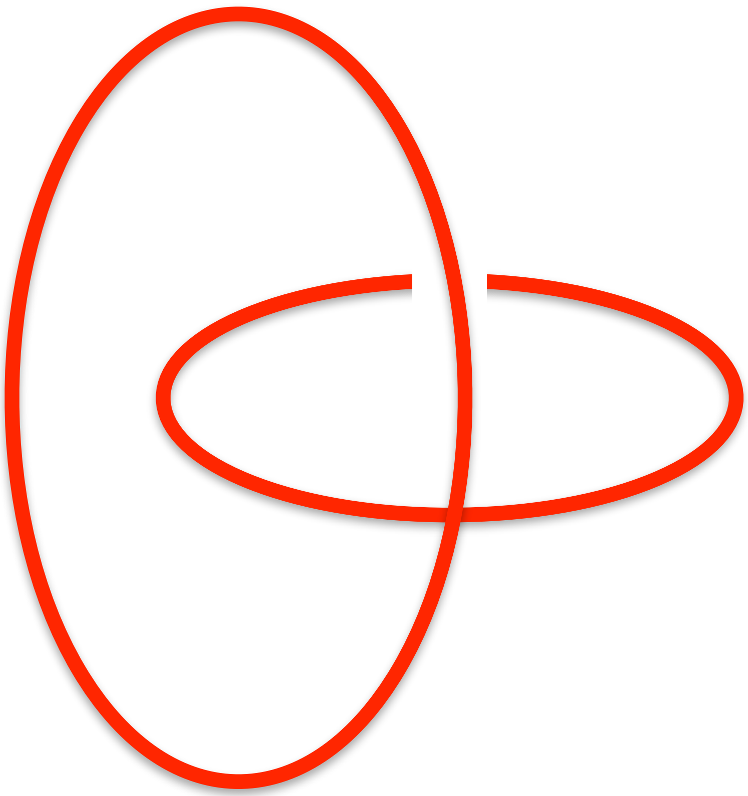
Going to a quantum (dilute) regime, one should take account of the commutator:

$$\boxed{\left[\hat{\rho}^a(x^+,\vec{x}),\hat{\rho}^b(x^+,\vec{y})\right] = -igf^{abc}\hat{\rho}^c(x^+,\vec{x})\;\delta^{(3)}(\vec{x}-\vec{y})}$$

This is compactly formulated with a Berry phase
***"In pursuit of Pomeron loops: The JIMWLK equation and the Wess-Zumino term"* by A. Kovner and M. Lublinsky**

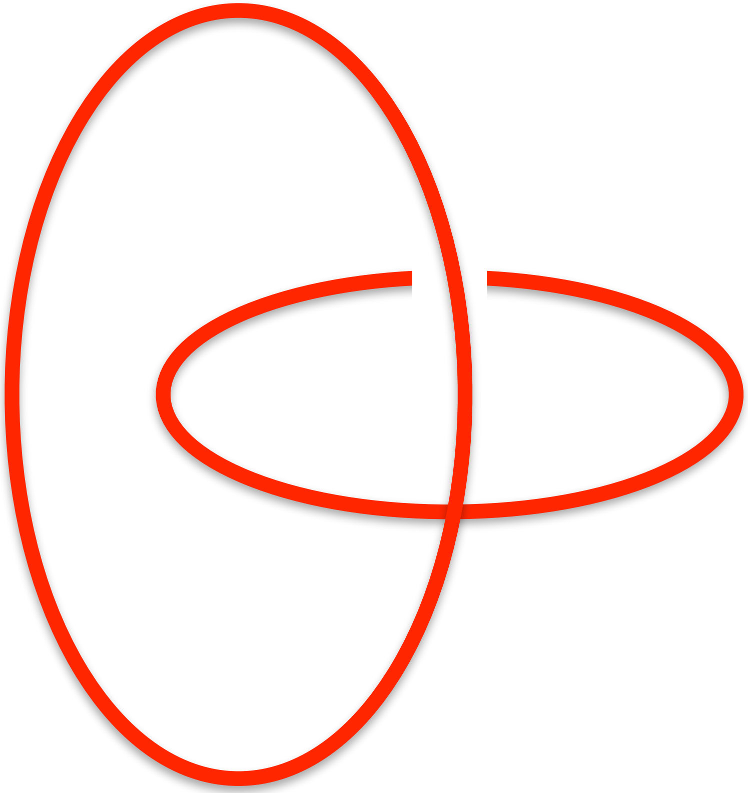
Another Example of Geometrical Inv.

Linkage Number

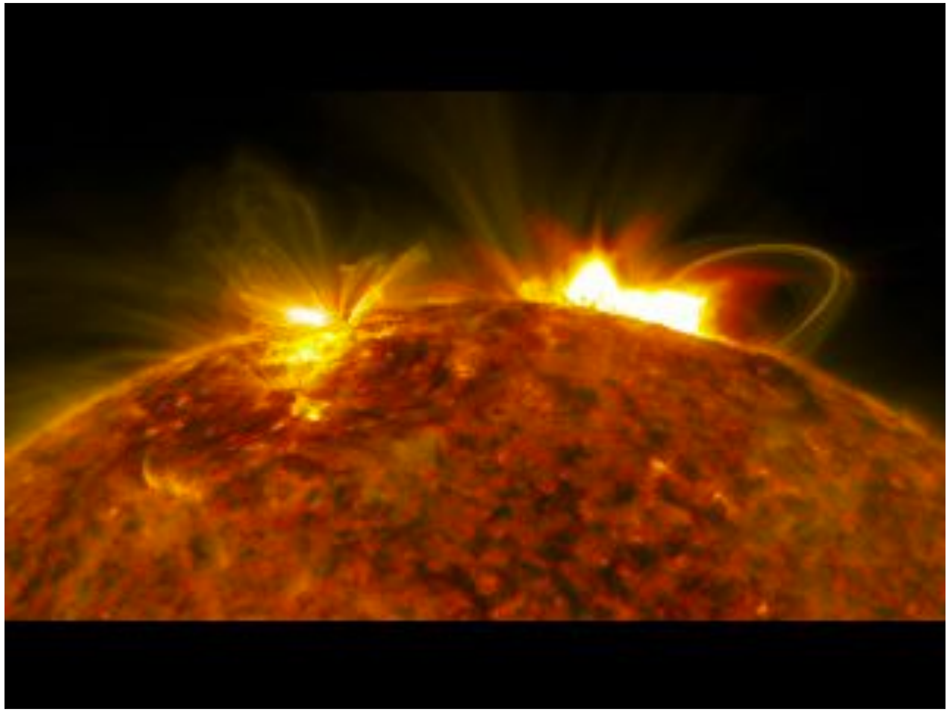


Another Example of Geometrical Inv.

Linkage Number



Magnetic Flux

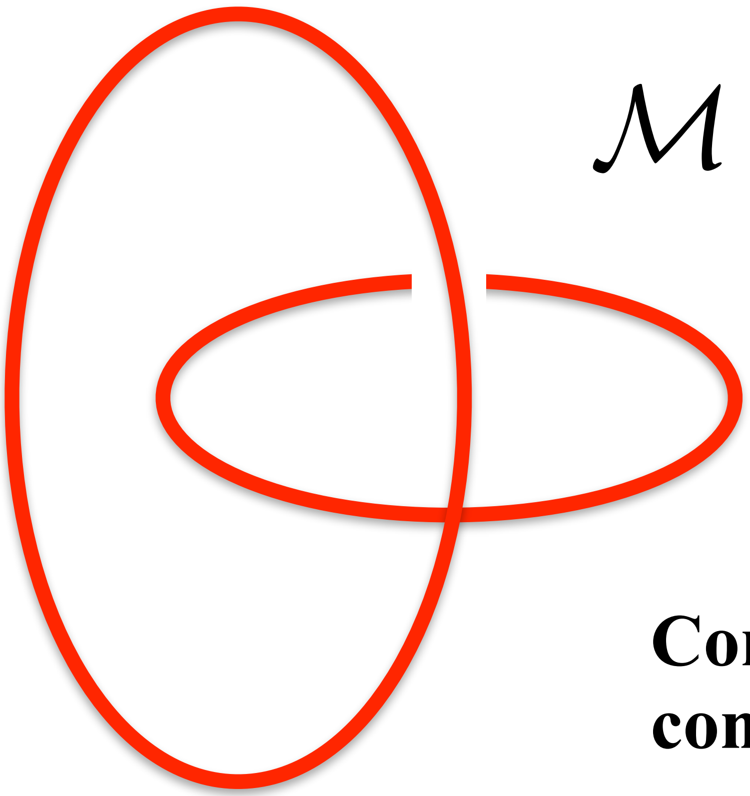


Another Example of Geometrical Inv.

Linkage Number

Magnetic Helicity

$$\mathcal{M} = \frac{e^2}{4\pi^2} \int d^3x \, \boldsymbol{A} \cdot \boldsymbol{B}$$



Magnetic Flux

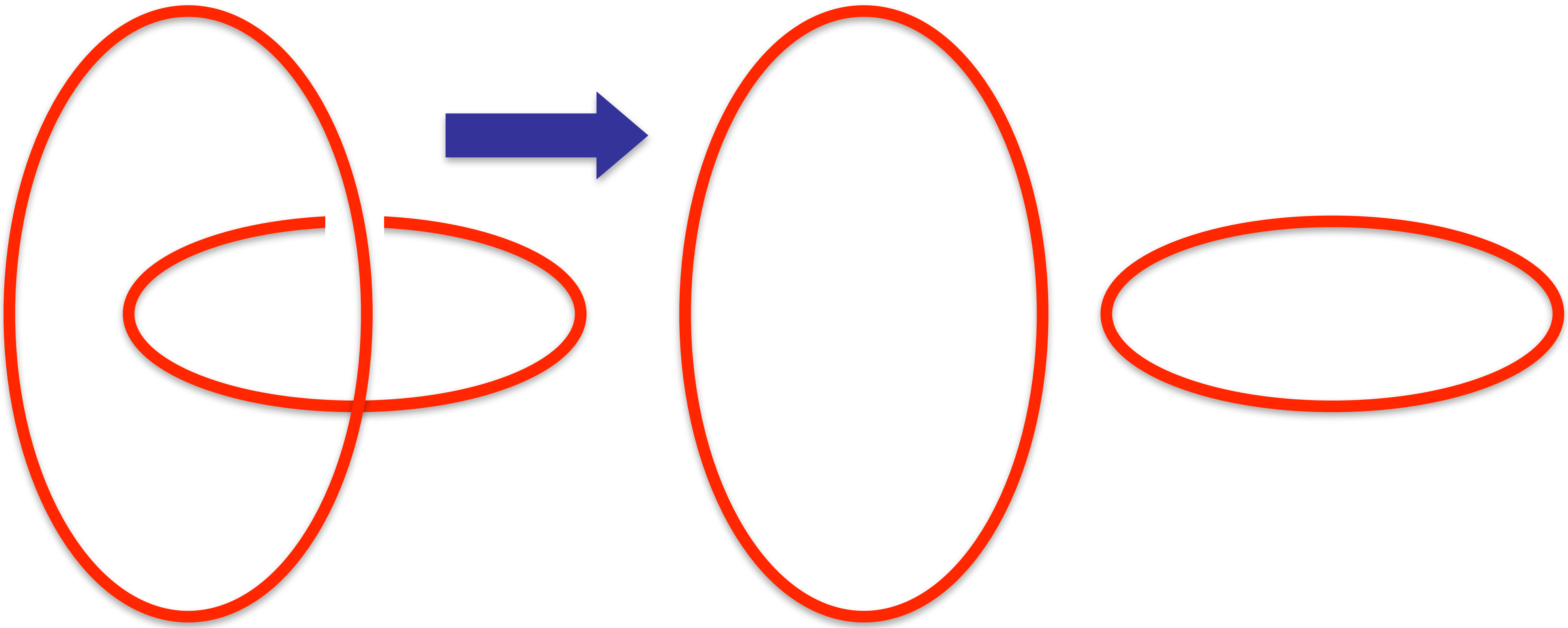
Gauge Invariant???
Yes, up to surface terms

Conserved charge associated with continuous duality transformation

Another Example of Geometrical Inv.

What happens?

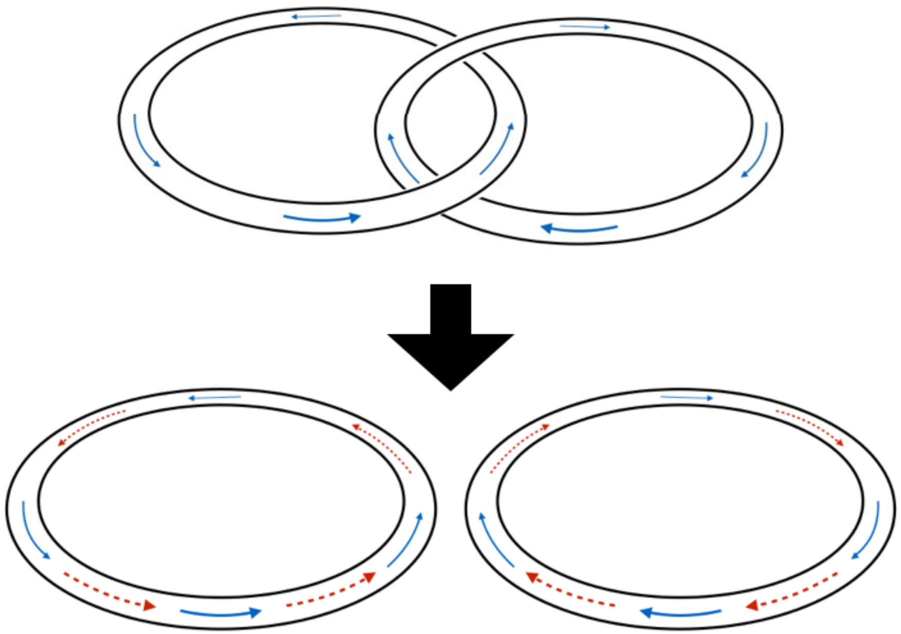
Possible with matter



Another Example of Geometrical Inv.

What happens?

This question was addressed in
“*Quantized chiral magnetic current from reconnection of magnetic flux*” by Hirono, Kharzeev, Yin (2016)



Anomaly Relation

Conservation Law (Talk by Schlichting)

$$\frac{d}{dt}(N_5 + \mathcal{M}) = 2m \int d^3x \, \langle \bar{\psi} i \gamma_5 \psi \rangle$$

"Chirality"

Magnetic Helicity

Negligible for massless theories usually… but some subtleties:
see "***Axial Ward identity and the Schwinger mechanism***"
by Copinger-Fukushima-Pu (2018)

Anomaly Relation

Pseudo-conserved Current (Chern-Simons)

$$\boldsymbol{E} \cdot \boldsymbol{B} = \partial_\mu K^\mu$$

$$= \frac{d}{dt} \underline{(\boldsymbol{A} \cdot \boldsymbol{B})} + \boldsymbol{\nabla} \cdot \underline{(A_0 \boldsymbol{B} + \boldsymbol{E} \times \boldsymbol{A})}$$

Magnetic Helicity **Optical Angular Momentum**

j_5^0 **Chirality of matter** $\boldsymbol{j}_5$ **Spin of matter**

Angular Momentum Decomposition

Side Remark

- proton spin problem (gluon helicity)
- rotating quark-gluon plasma (Λ polarization)
- laser physics (discussions by S. Barnett)

Jaffe-Manohar decomp.

$$\boldsymbol{L} = \int d^3x\, E^i(\boldsymbol{x}\times\boldsymbol{\nabla})A^i \qquad L^g$$

$$\boldsymbol{S} = \int d^3x\, \boldsymbol{E}\times\boldsymbol{A} \qquad \Delta G$$

"*The photon angular momentum controversy: Resolution of a conflict between laser optics and particle physics*" by E. Leader (2016)

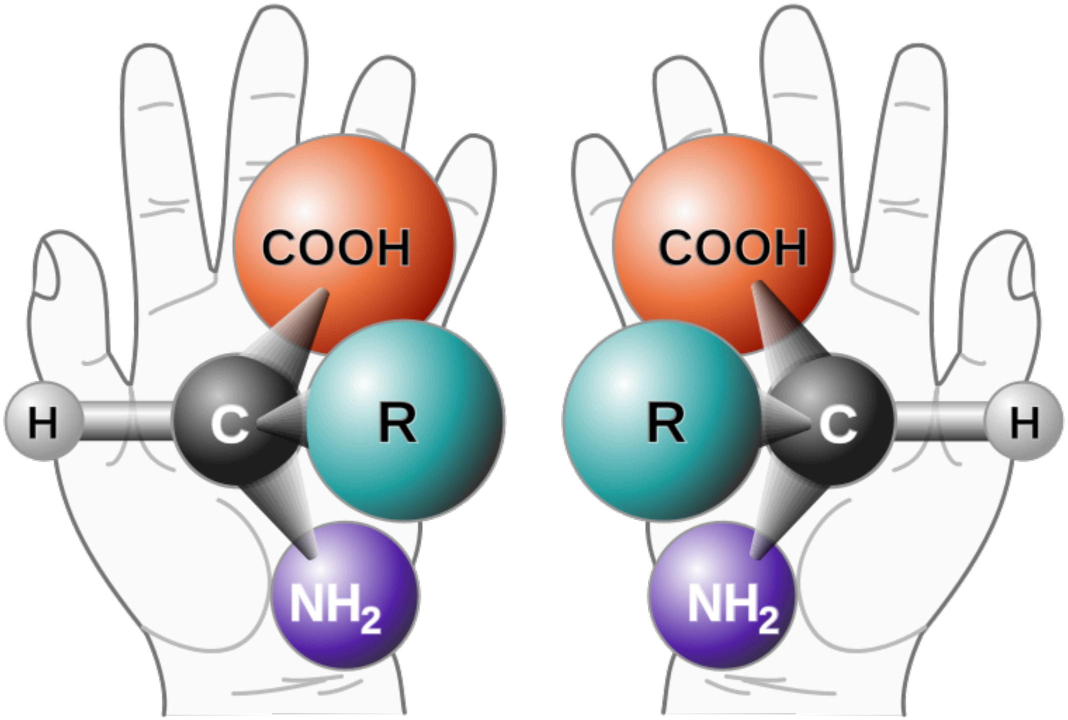
A pedagogical review (next month) by Fukushima-Pu

Chirality

Chirality of Oldies

Mirror image (enantiomer) not identical ← Old definition!

L.D. Barron : True and False Chirality

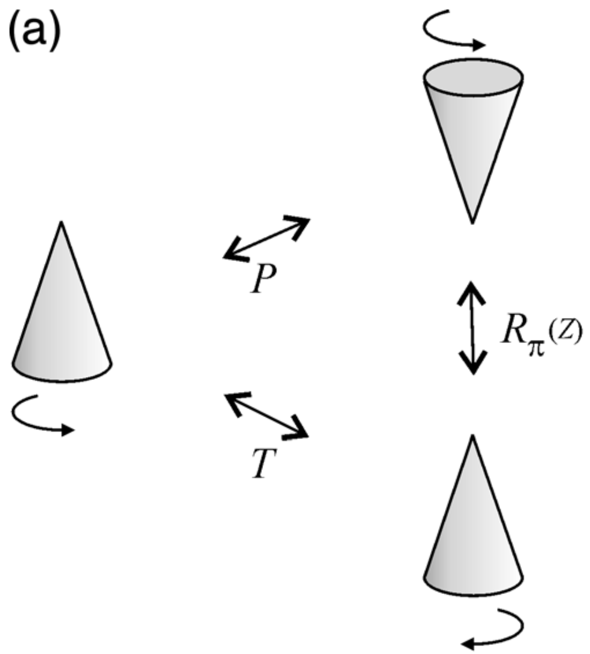


Chirality

Chirality of Today's Chemistry

L.D. Barron : True and False Chirality

spinning cones



True chirality is exhibited by systems existing in two distinct enantiomeric states that are interconverted by space inversion, but not by time reversal combined with any proper spatial rotation.

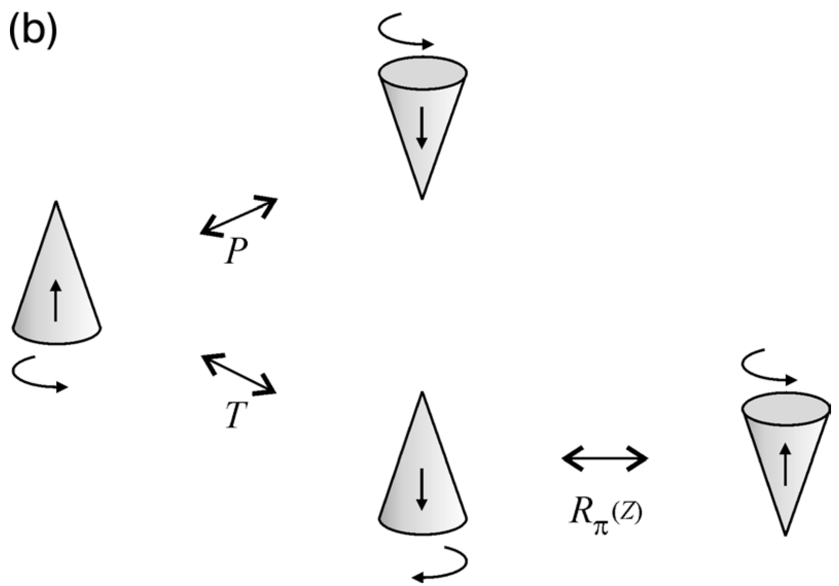
Laurence D. Barron
“An Introduction to Chirality at the Nanoscale"

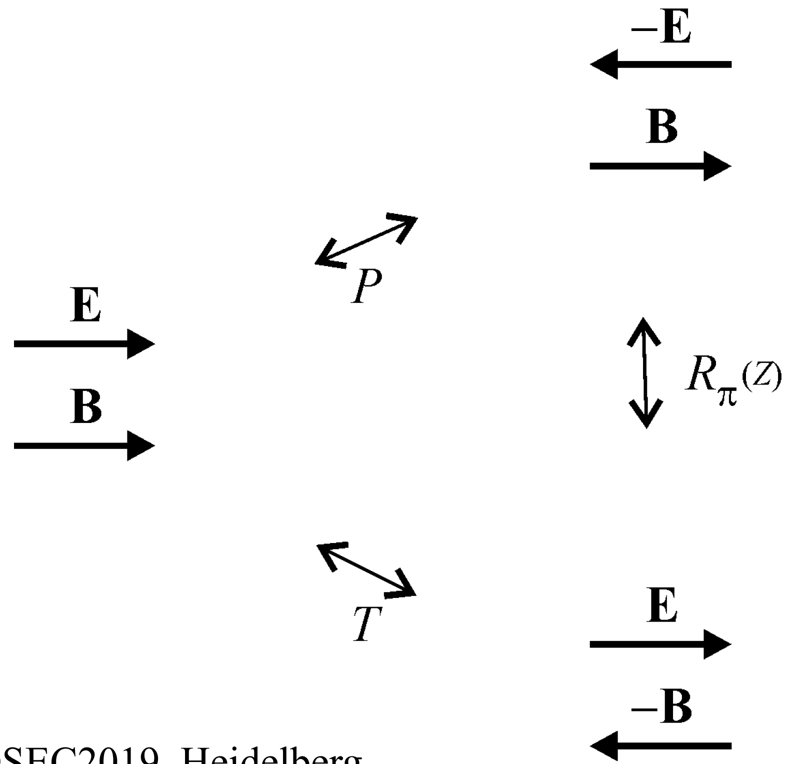
Chirality

Chirality of Today's Chemistry

L.D. Barron : True and False Chirality

translating spinning cones **collinear *E* and *B* = False Chirality**





Chirality

Side Remark

$$\boldsymbol{E} \cdot \boldsymbol{B} \neq 0$$ **This is not "chiral"**

What is "chiral" ? $\leftarrow$ Lipkin's Zilch

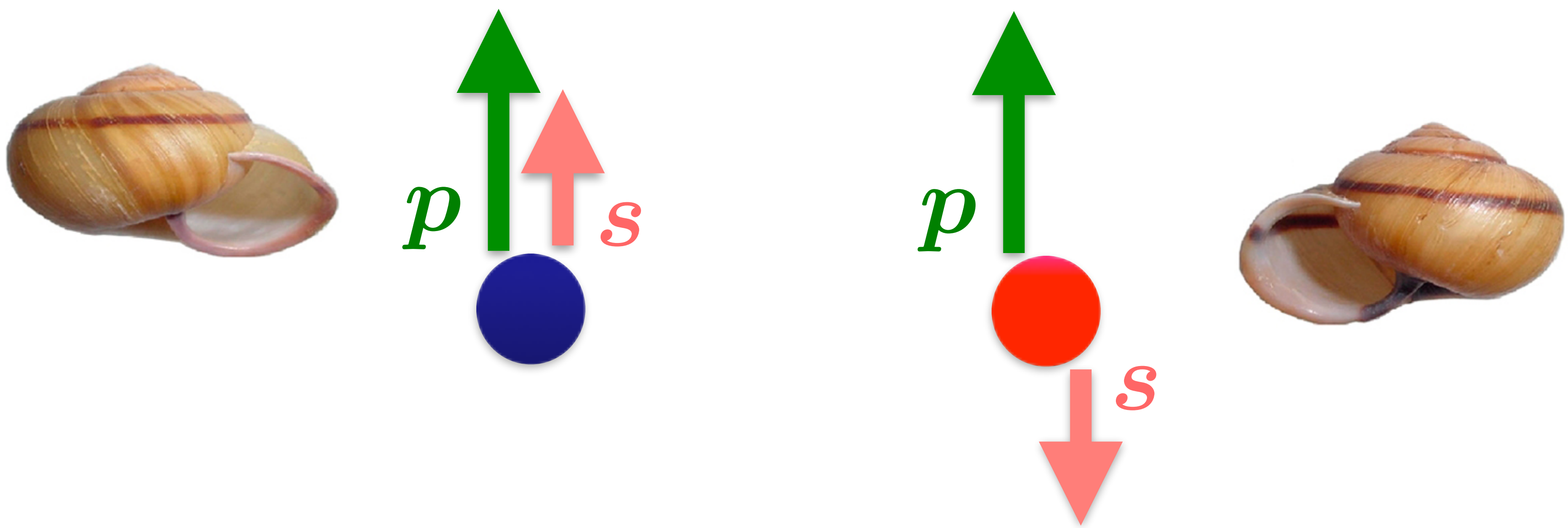
$$\chi = \frac{1}{2} \int d^3x \, (\boldsymbol{B} \cdot \partial_t \boldsymbol{E} - \boldsymbol{E} \cdot \partial_t \boldsymbol{B})$$

See: "*Zilch vortical effect*"
by Chernodub, Cortijo, Landsteiner (2018)

Chirality

Helicity (massless theory)

$$\boldsymbol{s} \cdot \hat{\boldsymbol{p}} = +\frac{1}{2} \qquad \boldsymbol{s} \cdot \hat{\boldsymbol{p}} = -\frac{1}{2}$$



Chiral Kinetic Theory (CKT)

Boltzmann (Vlasov) equation with spin effects

$$(1 + \boldsymbol{B}\cdot\boldsymbol{\Omega})\frac{\partial f}{\partial t} + \left[\boldsymbol{v}_p + (\boldsymbol{v}_p\cdot\boldsymbol{\Omega})\boldsymbol{B} + \boldsymbol{E}\times\boldsymbol{\Omega}\right]\cdot\frac{\partial f}{\partial \boldsymbol{x}}$$

$$+ \left[\boldsymbol{E} + \boldsymbol{v}_p\times\boldsymbol{B} + (\boldsymbol{E}\cdot\boldsymbol{B})\boldsymbol{\Omega}\right]\cdot\frac{\partial f}{\partial \boldsymbol{p}} = I_{\rm coll}[f]$$

$$\boldsymbol{\Omega} = \hbar\frac{\boldsymbol{p}}{2|\boldsymbol{p}|^3}$$ **Berry curvature**

$$\varepsilon = |\boldsymbol{p}| - \boldsymbol{B}\cdot p\boldsymbol{\Omega} \qquad \boldsymbol{v} = \frac{\partial\varepsilon}{\partial\boldsymbol{p}}$$

Son-Yamamoto (2012)

Stephanov-Yin (2012)

Chiral Kinetic Theory (CKT)

Boltzmann (Vlasov) equation with spin effects

$$\boxed{(1+\boldsymbol{B}\cdot\boldsymbol{\Omega})}\frac{\partial f}{\partial t}+\boxed{\left[\boldsymbol{v}_p+(\boldsymbol{v}_p\cdot\boldsymbol{\Omega})\boldsymbol{B}+\boldsymbol{E}\times\boldsymbol{\Omega}\right]}\cdot\frac{\partial f}{\partial \boldsymbol{x}}$$

$$+\left[\boldsymbol{E}+\boldsymbol{v}_p\times\boldsymbol{B}+(\boldsymbol{E}\cdot\boldsymbol{B})\boldsymbol{\Omega}\right]\cdot\frac{\partial f}{\partial \boldsymbol{p}}=I_{\mathrm{coll}}[f]$$

$$\boldsymbol{\Omega}=\pm\hbar\frac{\boldsymbol{p}}{2|\boldsymbol{p}|^3}$$ **Berry curvature**

$$\varepsilon=|\boldsymbol{p}|-\boldsymbol{B}\cdot p\boldsymbol{\Omega}\qquad \boldsymbol{v}=\frac{\partial\varepsilon}{\partial\boldsymbol{p}}$$

Son-Yamamoto (2012)

Stephanov-Yin (2012)

Chiral Kinetic Theory (CKT)

Boltzmann (Vlasov) equation with spin effects

$$\rho = \int_{\boldsymbol{p}} (1 + \boldsymbol{B} \cdot \boldsymbol{\Omega}) f(\boldsymbol{p}, \boldsymbol{x})$$

$$\boldsymbol{j} = \int_{\boldsymbol{p}} \left[\boldsymbol{v}_p + \underline{(\boldsymbol{v}_p \cdot \boldsymbol{\Omega})\boldsymbol{B}} + \boldsymbol{E} \times \boldsymbol{\Omega} \right] f(\boldsymbol{p}, \boldsymbol{x})$$

Electric Current ~ Magnetic Field

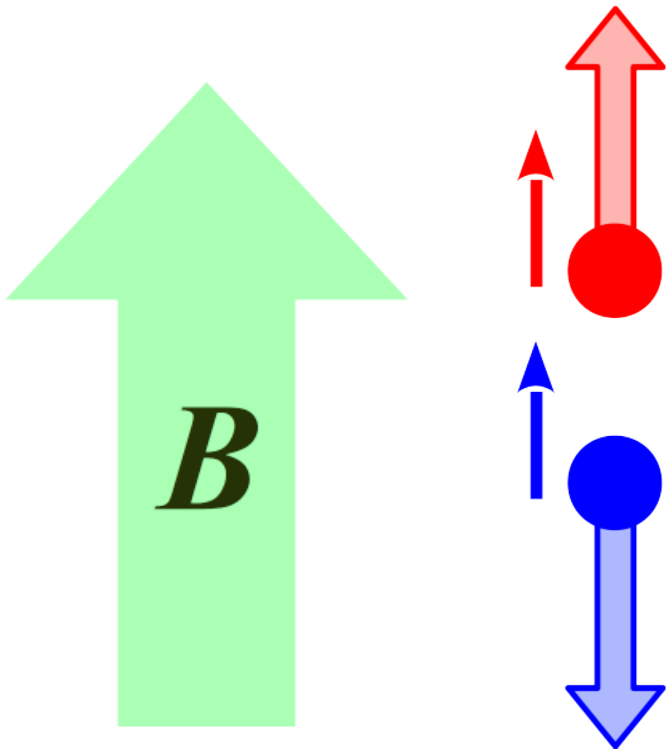
Chiral Magnetic Effect

"The effects of topological charge change in heavy ion collisions: 'Event by even P and CP violation'" **by Kharzeev, McLerran, Warringa**

Chiral Magnetic Effect

Chiral Magnetic Effect

"*The Chiral Magnetic Effect*" by Fukushima, Kharzeev, Warringa (2008)



Right-handed particles
Momentum parallel to Spin

Left-handed particles
Momentum anti-parallel to Spin

Chiral Magnetic Effect

Chiral Magnetic Effect

"***The Chiral Magnetic Effect***" **by Fukushima, Kharzeev, Warringa (2008)**

$$\boldsymbol{j} = \frac{e^2 \mu_5}{2\pi^2} \boldsymbol{B}$$

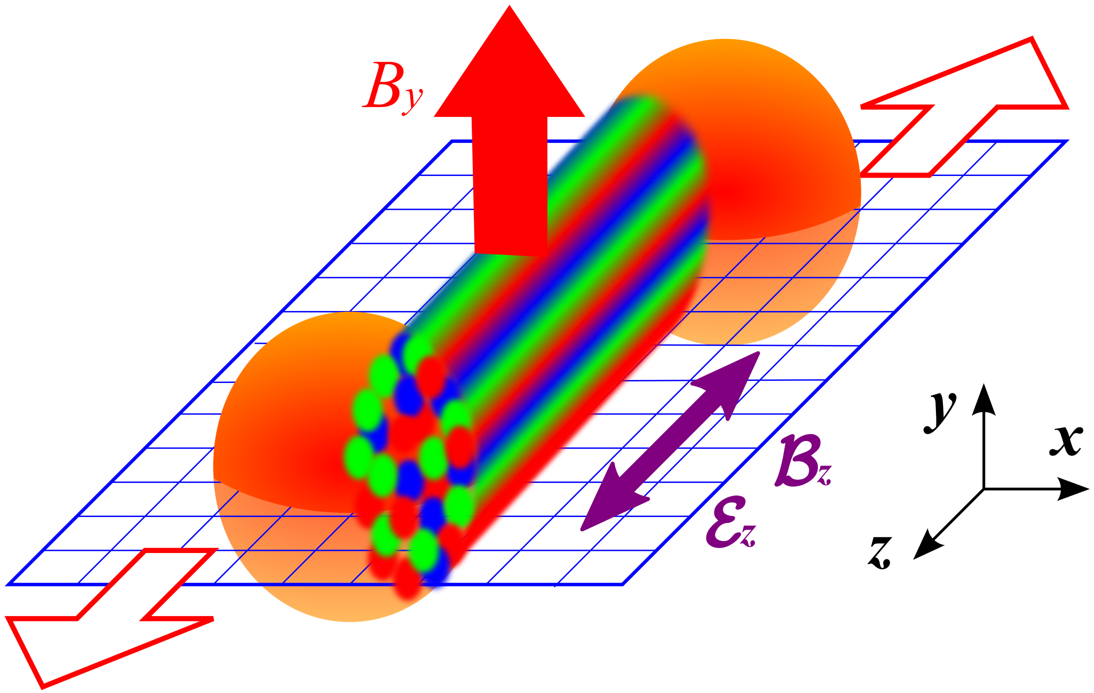
Chiral chemical potential is nonzero only out of equilibrium.

See: "*Axial Ward identity and the Schwinger mechanism*"
by Copinger, Fukushima, Pu (2018)

Chiral Magnetic Effect

Chiral Magnetic Effect

"*Real-time dynamics of the Chiral Magnetic Effect*"
by Fukushima, Kharzeev, Warringa (2010)



Chiral Magnetic Effect

Chiral Magnetic Effect

In heavy ion collisions isobar experiments will clarify!

"Examination of the observability of a chiral magnetically driven charge-separation difference…" **by Magdy, Shi, Liao, Liu, Lacey (2018)**

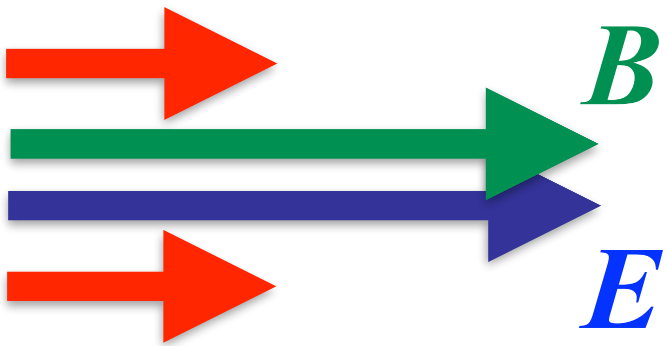
Classical statistical simulations by
J. Berges, S. Schlichting, N. Mueller, S. Sharma, M. Mace
R. Venugopalan, …

Experimentally confirmed in Weyl semimetals

Chiral Magnetic Effect

How to see it?

$$j_{\rm CME} = (\boldsymbol{E}\cdot\boldsymbol{B})\boldsymbol{B} \;\;\propto\;\; B^2$$



External

$$j_{\rm Ohm} = \sigma \boldsymbol{E}$$

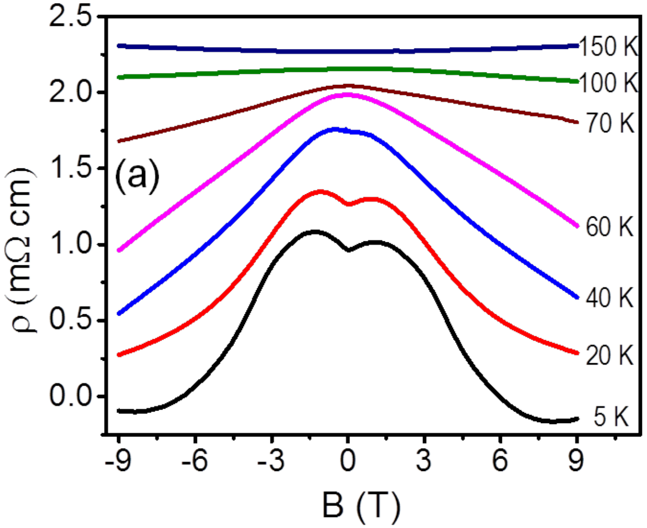
"Chiral anomaly and classical negative magnetoresistance of Weyl metals" by Son, Spivak (2013)

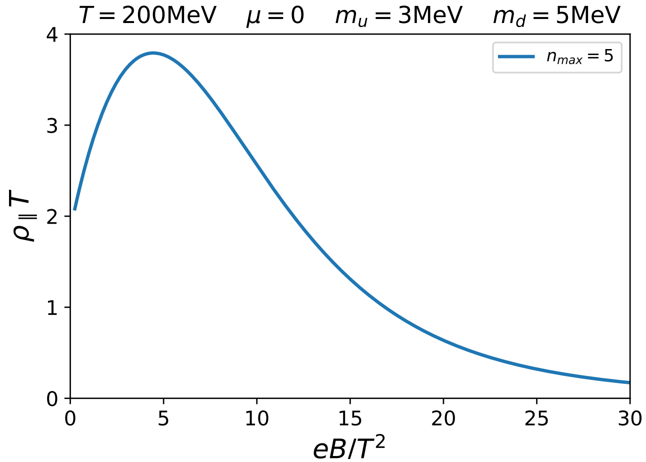
$$\sigma_{\rm CME} \propto B^2$$ **Chiral kinetic theory in the rel.-time approx.**

Chiral Magnetic Effect

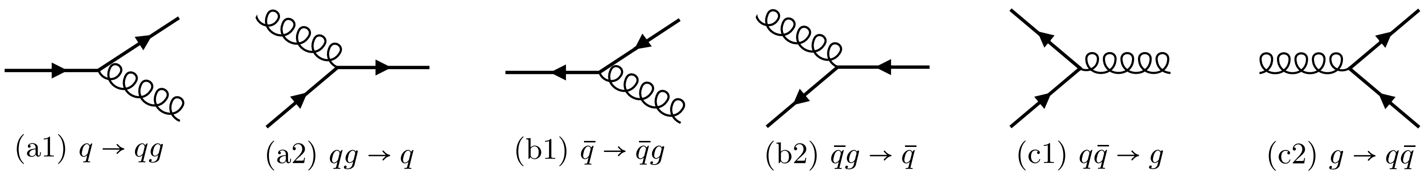
How to see it?

"*Chiral magnetic effect in ZrTe5*" by Li, Kharzeev, … (2015)





"*Electric conductivity of hot and dense quark matter in a magnetic field…*" by Fukushima, Hidaka (2017)



Chiral Kinetic Theory (CKT)

Universal scaling (nonthermal fixed point)?

"***Self-similar inverse cascade of magnetic helicity driven by the chiral anomaly***" **by Hirono, Kharzeev, Yin (2015)**

"***Scaling laws in chiral hydrodynamic turbulence***" **by N. Yamamoto (2016)**

Distribution func. not Lorentz scalar

$$\delta t = \boldsymbol{\beta} \cdot \boldsymbol{x}$$

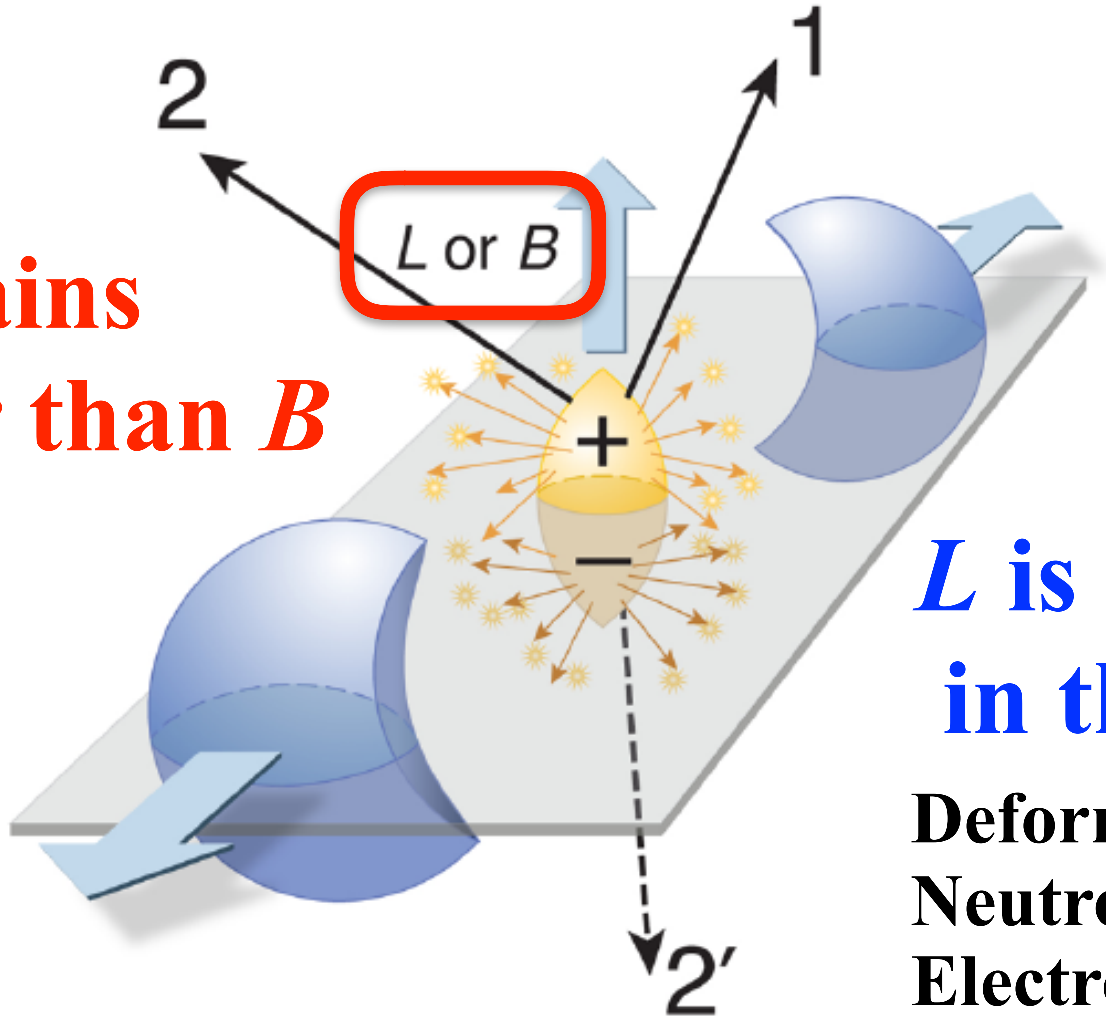
$$\delta \boldsymbol{x} = \boldsymbol{\beta} t + \boxed{\boldsymbol{\beta} \times p\boldsymbol{\Omega}}$$

$$\delta \boldsymbol{p} = \boldsymbol{\beta} \varepsilon_p + \boxed{(\boldsymbol{\beta} \times p\boldsymbol{\Omega}) \times \boldsymbol{B}}$$

"Lorentz invariance in Chiral Kinetic Theory" by Chen, Son, Stephanov, Yee, Yin (2014)

Rotation induced Phenomena

L* remains longer than *B

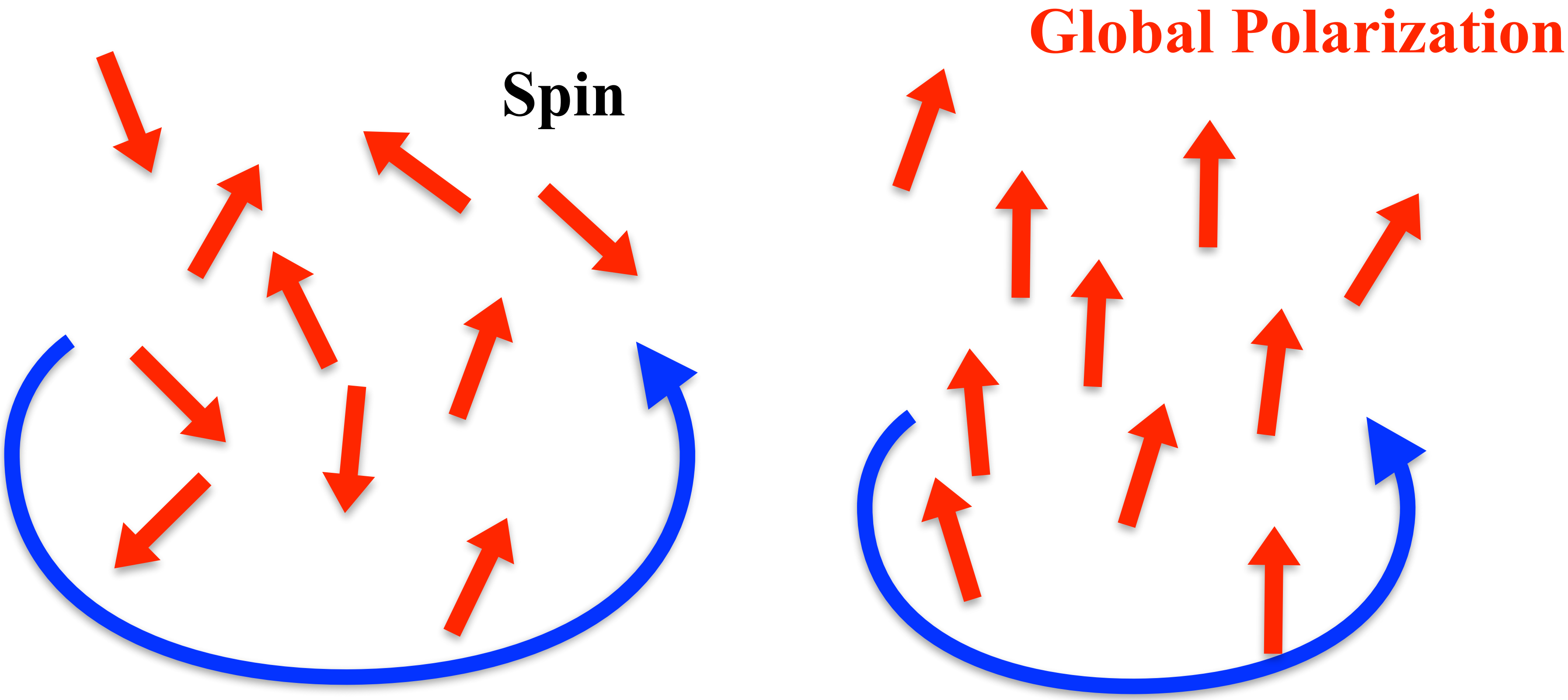


***L* is ubiquitous in the nature**

Deformed Nuclei
Neutron Stars
Electron Vortices …

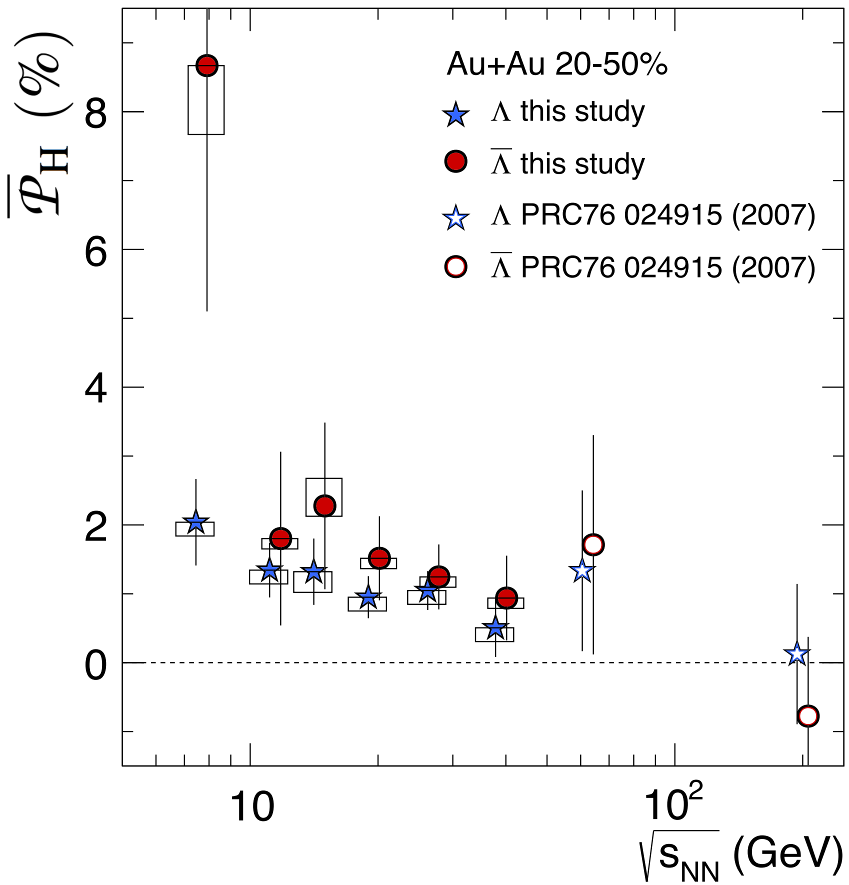
Rotation induced Phenomena

Polarization by Gyroscopic Motion



Rotation induced Phenomena

Global Polarization of Λ



$$P_{\text{Vortical}} = \frac{1}{2}\left(P_\Lambda + P_{\bar{\Lambda}}\right)$$

$$P_{\text{Magnetic}} = \frac{1}{2}\left(P_\Lambda - P_{\bar{\Lambda}}\right)$$

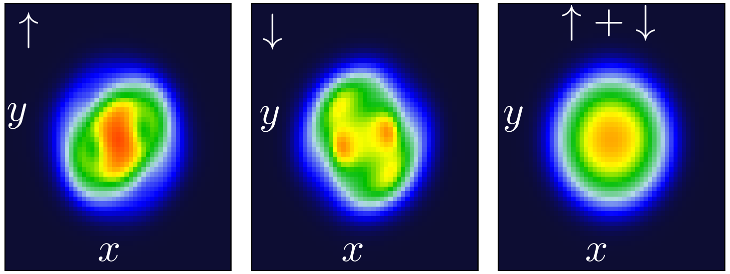
Becattini, Csernai, X.N.Wang, Q.Wang, Karpenko, …

STAR: 2016

Rotation induced Phenomena

Analogy in cold atomic systems

"Einstein-de Haas effect in a dipolar Fermi gas"
by U. Ebling, M. Ueda (2017)



Vlasov-Boltzmann simulation
Polarization → Mechanical Rotation
(*Einstein-de Haas effect*)

In heavy ion collision
Orbital angular momentum → Polarization
(*Barnett effect*)

Rotation induced Phenomena

Chiral Vortical Effect

CME $\boldsymbol{j} \sim \mu_5 \boldsymbol{B}$

CSE $\underline{\boldsymbol{j}_5} \sim \underline{\mu \boldsymbol{B}}$

Spin Polarization **Material under *B***

"Anomalous axion interactions and topological currents in dense matter"
by Metlitski, Zhitnitsky (2005)

Rotation

$\rightarrow \boldsymbol{B} \sim \mu \underline{\boldsymbol{\omega}} \rightarrow \boldsymbol{j}_5 \sim \mu^2 \boldsymbol{\omega}$

Angular Velocity

Rotation induced Phenomena

Chiral Vortical Effect

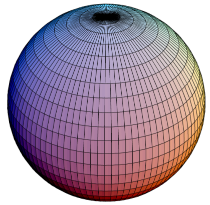
$$\boldsymbol{j}_{R/L} = \mp\boldsymbol{\omega}\int_{\boldsymbol{p}} f'_{R/L}(p)$$

$$= \pm\left(\frac{T^2}{12} + \frac{\mu^2_{R/L}}{4\pi^2} + \cdots\right)\boldsymbol{\omega}$$

"Quantum field theory at finite temperature in a rotating system"
by A. Vilenkin (1980)

Rotation induced Phenomena

Chiral Vortical Effect



$$\boldsymbol{j}_{R/L} = \pm\left(\frac{T^2}{12} + \frac{\mu_{R/L}^2}{4\pi^2} - \frac{m^2}{8\pi^2} - \frac{R}{96\pi^2} + \cdots\right)\omega$$

Scalar Curvature

Mass correction
Not anomaly protected

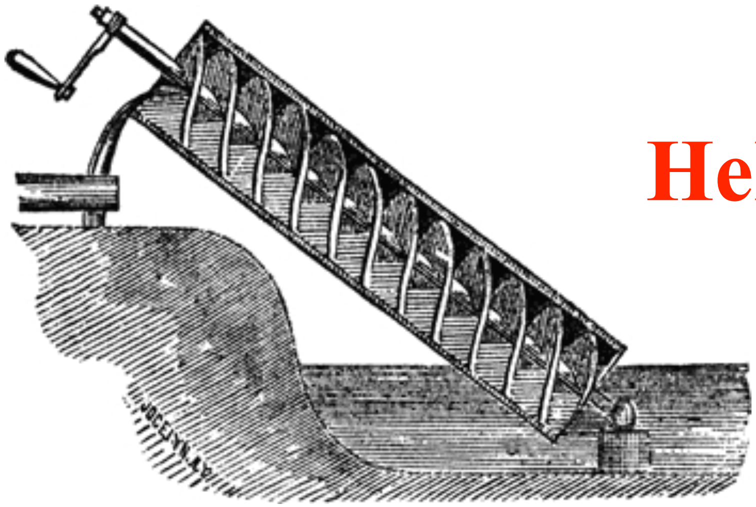
"*Chiral vortical effect with finite rotation, temperature, and curvature*" by A. Flachi, K. Fukushima (2018)

Rotation induced Phenomena

Rotation + Magnetic Field = Finite Density

$$n_V = \frac{e}{4\pi^2} \boldsymbol{B} \cdot \boldsymbol{\omega}$$

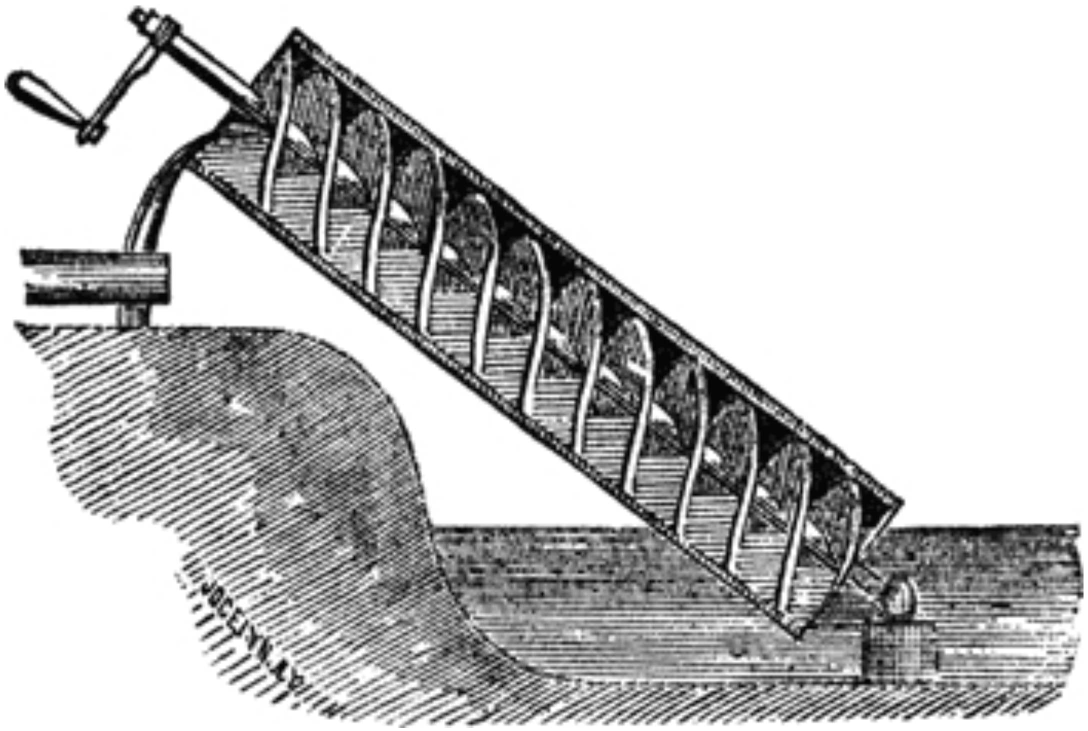
"Charge redistribution from anomalous magnetovorticity coupling"
by Hattori, Yin (2016)



Helical Structure + Rotation
→ Pumping

Rotation induced Phenomena

Where do you find a pump?



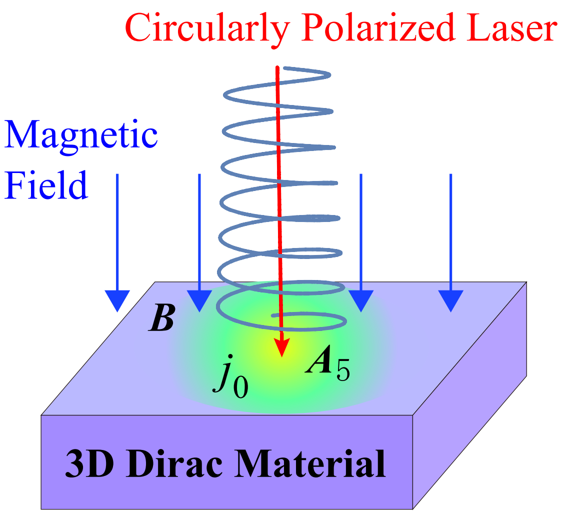
Everywhere…

Rotation induced Phenomena

Helical structure from external fields

You may think that rotating a whole system is a bit(?) difficult… don't worry!

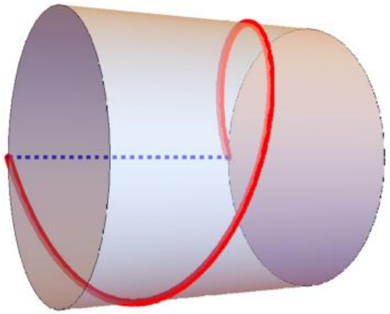
You can use circularly rotating electromagnetic fields to induce similar phenomena.



"Chiral pumping effect induced by rotating electric fields"
by Ebihara, Fukushima, Oka (2015)

Rotation induced Phenomena

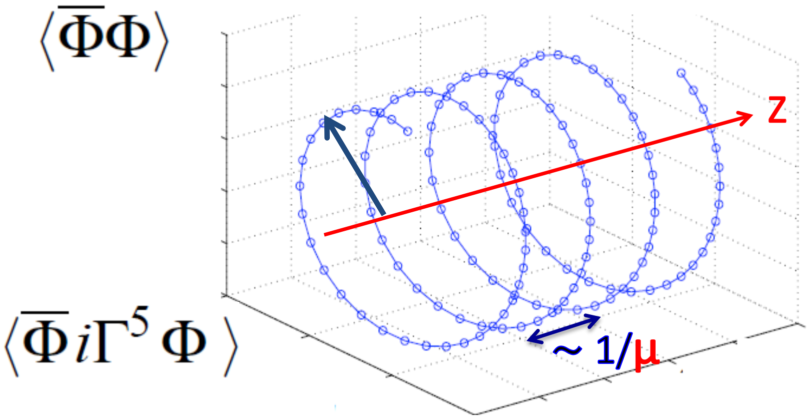
Thouless pump in Floquet Theory



The energy shifted by $2\pi/T = \omega$

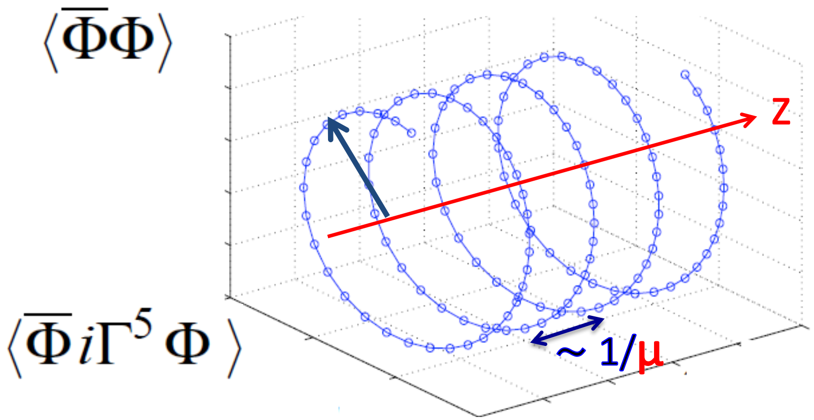
[fig. from Higashikawa]

QCD ground state at high density



Rotation induced Phenomena

Similar to QCD ground state at high density



High Density / Strong *B*
→ (1+1)D effectively

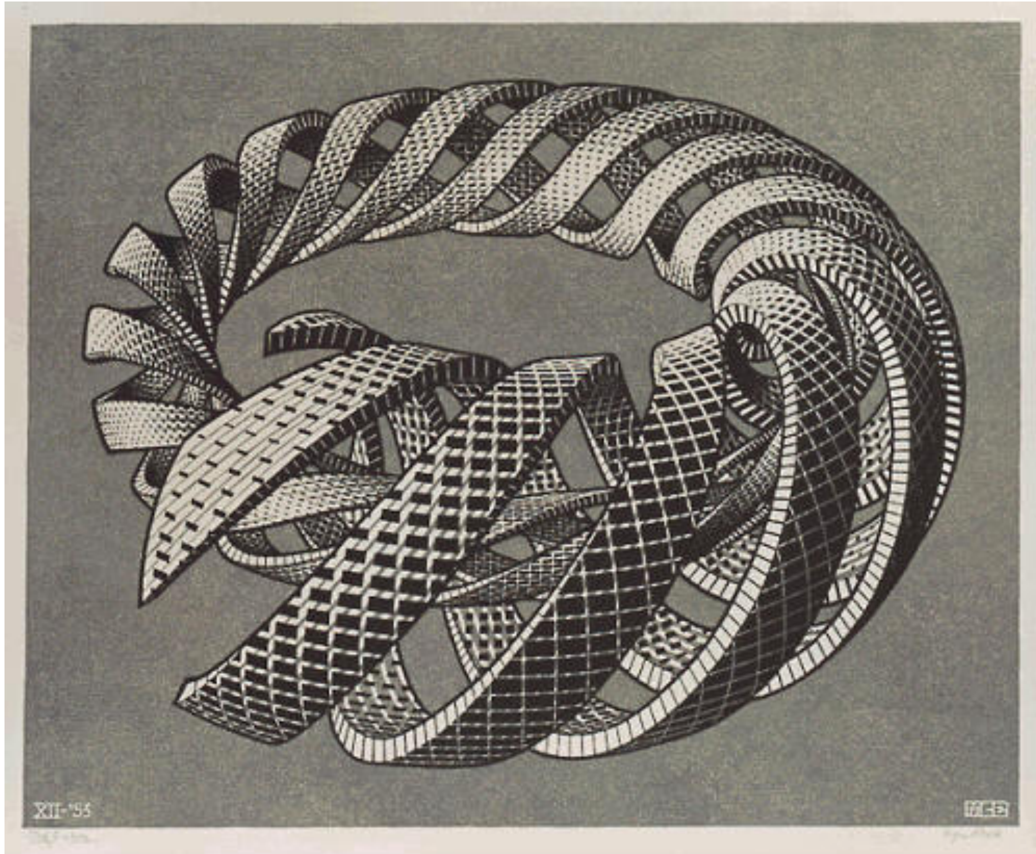
In (1+1)D the ground state has spiral condensates stabilized by the axial anomaly (chiral pump)!

"*Chiral magnetic spiral*" by Basar, Dunne, Kharzeev (2010)

Rotation induced Phenomena

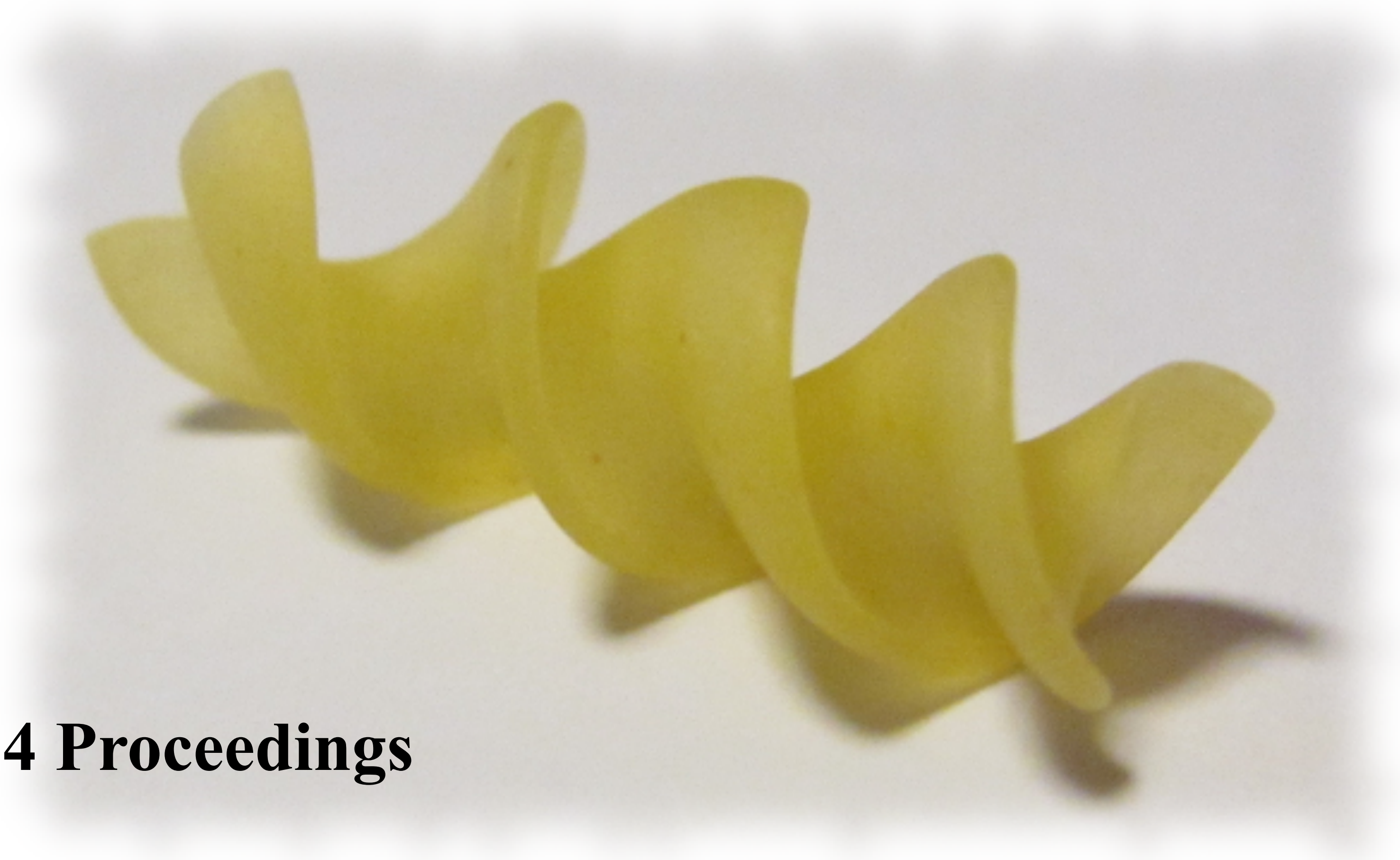
Similar to QCD ground state at high density

QM'14 Talk



Rotation induced Phenomena

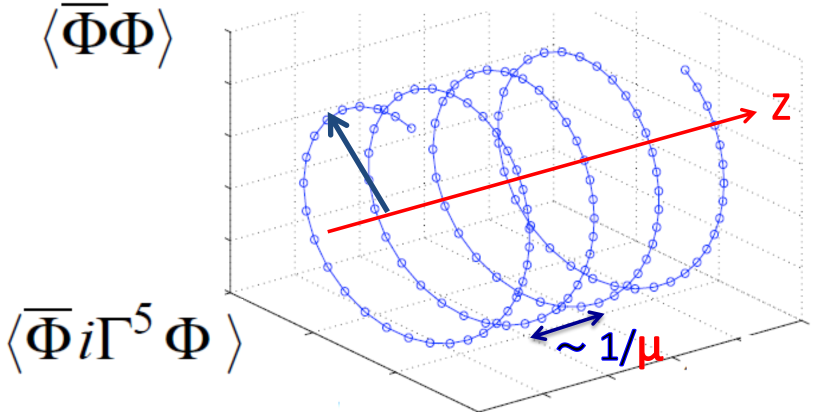
Similar to QCD ground state at high density



QM'14 Proceedings

Rotation induced Phenomena

Similar to QCD ground state at high density



High Density / Strong *B*
→ (1+1)D effectively

$$n_V = \underbrace{\frac{eB}{2\pi}}_{\text{Landau factor}} \cdot \underbrace{\frac{\mu}{\pi}}_{\text{Anomaly protected density}}$$

Landau factor **Anomaly protected density**

Rotation induced Phenomena

Simplest Floquet Example

[2-state model]

$$H(t) = B_z \sigma_z + B_{||}(\sigma_x \cos \omega t + \sigma_y \sin \omega t)$$

$$U(t_2, t_1) = V(t_2) e^{-iH_{\mathrm{rot}}(t_2 - t_1)} V^\dagger(t_1)$$

$$V(t) = e^{-i\frac{\sigma_z}{2}\omega(t-t_0)}$$

$$H_{\mathrm{rot}} = B_z \sigma_z + B_{||}(\sigma_x \cos \omega t_0 + \sigma_y \sin \omega t_0) - \underline{\frac{\sigma_z}{2}\omega}$$

$$\mu \sim \omega/2$$

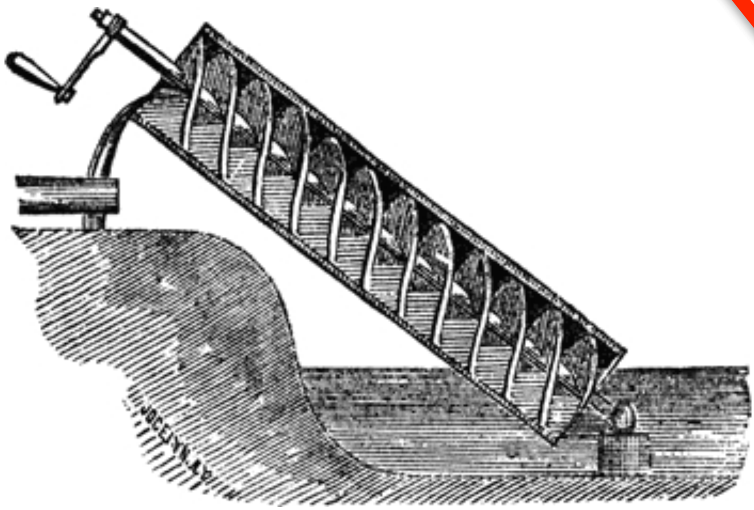
Rotation induced Phenomena

Rotation + Magnetic Field = Finite Density

$$n_V = \frac{e}{4\pi^2} \boldsymbol{B} \cdot \boldsymbol{\omega}$$

Thouless Pump

$\mu \sim \omega/2$

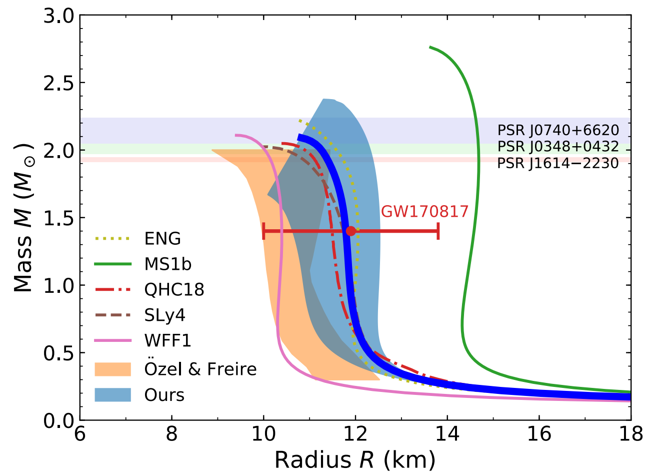


$$n_V = \frac{eB}{2\pi} \cdot \frac{\mu}{\pi}$$

The story continues more…

Anomaly plays an essential role in constructing an equation of state for neutron stars.

Nuclear matter ~ Quark matter



EFT for dense nuclear matter under *B* is identical to chiral magnets due to anomaly.

(Poster by Nishimura)

Summary

Chirality and helicity connect diff. fields!

- □ High-energy nuclear physics (HIC / spin)
- □ Weyl / Dirac semimetals
- □ Optical laser / Electron vortex beams

Magnetic field and/or Rotation

- □ Heavy ion collision has both!
  - ♦ Global (local) polarization measurements
  - ♦ Isobar experiments disentangle B effects
- □ Anomaly induces unusual phenomena
- □ Ideas importable / exportable bet. diff. fields