Real-time dynamics in quantum link gauge theories

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Quantum Systems in Extreme Conditions
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Outline

Introduction

Static Properties

Dynamical Properties

Outlook

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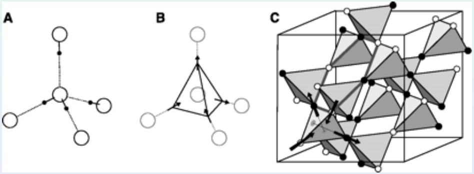
Confluence of CMT and PP

- Emergent gauge fields describe many (condensed matter) systems.
- Degenerate ground states in water-ice (H_2O) and spin-ice (pyrochlore materials, e.g. $Ho_2Ti_2O_7$) $\longrightarrow$ ice states.



- Tunneling between two ice states via ring exchange.
- Low energy spin liquid phases admit gauge theory description.

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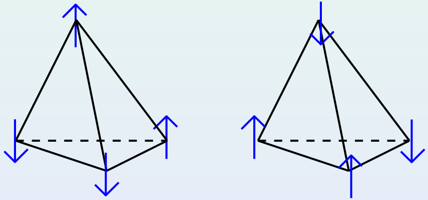

+ 4 more states

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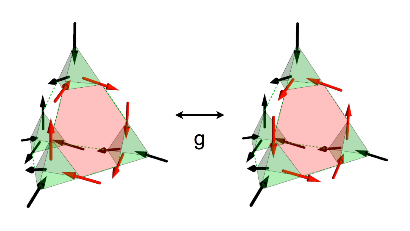

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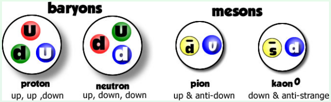
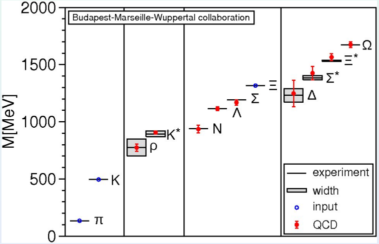
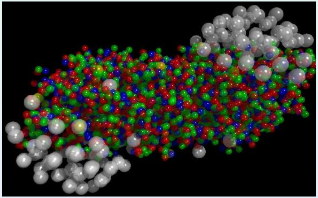
Confluence of CMT and PP

Properties of protons, neutrons and other particles (hadrons) made of quarks and gluons explained by quantum chromodynamics (QCD).





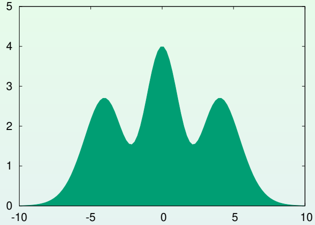
Hadron spectrum



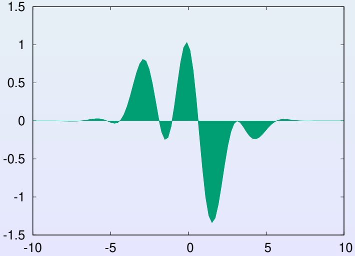
Quark Gluon Plasma

Emergent phenomena also possible here ?

Success stories of classical computers

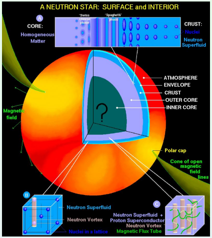


Monte Carlo methods on fast, reliable supercomputers.

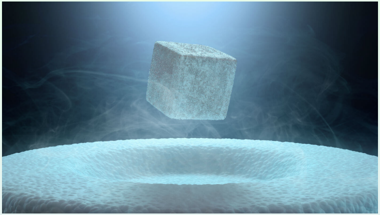


Importance sampling breaks down for rapidly oscillating integrands.

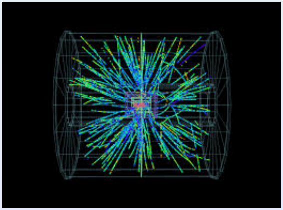
Classical nightmares for sign failures



Neutron Star



Superconductivity



Heavy-Ion Collisions

Hinders first principle studies of finite density systems, non-equilibrium phenomena.

Possible to use tensor networks, quantum simulators, effective theories?

What are Quantum Links?

- Adapt the existing (Wilson) formulation for new methods.
- Microscopic description need not be identical to produce the same low-energy (IR) physics: Quantum Links.
- Generalized lattice gauge theories:
  - Horn (1981); Orland, Rohrlich (1990);
  - Chandrasekharan, Wiese (1997); + Brower (1999)
  - Rokhsar, Kivelson (1988); Moessner, Sondhi, Fradkin (2002)
- In this talk: Abelian pure gauge theory.
- Gauge fields ($\mathrm{U}, \mathrm{U}^\dagger$) and electric fields ($\mathrm{E}$) act on finite dimensional Hilbert space of a quantum spin S_{xy}.
- $\mathrm{U} = \mathrm{S}^+$; $\mathrm{U}^\dagger = \mathrm{S}^-$; $\mathrm{E} = \mathrm{S}^z$ and satisfy

$$[\mathrm{E}_{xy}, \mathrm{U}_{xy}] = \mathrm{U}_{xy}; \quad [\mathrm{E}_{xy}, \mathrm{U}^\dagger_{xy}] = -\mathrm{U}^\dagger_{xy}; \quad [\mathrm{U}_{xy}, \mathrm{U}^\dagger_{xy}] = 2\mathrm{E}_{xy}$$

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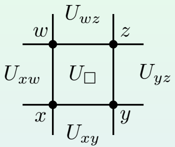
Quantum Links in $(2+1)$-d

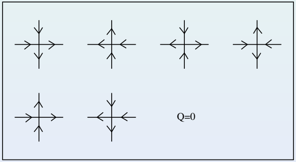
- Spin $S = \frac{1}{2}$ links on a square lattice; relevant for spin ice.

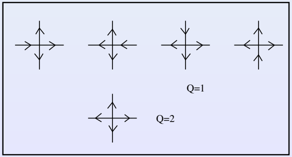
$$\mathrm{H} = -J \sum_{\square} \left(U_{\square} + U_{\square}^{\dagger} \right) + \lambda \sum_{\square} \left(U_{\square} + U_{\square}^{\dagger} \right)^2 ;$$

$$G_x = \sum_i \left(\mathrm{E}_{x,x+i} - \mathrm{E}_{x-i,x} \right)$$



$U_{\square} = U_{xy} U_{yz} U_{wz}^{\dagger} U_{xw}^{\dagger}$



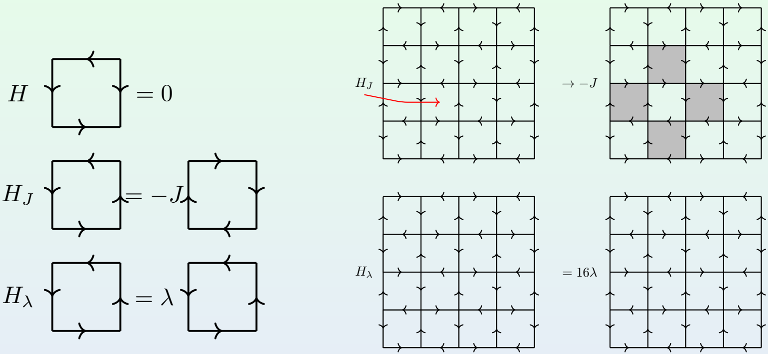


- E_{xy}^2 is a constant for $S = \frac{1}{2}$: drops in H, but enters via G_x.

$$[G_x, H] = 0; \; V = \prod_x \mathrm{e}^{-i\theta_x G_x}$$

$$\tilde{\mathrm{H}} = V^{\dagger} \mathrm{H} \, V = H$$

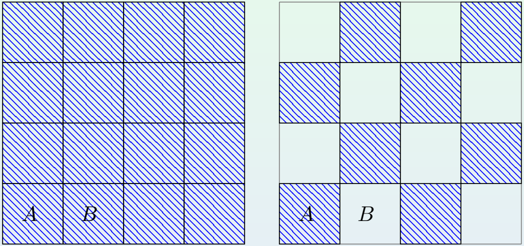
Competing Orders



- $Z = \mathrm{Tr}\ \left[e^{-\beta H}\mathbb{P}_{\mathbb{G}}\right]$; $\mathbb{G} = \prod_x \delta(G_x)$
- Symmetry breaking patterns change as a function of $\frac{\lambda}{J}$. Exact Diagonalization calculations upto 96 links.

Crystalline Confined Matter

Nature of the phase transition studied with a cluster QMC algorithm. Banerjee, Jiang, Widmer, Wiese (2013).

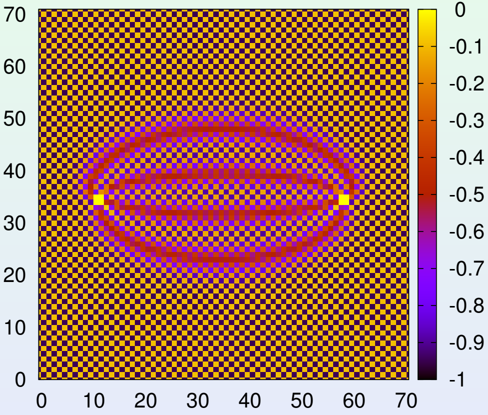


Flippability for $\lambda < \lambda_c$ $(\pm 1, \pm 1)$, $\lambda > \lambda_c$ $(\pm 1, 0)$.

- weak first order phase transition separates the two new phases of crystalline confined quantum matter.
- Spontaneously broken translation and charge conjugation.
- At λ_c, an approximate global SO(2) symmetry is emergent.

Crystalline Confined Matter

Energy density of static $Q = \pm 2$ charges.



The flux strands carry fractional $Q = \frac{1}{2}$, which can be identified with domain walls using effective field theory methods.

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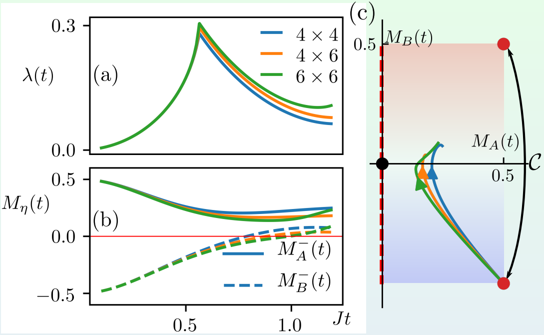
Dynamical Quantum Phase Transitions



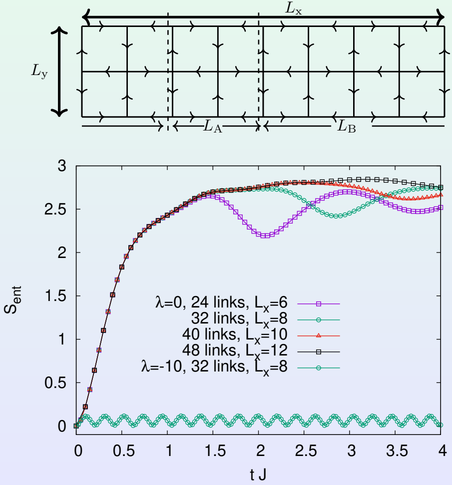
- Displays non-analytic behavior of the Loschmidt Echo:

$$\mathcal{G}(t) = \langle \psi_0 | e^{-iHt} | \psi_0 \rangle; \quad \lambda(t) = -\frac{1}{N} \log |\mathcal{G}(t)|^2$$

- Quench via by the Hamiltonian with $\lambda = 0$.
  Huang, Banerjee, Heyl (PRL, 2019).
- DQPT in Schwinger model studied in Zache, Mueller, Schneider, Jendrzejewski, Berges, Hauke (PRL, 2019).

Thermalization or not

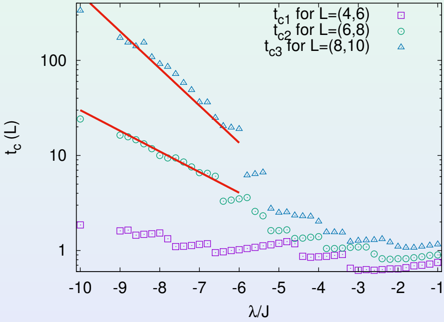
- Entanglement Entropy: $\rho_A = \mathrm{Tr}_B[\rho_{AB}]$; $S_A = -\mathrm{Tr}[\rho_A \log \rho_A]$.



- Theoretical expectations (Calabrese,Cardy) indicate a linear growth before saturation.
- Our results indicate the existence of different dynamical regimes. Banerjee, Huang, Heyl, Sen (in preparation)

Thermalization or not

- Entanglement Entropy: $S_A = -\mathrm{Tr}[\rho_A \log \rho_A]$; $\rho_A = \mathrm{Tr}_B[\rho_{AB}]$.



- Calculate the smallest $t_c(L_1, L_2)$ for which $|S_E(L_1, t) - S_E(L_2, t)| = \Delta S = 0.05$.
- Our results indicate the existence of different dynamical regimes.

  Banerjee, Huang, Heyl, Sen (in preparation)

Thermalization aspects

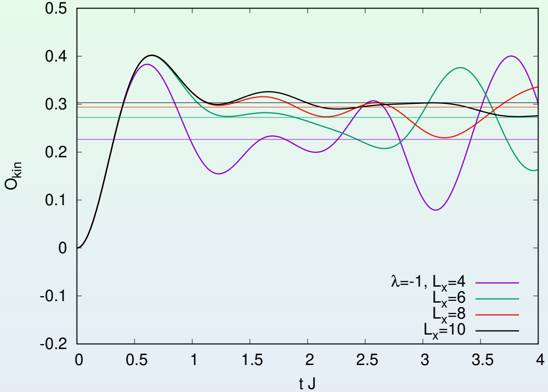
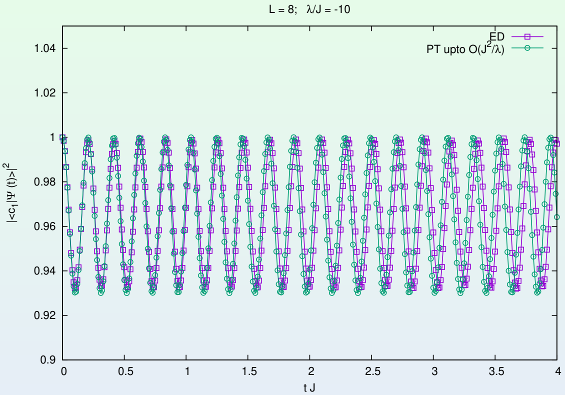


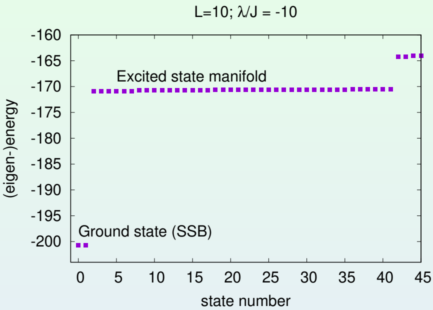
Figure: $\mathcal{O}_{\mathrm{kin}} = 1/V \sum_{\Box}(U_{\Box} + U_{\Box}^{\dagger})$.
$\langle \mathcal{O}_{\mathrm{kin}} \rangle (t) = \sum_{k,l,m,n} \langle \Phi | \Psi_m \rangle \Psi^{\star}_{m,k} \mathcal{O}_{\mathrm{kin}}(k,l) \Psi_{n,l} \langle \Psi_n | \Phi \rangle \mathrm{e}^{i(E_m - E_n)t}$
Fast approach of local observables to diagonal ensemble results ($E_m = E_n$).

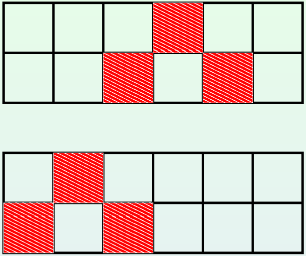
Thermalization aspects



- Time-dependent PT describes the system well for $\lambda \to -\infty$.
- $\lambda/J = -10$, dominant frequency matches to 1% for $L = 8$.
- Exponentially long time is required for tunneling to the symmetry broken partner.

Thermalization aspects





$+(4L-2)$ others.

- $\Delta E = \langle \Phi | E | \Phi \rangle (L) - E_0(L) = \alpha L = \frac{cL}{\lambda}$,
  E_0 is the ground state energy of the $L \times 2$ system.
- When $\Delta E \simeq 3\lambda$, PT breaks down and the system thermalizes;
  Prediction: $L^\star = \frac{3\lambda^2}{c}$.
- For small lattices $c \approx 0.665$ and $L^\star \sim 450$ for $\lambda/J = -10$.
  Tensor network approaches?

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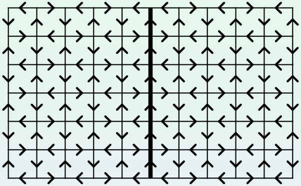
- Extend current studies to interface dynamics, string excitations in pure gauge theories.



- Quantum Link Models: fresh viewpoint for many problems in strongly interacting theories.
- Experiments with quantum simulators and quantum computers.

Thanks to my collaborators, the DFG, and the sources of the pictures which are not made by me.

THANK YOU!