Lattice study of QCD phase diagram in (B, T, μ) space

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in collaboration with

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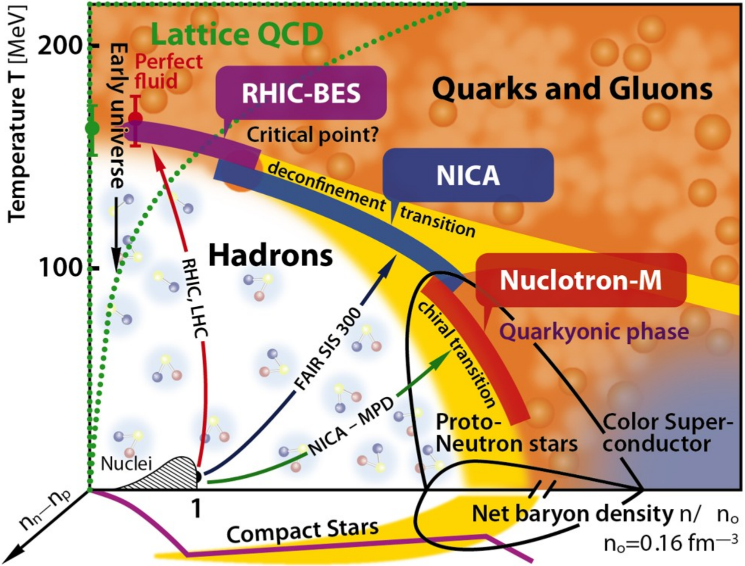
Based on arXiv:1909.09547

Quantum Systems in Extreme Conditions 2019

23-27 September 2019

QCD phase diagram

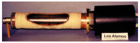


Magnetic fields in nature

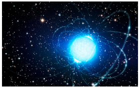
Souvenir magnet — 5×10^{-3} T

Max permanent magnet — 1.25 T

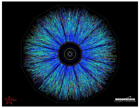
Magnetic field to levitate a frog — 16 T



Human produced pulsed magnetic field — 2.8×10^{3} T



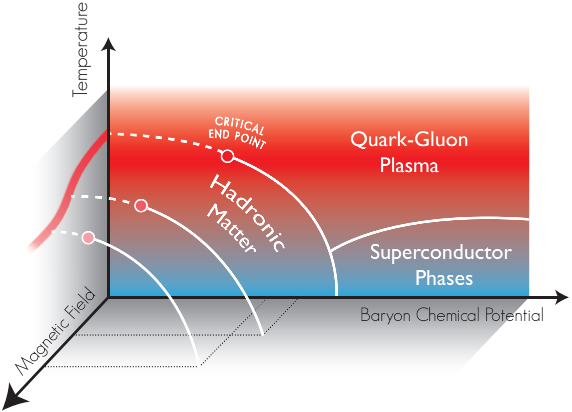
Magnetars — $10^{8} - 10^{11}$ T



Heavy ion collisions — $10^{14}\mathrm{T} \sim m_{\pi}^{2}$

Wikipedia

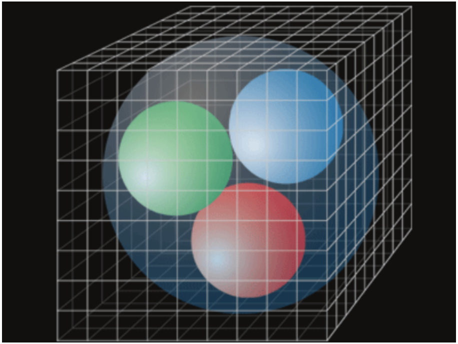
QCD and Magnetic Field



[P.Costa, M.Ferreira, C. Providência, 2018]

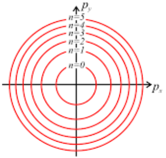
Can be studied in effective models
What can Lattice tell us about the phase diagram in μ, B, T?

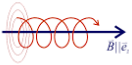
Lattice QCD



- (Euclidean) path integral: $Z = \int DA_\mu D\bar{\psi} D\psi e^{-S[\bar{\psi},\psi,A_\mu]}$
- Discretize it
- Calculate using Monte Carlo methods

Lattice QCD in $T - B$ plane

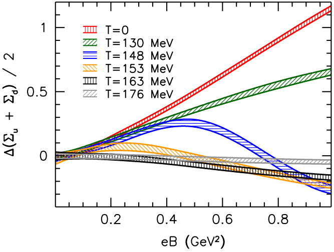




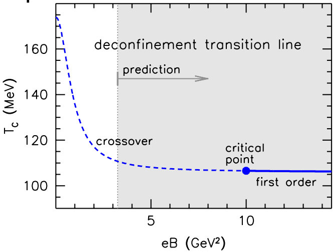
- Magnetic Catalysis

  [Gusynin V., Miransky V., Shovkovy I., 1994]

- Inverse Magnetic Catalysis
- CEP?
- Exotic phases?



[F.Bruckmann et al., 2013]



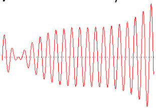
[G.Endrodi, 2015]

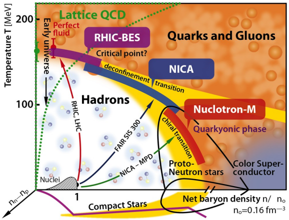
Lattice QCD in $T - \mu$ plane

Sign problem!

$$Z = \int DA_\mu D\bar{\psi} D\psi e^{-S[\bar{\psi},\psi,A_\mu]}$$

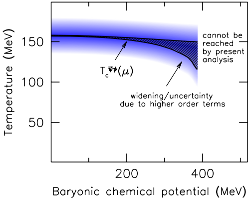
$$\mu_B \Rightarrow S \notin \mathbb{R}$$

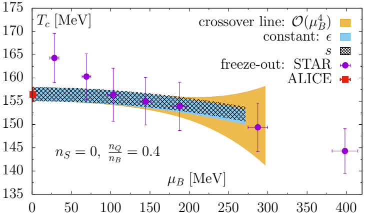




- Curvature of pseudocritical line

$$T_c(\mu_B) = T_c(0) - A_2\mu_B^2 + O(\mu_B^4)$$





[R. Bellwied et al., 2015] [HotQCD, 2018]

Numerical setup

- u, d, s quarks
- $\mu_s = 0$, $\mu_u = \mu_d = \mu_I \to i\mu_I$
- Crossover: analytical continuation
  $M(\mu_I) = A + B\mu_I^2 + \ldots \Rightarrow M(\mu_B) = A - B\mu_B^2 + \ldots$

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- $eB \in [0.1, 1.5]$ GeV^2
- $\frac{\mu_I}{\pi T} \in [0.00, 0.275]$
- Scan in $T \implies T_c, \delta T_c(\mu_B, eB)$

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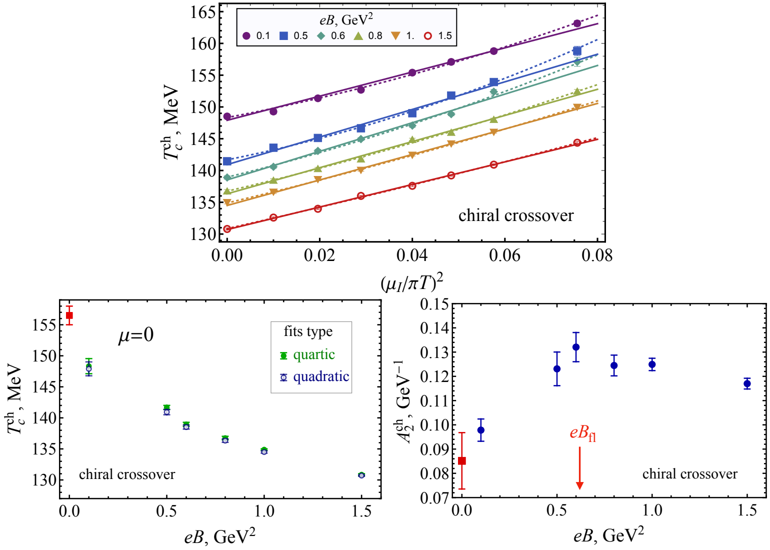
Observables:

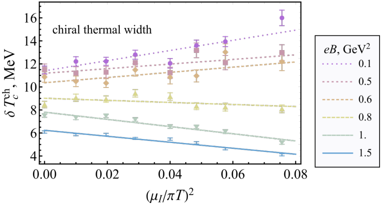
- Light quark chiral condensate $\langle\bar{\psi}\psi\rangle_I^r \Leftrightarrow$ Chiral symmetry breaking/restoration
- Polyakov loop $L^r \Leftrightarrow$ Confinement/Deconfinement

Chiral phase transition vs $\mu_I/\pi T$



$$T_c(\mu_B, B) = T_c(0, B) - A_2(B)\mu_B^2 + O(\mu_B^4)$$

Chiral thermal width



<u>CEP at $eB = 0$</u>

$(T, \mu_B) \sim$

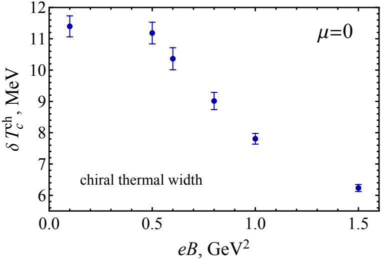
$(100(25), 800(140))$ MeV

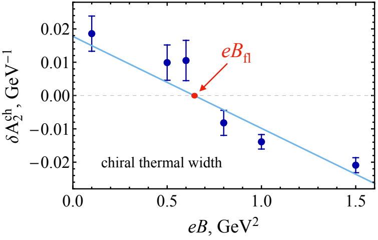
FRG: $(107, 635)$ MeV

[W. Fu, J. Pawlowski, F. Rennecke, 2019]

Holography: $(89, 724)$ MeV

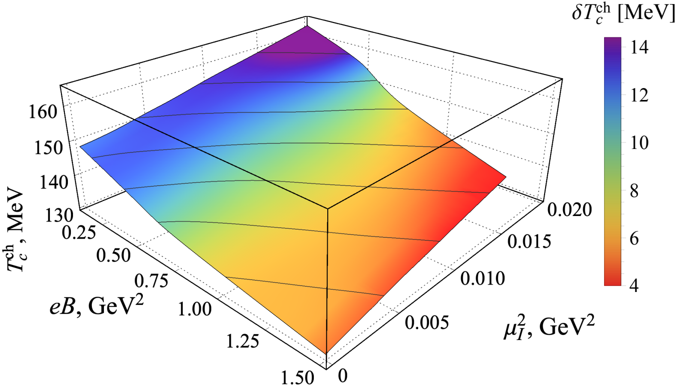
[R.Critelli et al., 2017]



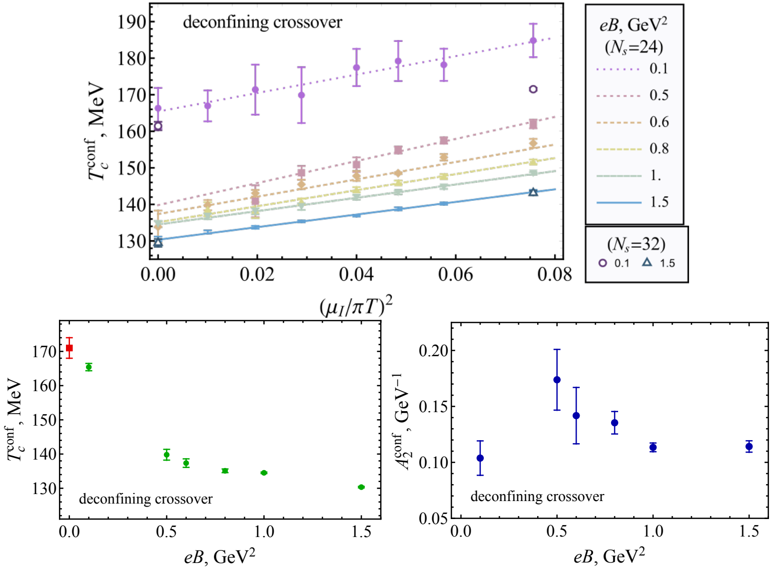


$$\delta T_c^{\rm ch}(\mu_B, B) = \delta T_c^{\rm ch}(0, B) - \delta A_2^{\rm ch}(B)\mu_B^2 + O(\mu_B^4)$$

Critical temperature and width of the chiral crossover

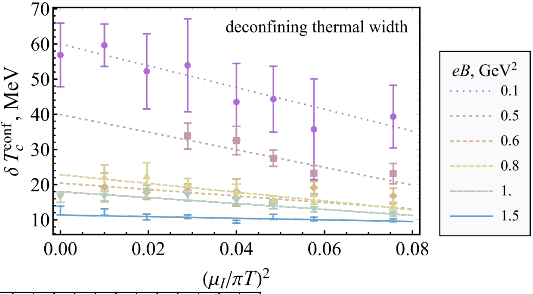


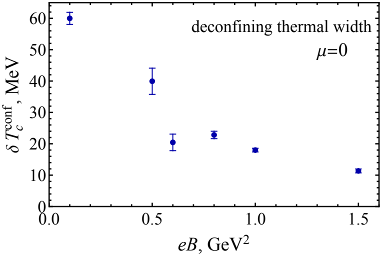
Confining crossover, critical temperature T_c

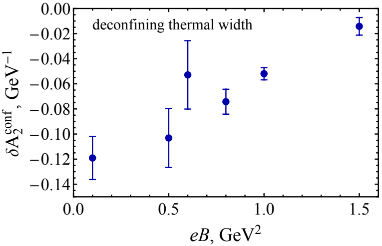


$$T_c(\mu_B, B) = T_c(0, B) - A_2(B)\mu_B^2 + O(\mu_B^4)$$

Confining crossover, width δT_c

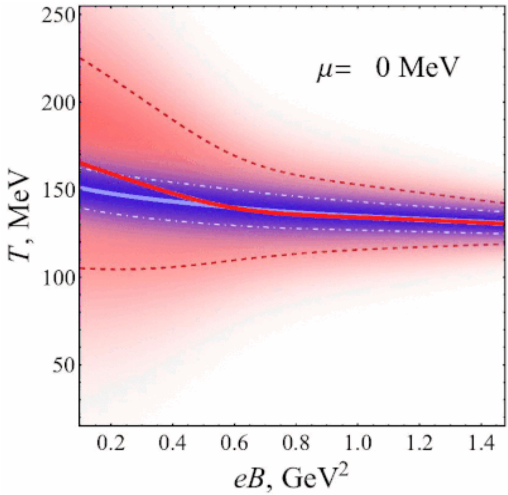






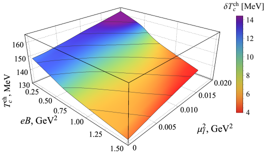
$$\delta T_c^{\rm conf}(\mu_B, B) = \delta T_c^{\rm conf}(0, B) - \delta A_2^{\rm conf}(B)\mu_B^2 + O(\mu_B^4)$$

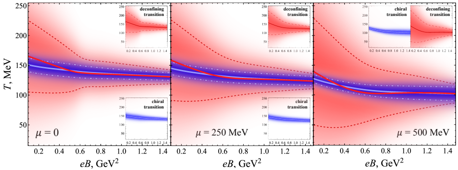
Chiral and deconfining crossover at real μ_B



Results and conclusions

- QCD phase diagram with nonzero T, eB, μ
- Simulations with imaginary $\mu_u = \mu_d = i\mu_I$, $\mu_s = 0$
- Critical temperature: mild interplay between eB and μ_B:
  - Inverse Magnetic Catalysis
  - Mild dependence of curvature A_2 on eB, peak $eB \approx 0.6\text{GeV}^2$
- Width of the transition:
  - CEP at $eB = 0$: $(T, \mu_B) \sim (100(25), 800(140))$ MeV
  - Magnetic field makes the transition sharper (chiral and deconfining transitions merge at large eB) at μ_I, small μ_B
  - Chiral thermal width (Behaviour changes at $eB_c \approx 0.6\text{GeV}^2$):
    - $eB < eB_c$: δT_c^{ch} slightly decreases with μ_B
    - $eB > eB_c$: δT_c^{ch} increases with μ_B
  - Baryonic matter always weakens the deconfining crossover





Backup

Details of lattice setup

- Lattice size: $\underline{6 \times 24^3}$, 6×32^3, 8×32^3
- Stout improved staggered $N_f = 2 + 1$ fermions
- Tree level Symanzik gauge action
- Physical masses of u, d, s quarks
- $q_u = 2/3e$, $q_d = q_s = -1/3e$
- Data for $eB = 0$ are taken from [C. Bonati et al., 2014]
- $\mu_s = 0$, $\mu_u = \mu_d = \mu_I \to i\mu_I$
- $O(100)$ configurations per each T, μ_I, eB

Obserbables and renormalization

Light quark chiral condensate:

$$\langle\bar{\psi}\psi\rangle_l = \frac{T}{V}\frac{\partial \log Z}{\partial m_l} = \langle\bar{u}u\rangle + \langle\bar{d}d\rangle$$

$$\langle\bar{\psi}\psi\rangle_r = \frac{\left[\langle\bar{\psi}\psi\rangle_l - \frac{2m_l}{m_s}\langle\bar{s}s\rangle\right](T, eB, \mu)}{\left[\langle\bar{\psi}\psi\rangle_l - \frac{2m_l}{m_s}\langle\bar{s}s\rangle\right](0,0,0)}$$

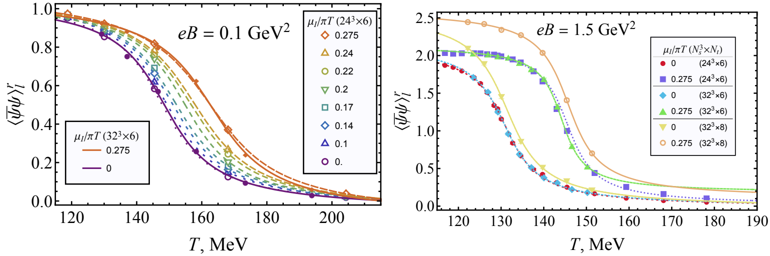
Polyakov loop ($\langle L\rangle = exp(-F_Q/T)$):

$$L(\vec{x}) = \frac{1}{3V}\,\mathrm{tr}\prod_{\tau=1}^{N_\tau} U_4(\vec{x},\tau)$$

Renormalized with GF [P.Petreczky and H.-P. Schadler, 2016]

$$L^r(\vec{x}) = L(\vec{x})[V_{t=f}(x,\mu)]$$

Chiral condensate

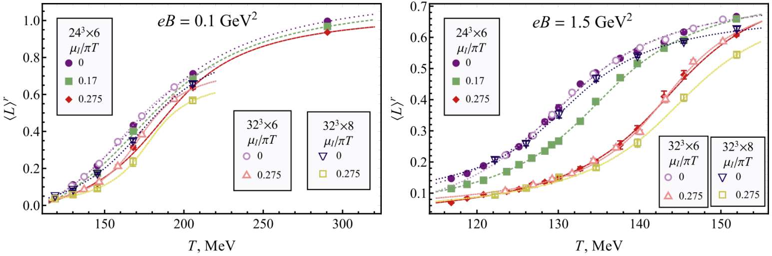


Inflection point:

$$\langle\bar{\psi}\psi\rangle_l^r(T) = A_1 + B_1 \arctan\left(\frac{T - T_c}{\delta T_c}\right)$$

- eB grows, T_c decreases
- Large eB - transition is sharper, but still a crossover

Polyakov loop



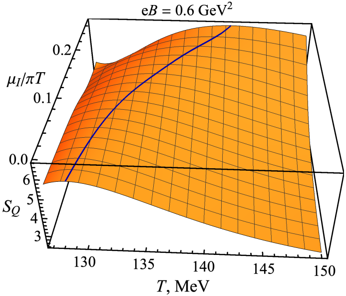
$$\langle L \rangle^r(T) = A_2 - B_2 \arctan\left(\frac{T - T_c}{\delta T_c}\right)$$

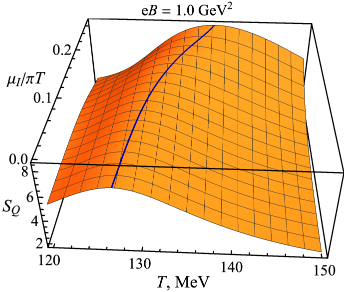
Thermodynamic properties of heavy quarks

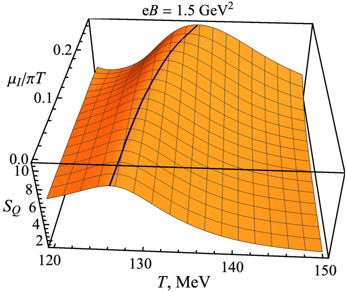
$$|\langle P \rangle| = e^{-\Omega_Q/T}$$

$$d\Omega_Q = -S_Q dT - N_Q d\mu - M_Q dB$$

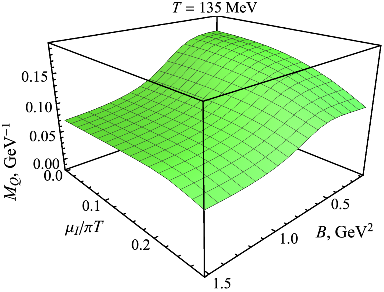
Single-quark entropy

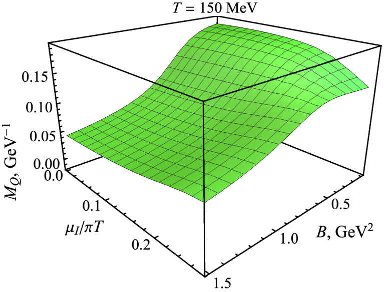


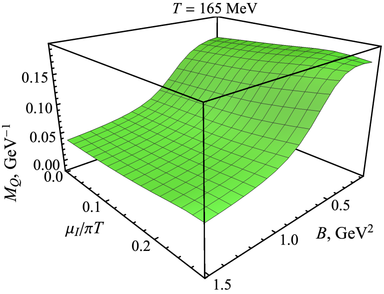




Single-quark magnetization







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